三次元座標計測(第2回)
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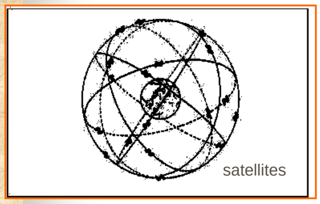
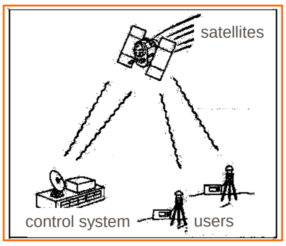
Surveying by GPS: What is “Traceability”?

Surveying by GPS

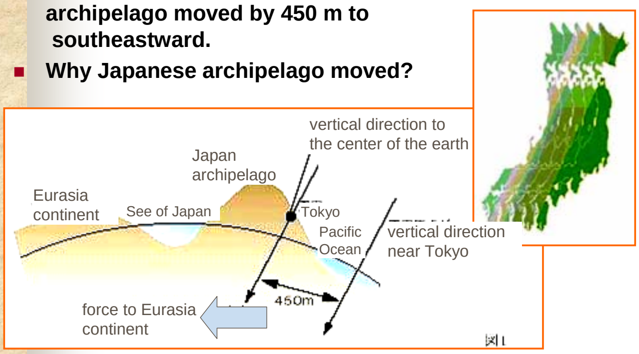
- Three dimensional position and time at any place in the earth can be surveyed by GPS(Global Positioning System) using artificial earth satellites.





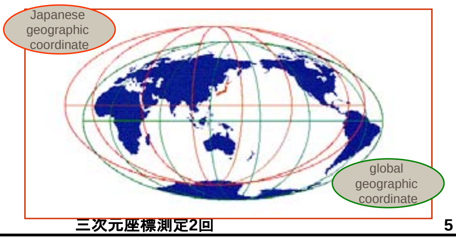
Movement of Japanese Archipelago ?

- By the positional measurement by GPS, Japanese archipelago moved by 450 m to southeastward.
- Why Japanese archipelago moved?



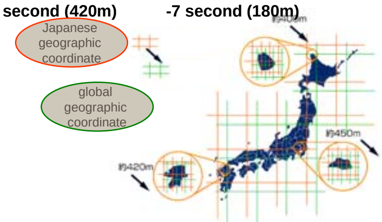
Global Geographic Coordinate System

- Longitude and latitude are the scale to measure the earth
  - The absolute scale in Japan is deferent from the global scale.
  - The high accuracy of scale does not mean the absolute correctness of the scale.
- Japanese geographic coordinate system changed to the global geographic coordinate system on 1st April 2002.



Differences of geographic coordinate systems

- Differences between global and Japanese geographic coordinate systems
- Latitude differences longitude differences
  - Wakkanai +8 second (240 m) -14 second (350m)
  - Tokyo +12 second (360 m) -12 second (300m)
  - Fukuoka +12 second (360m) -8 second (200m)
  - Naha +14 second (420m) -7 second (180m)



Japanese Surveying History

- Mr. Tadataka Inoh (1745-1818)
  - Best accurate surveying for its time



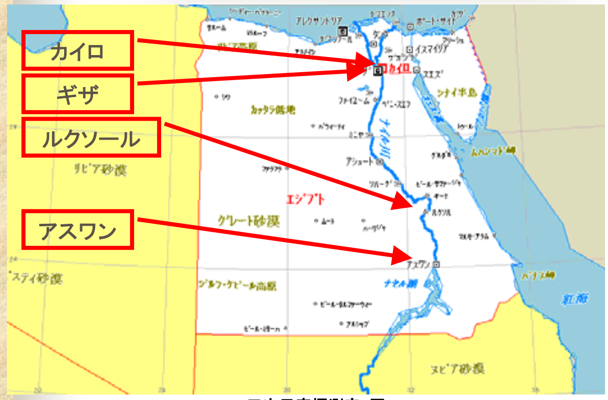
Conclusion in Surveying

- High accuracy and high repeatability can be performed without comparisons and/or standards.
- However, absolute accuracy can not be performed without comparisons and/or standards.
- It is deference between
  - high accuracy and
  - absolute accuracy.
- For the absolute accuracy, we need standards and global comparisons.
- Concept of “Traceability”

計測の歴史

エジプト
SI単位

エジプト



清野先生 in Egypt

ISMTII2001

- International Symposium on Measurement Technology and Intelligent Instruments

クフ王のピラミッド2

NIS 1

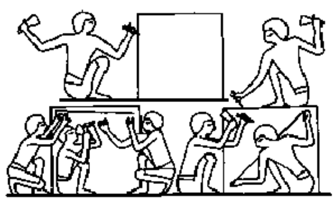
- The National measurement system of Egypt instituted 1500 years B.C. and re-instituted in modern times in the first decade of the twentieth-century took its formal modern shape when the National Institute for Standards was established in 1963.

NIS 2

カイロ博物館

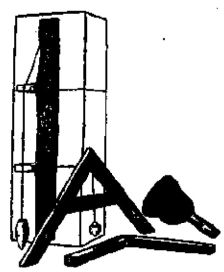
はじめに糸と水面があった

- BC1450年：水準器，糸を使っている.



エジプトのテーベの墳墓の壁画に見られる石工の加工風景 紀元前1450年頃

図1.1　テーベの墳墓の壁画[1)]

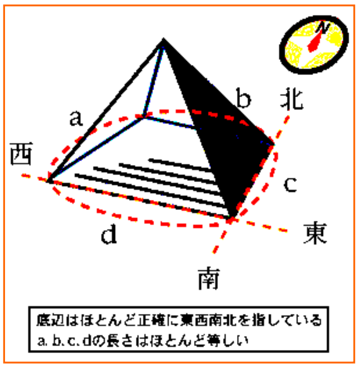


テーベ出土の曲尺，水準器と下げ振り定規

図1.2　テーベ出土の測定具[1)]

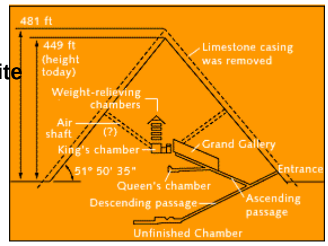
ピラミッドの正確さ

- 各辺の長さは数十センチメートルしか違わない.
- 平面性は，良質の糸を使うと，0.1 mm 程度の誤差を検出できる.



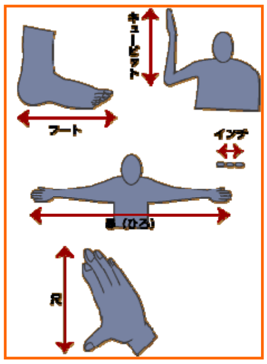
クフ王のピラミッド1

- 2589-2566 B.C.
- Total Blocks: over 2,300,000
- Base 227 m
- Average Weight: 2.5 tons
- Height: 137 m.
- Angle of Incline: 51°50' 35"
- Material: limestone, granite



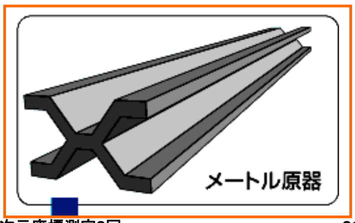
王様が基準

- 手足を基準として使う.
- ヤード:イギリス王ヘンリー1世の鼻から親指の先まで
- キュービット×2
  = ダブルキュービット
  = 1 m



王様から地球へ

- 1790:ダンケルクからバルセロナまで測量，子午線の1/4の1000万分の1を 1 m
- 1875:メートル条約，メートル原器(Pt 90%, Ir 10%)
- この形はフランスの“トレスカ”が考案した.
  曲げに強く，温度変化による伸び縮みも極めて少ない.



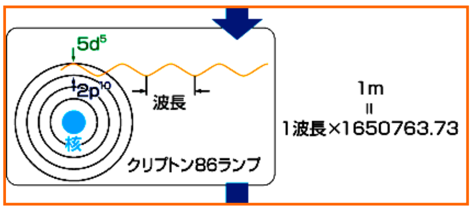
- ダンケルク:北緯51度
- バルセロナ:北緯41度

メートル条約の広がり

- 歴史的な背景
  - 18世紀末～:産業革命，工業化
  - フランス革命:神，王も死ぬ
  - 1793～1795:ルイ17世
  - 1793:ルイ16世，マリーアントワネットがギロチンで処刑
- メートル条約
  - 1875:フランス，ドイツ，イタリアなど19カ国
  - 1885:日本
  - 1960:SI単位系(日本は1993)

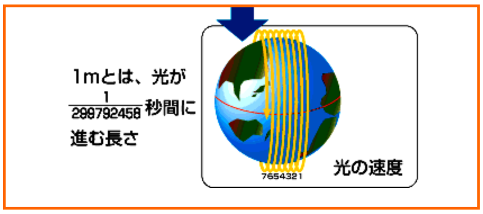
光の波長を基準としたメートル

- 1961:メートルはクリプトン86(86Kr)の原子の準位 2p10 と 5ds との間の遷移に対応する光の真空のもとにおける波長の1650763.73 倍に等しい長さ.
- 光の波長が基準．量子化された.



光の速度を定数としたメートル

- 1983：メートルは，1秒の299792458分の1の時間に，光が真空中を伝わる工程の長さである．
- 光の速度を定数．時間から長さを求める．



Traceability of Measurements

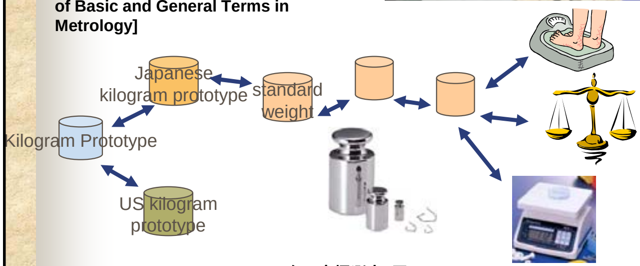
- What is “Traceability” ?
- Traceability of length

Japanese 1 kg and US 1 kg

- How to compare Japanese and US 1 kg ?
- The definition of mass is kilogram prototype in Paris.

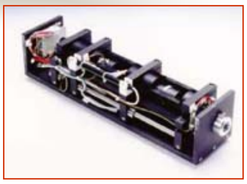
Traceability

- Property of the result of a measurement or the value of a standard whereby it can be related to stated references, usually national or international standards, through an unbroken chain of comparisons all having stated uncertainties. [International Vocabulary of Basic and General Terms in Metrology]



Traceability of Length

- Definition of Metre (1983, BIPM)
  - The metre is the length of the path traveled by light in vacuum during a time interval of 1/299 792 458 of a second
- Iodine stabilized laser 10^{-10} - 10^{-11}
- Laser interferometer 10^{-8} - 10^{-9}
- Gauge block
- Standard scale
- Rule



Iodine stabilized laser

HP (Hewlet Packard) laser interferometer

- BIPM: International Bureau of Weights and Measures

Gauge blocks and standard scales

- Size deviation of 100 mm gauge blocks
  - K class: 0.07 µm, 2nd class: 0.35 µm
- Standard scales
  - measuring range: 1000 mm, scale intervals: 1 mm, resolution: 1 µm
  - measuring range: 100 mm, scale intervals: 0.5 mm, resolution: 0.1 µm

Conclusion in Traceability

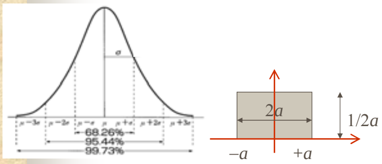
- Traceability can be established by an unbroken chain of comparisons all having stated uncertainties.
- Global comparisons are key to establish traceability.
- Then, we have to estimate uncertainties.

Uncertainty of measurement

- Uncertainty of measurement
- Estimation method of uncertainty
- Uncertainty of length measurement

Uncertainty of Measurement

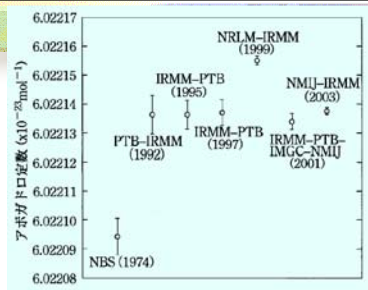
- Uncertainty of measurement *U*: parameter, associated with the result of a measurement, that characterizes the dispersion of the values that could reasonably be attributed to the measurand.
- The expanded uncertainty *U* is assumed to provide a high level of coverage (95 %) for the unknown true value of the measurement result *Y*
  - $Y - U \leq TrueValue \leq Y + U$ (with probability 95 %)
  - Gaussian distribution: 95 % by standard deviation
  - Rectangular distribution: standard deviation is $a/\sqrt{3}$



$$s^2 = \int_{-a}^{+a} \frac{1}{2a}(x-0)^2 dx = \frac{1}{2a}\int_{-a}^{+a} x^2 dx$$
$$= \frac{1}{2a}\left[\frac{x^3}{3}\right]_{-a}^{+a} = \frac{a^2}{3}$$
$$s = \frac{a}{\sqrt{3}}$$

True Value

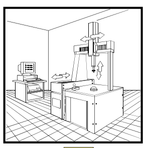
- Theoretically we can not use "True Value". Because no one knows "True Value".
- Practically we can estimate true value by higher precise measurement.

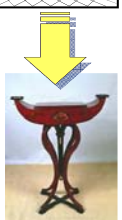


- Physical constants: Avogadro constant N_A mol^{-1}
  - 1973 6.022045 ± 0.00003 x 10^{23}
  - 1986 6.0221367 ± 0.0000035 x 10^{23}
  - 1998 6.02214199 ± 0.0000048 x 10^{23}
  - 2003 6.02214150 ± 0.0000010 x 10^{23}

Without Uncertainty:

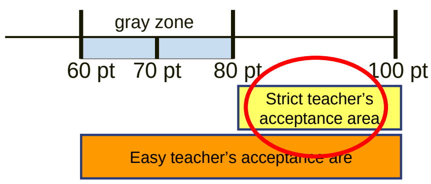
- Without uncertainty:
  - measuring result: is 10.256 mm without uncertainty and no knowledge of measuring method, measuring instrument ...
  - 10.256 mm ±0.03 mm or
  - 10.256 mm ±0.2 mm or
  - 10.256 mm ±5 mm or ...
- Measuring instrument without uncertainty is only a furniture. (Dr. Sartori, IMGC)





Decision with Uncertainty

- Examination results
  - uncertainty of rating of exam. : ±10 points
  - pass limit: over 70 points
- Decision of passing status
  - Student who gets 70 points is 50% should be rejected.
  - Clarify where responsibility lie !



Estimation Method of Uncertainty

- Describe the measuring and calibration method.
- Mathematical model
  - describes mathematical equations, lists all components, uses design of experiments method and correct measured values
- Evaluation of components of uncertainty
  - Type A. those which are evaluated by statistical methods (experimental methods),
  - Type B. those which are evaluated by other means (certification sheet, standards, other experimental knowledge)
- Combined uncertainty
  - u_c = Root Sum Square
- Expand uncertainty
  - $U = k\,u_c$（normally k = 2）

Example 1: Jug of Beer (1)

- Measure Jug of Beer
  - Volume of jug of beer is measured by graduated cylinder
- Using Budget Sheet
  - Repeatability
  - Graduated cylinder
  - Temperature

Example 1: Jug of Beer (2)

- Budget Sheet

Symbol	Component	Value ±	Evaluation type	Distribution	Divisor	Standard U.	Sensitivity coefficient	Standard U. (measured)
u_R	repeatability		A measure					
u_s	graduated cylinder		B certification					
u_T	temperature		B theory					
u_c	combined standard U.							
U	expand U.							

Example 1: Jug of Beer (3)

- u_R: repeatability of measurement
  - 10 times measurements by graduated cylinder
  - Average: 633.5 mL
  - Standard deviation: 3.598 mL
  - Distribution: Gaussian distribution
  - Divisor: 1

tim	1	2	3	4	5	6	7	8	9	10
mL	632	629	639	635	627	636	633	637	634	644

Symbol	Component	Value ±	Evaluation type	Distribution	Divisor	Standard U.	Sensitivity coefficient	Standard U. (measured)
u_R	repeatability	3.598 mL	A measure	Gaussian	1	3.598 mL	1	3.598 mL

Example 1: Jug of Beer (4)

- u_s: U. of graduated cylinder
  - by Certification sheet of graduated cylinder
  - Expand uncertainty: 3.0 mL (k = 2)
  - Distribution: Gaussian distribution
  - Divisor: 2

Symbol	Component	Value ±	Evaluation type	Distribution	Divisor	Standard U.	Sensitivity coefficient	Standard U. (measured)
u_s	graduated cylinder	3.0 mL	B certification	Gaussian	2	1.5 mL	1	1.5 mL

Example 2: Jug of Beer (5)

- u_T: Temperature
  - Resolution of digital thermometer: 1 deg (degree of Celsius)
    - Value: ±0.5 deg
    - Rectangular probability distribution: divisor $\sqrt{3}$
    - Standard uncertainty: 0.5 / $\sqrt{3}$ = 0.299 deg
  - Sensitivity coefficient
    - Coefficient of volume expansion: 5.23 x 10^{-3} (deg^{-1})
    - Volume: 633.5 mL
    - Sensitivity coefficient: 5.23 x 10^{-3} x 633.5 mL = 3.313 (mL/deg)
  - Standard U. (measured)

Symbol	Component	Value ±	Evaluation type	Distribution	Divisor	Standard U.	Sensitivity coefficient	Standard U. (measured)
u_T	temperature	0.5 deg	B theory	rectangular	$\sqrt{3}$	0.299 deg	3.313 mL/deg	0.956 mL

Example 2: Jug of Beer (6)

- u_c: Combined standard uncertainty
  - Root sum square of standard uncertainties
- U: Expand uncertainty (k = 2)
  - 2 x u_c
- Result: 633.5 mL ± 8.02 mL (k = 2)

Symbol	Component	Value ±	Evaluation type	Distribution	Divisor	Standard U.	Sensitivity coefficient	Standard U. (measured)
u_R	repeatability	3.598 mL	A measure	Gaussian	1	3.598 mL	1	3.598 mL
u_s	graduated cylinder	3.0 mL	B certification	Gaussian	2	1.5 mL	1	1.5 mL
u_T	temperature	0.5 deg	B theory	rectangular	$\sqrt{3}$	0.299 deg	3.313 mL/deg	0.956 mL
u_c	combined standard U.			Gaussian				4.01 mL
U	expand U.			Gaussian				8.02 mL

Root Sum Square (RSS)

- RSS

$$RSS = \sqrt{u_1^2 + u_2^2 + \cdots u_n^2}$$

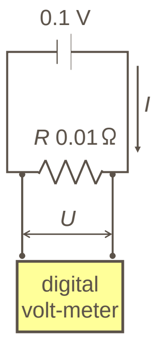
$$RSS = \sqrt{u_1^2 + u_2^2}$$

u_1	u_2	RSS
1	1	1.414
1	0.9	1.345
1	0.8	1.281
1	0.7	1.221
1	0.6	1.166
1	0.5	1.112
1	0.4	1.077
1	0.3	1.044
1	0.2	1.020
1	0.1	1.005

- Magic of RSS
  - No effect (less than 10%) by components less than 50% of the biggest component
  - We can estimate U. by only bigger components

Example 2: Current Measurement (1)

- Measurement circuit
  - Current I (about 10 A) is applied to resistance R (nominal 0.01 Ω, measurement 0.001018 Ω)
  - measure voltage drop U by digital volt-meter (DVM)
  - DVM's input resistance is over 10^9 Ω, leakage current is negligible small
  - equation of measured value and inputs
    $I = U/R$
- Type A uncertainties
  - repeatability of measurement: u_R
    for 12 times measurements, average and standard deviation
    average: 100.03 mV
    standard deviation: 0.028 mV



Example 2: Current Measurement (2)

- Type B uncertainties
  - U by DVM (from specifications by manufacture): u_{DVM}
    - 0.045 % at 100 mV, assumption of rectangular distribution
    - 0.045/100 x 100 / $\sqrt{3}$
    - Standard deviation: 0.026 mV
  - U by calibration value of R (from calibration table): u_{Reg}
    - relative U: 6 x 10^{-4} (k=2)
    - standard deviation 3 μΩ

Example 2: Current Measurement (3)

- Measured value: $I = U/R$ = 0.10003/0.010018 = 9.985 A
- Sensitivity coefficients: by partial differentials
  - Voltage
    - partial differential: $dI / dU = 1/R$
    - sensitivity: $1/R$ ≈ 100 (A/V)
  - Registance
    - partial differential: $dI / dR = U/R^2$
    - sensitivity: U/R^2 = 1000 (A/Ω)
  - Combined uncertainty u_c
    $u_c^2 = (1/R)^2 \times (u_R^2 + u_{DVM}^2) + (U/R^2)^2 \times (u_{Reg}^2) = 24.6 \times 10^{-6}$ (A²)
    $u_c = 5 \times 10^{-3}$ (A) = 0.005 (A)
  - Expand uncertainty U
    $U = 2 \times u_c$ = 0.01 (A) (k = 2)
  - Expression of measuring result
    9.985 ± 0.01 A

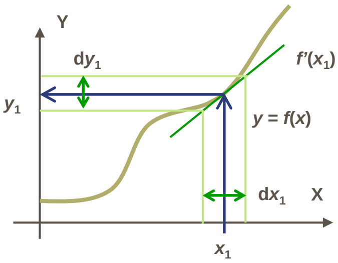
Example 2: Current Measurement (4)

- Budget Sheet
- Expression of measuring result
  9.985 ± 0.01 A

Symbol	Component	Value ±	Evaluation type	Distribution	Divisor	Standard U.	Sensitivity coefficient	Standard U. (measured)
u_R	repeatability	0.028 mV	A measure	Gaussian	1	0.028 mV	100 A/V	0.0028 A
u_{DVM}	DVM	0.045 mV	B certification	rectangular	$\sqrt{3}$	0.026 mV	100 A/V	0.0026 A
u_{Reg}	registance	6 μΩ	B certification	Gaussian	2	3 μΩ	1000 A/Ω	0.003 A
u_c	combined standard U.			Gaussian				0.005 A
U	expand U.			Gaussian				0.01 A

Example 2: Current Measurement (5)

- Propagation of Error: Propagation of Uncertainty
  - Evaluate sensitivity coefficients by partial differentials
  - Relationship (function) *f* for parameter *x* and measurement result *y*
    $y = f(x)$
  - When uncertainty of parameter (x_1) is dx_1 , uncertainty of measuring result (y_1) dy_1 is calculated by partial different of function *f*
    $dy_1 = f'(x_1)\, dx_1$



誤差の伝播(2)

- 誤差とその分散
  - 誤差は正負により打ち消しあったりする.
  - 分散(標準偏差の二乗)はそれぞれの誤差の二乗和で計算できる.
    - 互いに独立な場合
    - 独立でない場合は共分散を考慮する必要がある

$$y = f(x_1, x_2, x_3 \cdots)$$

$$\mathrm{d}y = \frac{\partial f}{\partial x_1}\mathrm{d}x_1 + \frac{\partial f}{\partial x_2}\mathrm{d}x_2 + \frac{\partial f}{\partial x_3}\mathrm{d}x_3 + \cdots$$

$$s_y^2 = \left(\frac{\partial f}{\partial x_1}\right)^2 s_{x_1}^2 + \left(\frac{\partial f}{\partial x_2}\right)^2 s_{x_2}^2 + \left(\frac{\partial f}{\partial x_3}\right)^2 s_{x_3}^2 + \cdots$$

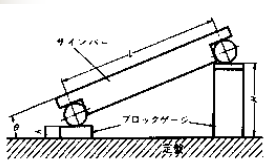
誤差の伝播(3)

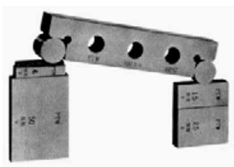
- 簡単な例
  - 誤差 3 μm と誤差 4 μm のブロックゲージを2つくっつけると: 誤差は 5 μm
  - サインバー

$$L\sin\alpha = H - h = E$$

$$\alpha = f(E, L) = \arcsin\left(\frac{E}{L}\right)$$

$$s_\alpha^2 = \left(\frac{\partial f}{\partial E}\right)^2 s_E^2 + \left(\frac{\partial f}{\partial L}\right)^2 s_L^2$$





誤差の伝播(4)

- サインバーの誤差
  - *E* と *L* の誤差比が定数の場合,
  - tan *α* が大きいと誤差が大きくなる.
  - *α* が45°以下で使う.

$$y = \arcsin x$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{\frac{\mathrm{d}x}{\mathrm{d}y}} = \frac{1}{\cos y} = \frac{\pm 1}{\sqrt{1-x^2}}$$

$$s_\alpha^2 = \left(\frac{\partial f}{\partial E}\right)^2 s_E^2 + \left(\frac{\partial f}{\partial L}\right)^2 s_L^2$$

$$= \left(\frac{\partial \arcsin\left(\frac{E}{L}\right)}{\partial E}\right)^2 s_E^2 + \left(\frac{\partial \arcsin\left(\frac{E}{L}\right)}{\partial L}\right)^2 s_L^2$$

$$= \left(\frac{1}{L\cos\alpha}\right)^2 s_E^2 + \left(\frac{E}{L^2\cos\alpha}\right)^2 s_L^2$$

$$= \left(\frac{\sin\alpha}{E\cos\alpha}\right)^2 s_E^2 + \left(\frac{\sin\alpha}{L\cos\alpha}\right)^2 s_L^2$$

$$= \tan^2\alpha\left(\left(\frac{s_E}{E}\right)^2 + \left(\frac{s_L}{L}\right)^2\right)$$

誤差の伝播(5)

- 積で表されるとき
  - 測定値 *y* が *n* 個の測定値 x_1, x_2 ..., x_n のべき乗の積で表されるとき
  - それぞれの測定値の誤差の標準偏差 s_{x1}, s_{x2} ... s_{xn} とし
  - べき乗を p_1, p_2..., p_n とする.

$$y = x_1^{p_1} \cdot x_2^{p_2} \cdots x_n^{p_n}$$

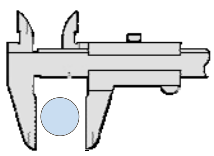
$$s_y^2 = \left(\frac{\partial y}{\partial x_1}\right)^2 s_{x_1}^2 + \left(\frac{\partial y}{\partial x_2}\right)^2 s_{x_2}^2 + \cdots$$

$$= \left(p_1 x_1^{p_1-1} \cdot x_2^{p_2} \cdots x_n^{p_n}\right)^2 s_{x_1}^2 + \left(x_1^{p_1} \cdot p_2 x_2^{p_2-1} \cdots x_n^{p_n}\right)^2 s_{x_2}^2 + \cdots$$

$$= \left(\frac{p_1 y}{x_1}\right)^2 s_{x_1}^2 + \left(\frac{p_2 y}{x_2}\right)^2 s_{x_2}^2 + \cdots$$

Example 3: Measurement by Caliper (1)

- Measuring conditions
  - Vernier caliper: measuring range is 150 mm, resolution: 0.05 mm
  - Diameter of brass cylinder: nominal diameter is 50 mm
  - Simple air conditioning room
- Components
  - Vernier caliper
  - Temperature
    - Deviation from 20 deg (degree of Celsius): from 20 deg to 26 deg
    - U of coefficient of thermal expansion
    - U of temperature difference between caliper and workpiece: ±0.2 deg
  - Repeatability of measurement



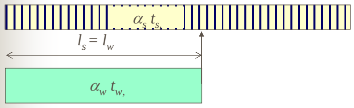
Example 3: Measurement by Caliper (2)

- Components and values (standard deviations)
  - Vernier caliper (B type): instrumental error is 0.05 mm and assumption of rectangular distribution, 0.05/ $\sqrt{3}$ = 0.029 mm
  - Temperature (B type): from equations: under 0.001 mm, negligible small
  - Repeatability by inspector (A type): from experiment: 0.015 mm
- Combined Uncertainty and Expand Uncertainty
  - u_c: Root sum square: 0.033 mm
  - U = 0.07 mm (k = 2)

Symbol	Component	Value ±	Evaluation type	Distribution	Divisor	Standard U.	Sensitivity coefficient	Standard U. (measured)
u_v	vienier	0.05 mm	B	rectangular	$\sqrt{3}$	0.029 mm	1	0.029 mm
u_T	temperature		B	Gaussian				small
u_r	repeatability	0.015 mm	A	Gaussian	1	0.015 mm	1	0.015 mm
u_c	combined			Gaussian				0.033 mm
U	expand U.			Gaussian				0.07 mm

Example 3: Measurement by Caliper (3)

- Temperature Effect
  - Scale: reading l_{s20}, coefficient of thermal expansion α_s, temperature t_s
  - Work: length l_{w20}, coefficient of thermal expansion α_w, temperature t_w



$$l_s = l_{s20}(1+\alpha_s(t_s-20))$$
$$l_w = l_{w20}(1+\alpha_w(t_w-20))$$
$$l_{w20} = \frac{l_{s20}(1+\alpha_s(t_s-20))}{1+\alpha_w(t_w-20)} \approx l_{s20}(1+\alpha_s(t_s-20)-\alpha_w(t_w-20))$$

Approximation

$$x \prec\prec 1$$
$$\frac{1}{1+x} = \frac{(1-x)}{(1+x)(1-x)} = \frac{1-x}{1-x^2} \approx 1-x$$
$$\frac{1}{1-x} = \frac{(1+x)}{(1-x)(1+x)} = \frac{1+x}{1-x^2} \approx 1+x$$

x	1/(1+x)	1-x
0.0001	0.999900010	0.9999
0.001	0.999000999	0.999
0.01	0.990099010	0.99
0.1	0.909090909	0.9

Example 3: Measurement by Caliper (4)

- Conditions
  - Difference from 20 deg: 3 deg
  - Coefficient of thermal expansion: 10x10^{-6} /deg
  - Diameter: 50 mm
  - U. of thermometer: 0.1 deg
  - U. of coefficient of thermal expansion: 1x10^{-6} /deg
- Standard Uncertainties
  - U. by thermometer: 50 x 10 x 10^{-6} x 0.1 = 0.05 μm = 50 nm
  - U. by CTE: 50 x 3 x 1 x 10^{-6} = 0.15 μm = 150 nm
- It is negligible small for vernier caliper, however, it is very large in nano metrology

$$e_l = \frac{\partial l_{w20}}{\partial \alpha_s} e_{\alpha_s} = l_{s20}(t_s-20)e_{\alpha_s}$$
$$e_l = \frac{\partial l_{w20}}{\partial t_s} e_{\alpha_s} = l_{s20}\alpha_s e_{t_s}$$

Conclusion in Length Measurement Uncertainty

- Contributors
  - Measuring instrument
  - Calibration of measuring instrument
  - Environment
- Good condition, good measurement
  - U. by instrument is the biggest U.
- High accuracy (nano-metrology)
  - U. by environment (temperature) is the biggest U.
- Complicated measuring instruments (CMM, height gauge, video CMM): U. by strategy of measurement is the biggest U.