三次元座標計測(第7回)
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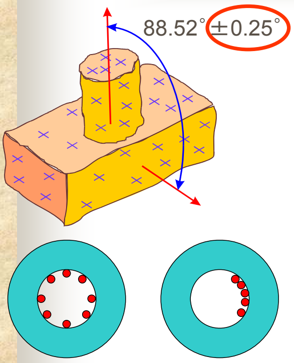
三次元測定の不確かさ

機械計測破たんのシナリオ

- 機械計測の多くの部分に三次元測定機(CMM)が必要.
- 測定の不確かさ，ISO9000によれば，測定機には測定の不確かさが必要.
- 現在，CMMによる複雑な測定の不確かさを推定することが難しい.
- 機械計測の多くの部分が破たんして，機械部品の品質保証ができない.

座標計測における不確かさ

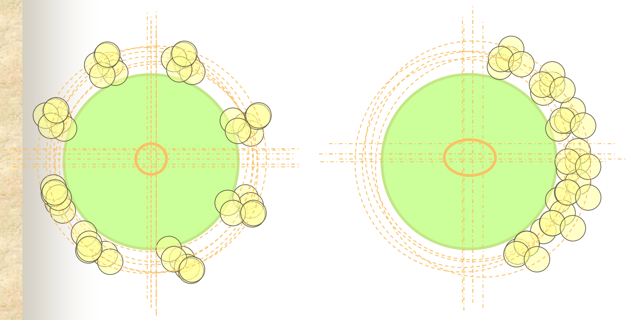


均等に 8 点で測定　　一部を 5 点で測定

- 1点測定の不確かさ，
  寸法測定の不確かさ
- 複雑な測定による測定結果の不確かさは，どのような関係か？
- 測定戦略の違い：
  同じ測定機，プロービングシステムで同じ測定物を測定しても不確かさは同じではない.

測定戦略の違い

- 同じ測定機で同じ測定物を測定しても，測定の不確かさは異なる



不確かさの要因

- 三次元測定機本体の誤差
  - 幾何的な誤差，繰り返し誤差
  - スケールの分解能
- プロービングシステムの誤差
  - 繰り返し誤差
- 環境の誤差
  - 温度の影響
  - 振動の影響
- 測定物
  - 固定方法，ハンドリング
  - 表面粗さ，形状誤差
- 測定戦略

座標計測の不確かさの推定

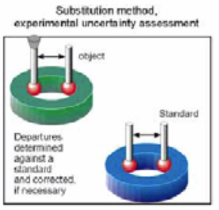
- 長さ測定に対する評価手法
- 置き換え法 (PTB)

- シミュレーション法
  - 統計的な推定 (PTB, NPL, UT ...)
  - 形体計測 (UT)
  - 拘束条件によるシミュレーション(NIST)
  - バーチャルCMM (PTB)

置き換え法（比較測定）

- CMMをコンパレータとして，ワークピースと校正されたワークピースの比較を行う
- 実験による不確かさの推定
  - 測定するワークピースの代わりに校正されたワークピースを使う
  - 同じ測定の手順を行う
  - これを数回繰り返す
  - 不確かさの評価を行う

Substitution method, experimental uncertainty assessment


object
Standard
Departures determined against a standard and corrected, if necessary

シミュレーション法

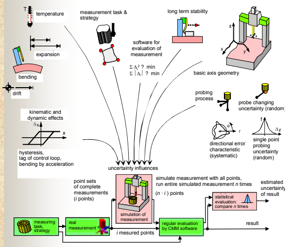
シミュレーション法

- 計算機シミュレーションによる不確かさ推定
- ISO 15530シリーズの主たる手法
- 規格は以下のことを含む
  - シミュレーションの原理の定義
  - ソフトウェアに対する最小限の要求事項
  - どの不確かさを考慮しているかを示すチェックリスト
  - どんなモデルを使っているか
  - 市場における全ての技術的な解決に開かれている
  - シミュレーションソフトを検査する手法の提案

シミュレーション法の基本概念

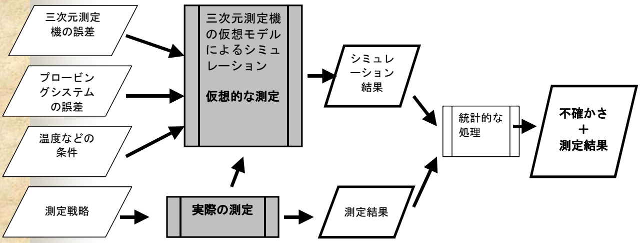
- CMMから測定の不確かさを推定する
  - 繰返しのあるCMMの幾何誤差，プローブの誤差をアーティファクトで評価する
  - 短い周期のCMMの幾何誤差，温度の影響をアーティファクトで評価する
  - プローブの偶然誤差を球測定で評価する
- 測定戦略に関係する不確かさの影響をシミュレーションで評価する
  - 測定点の数
  - 測定点の位置
- 他の要因の評価
  - ワークピースの形状誤差
  - ワークピースの固定方法
- 最小二乗和の平方を計算する

バーチャルCMM (PTB)

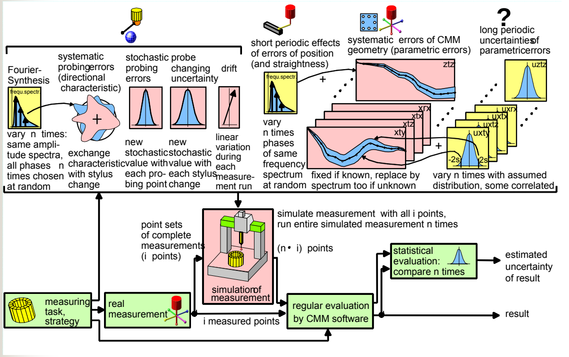

temperature
measurement task & strategy
long term stability
expansion
software for evaluation of measurement
bending
Σ Δ_i² ? min
Σ |Δ_i| ? min
basic axis geometry
drift
probing process
probe changing uncertainty (random)
kinematic and dynamic effects
directional error characteristic (systematic)
single point probing uncertainty (random)
hysteresis, lag of control loop, bending by acceleration
uncertainty influences
point sets of complete measurements (i points)
simulate measurement with all points, run entire simulated measurement n times
(n · i) points
estimated uncertainty of result
statistical evaluation: compare n times
simulation of measurement
measuring task, strategy
real measurement
regular evaluation by CMM software
result
i mesured points

- CMMの実際のモデルを使う．(全パラメタモデル)
- ボールプレート法でCMMの幾何誤差を推定する．
- 参照球でプローブ誤差を推定する．
- オンラインVCMMとオフラインVCMM.

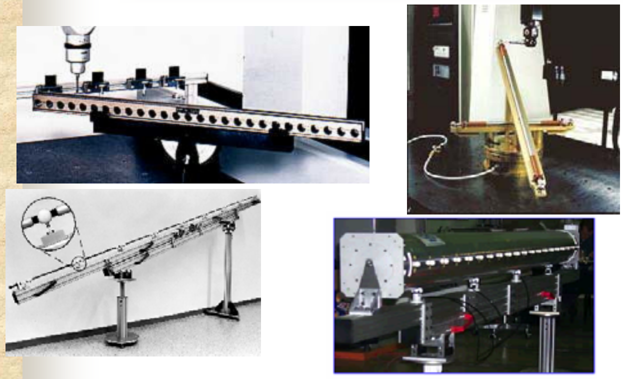
バーチャルCMMの基本構成


三次元測定機の誤差
プロービングシステムの誤差
温度などの条件
測定戦略
三次元測定機の仮想モデルによるシミュレーション
仮想的な測定
実際の測定
シミュレーション結果
測定結果
統計的な処理
不確かさ
+
測定結果

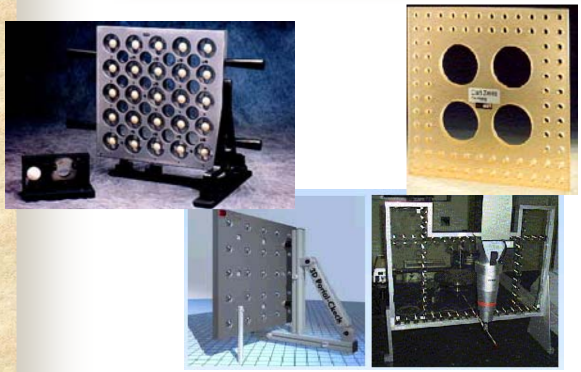
バーチャルCMMソフトウェア


Fourier-Synthesis
frequ.spectr.
vary n times: same amplitude spectra, all phases n times chosen at random
systematic probing errors (directional characteristic)
exchange characteristic with stylus change
stochastic probe probing errors
new stochastic value with each probing point
changing uncertainty
new stochastic value with each stylus change
drift
linear variation during each measurement run
short periodic effects of errors of position (and straightness)
frequ.spectr.
vary n times phases of same frequency spectrum at random
systematic errors of CMM geometry (parametric errors)
ztz
xrx
xtx
xtz
xty
fixed if known, replace by spectrum too if unknown
?
long periodic uncertainties of parametric errors
uztz
uxrx
uxtx
uxtz
uxty
-2s
2s
vary n times with assumed distribution, some correlated
simulate measurement with all i points, run entire simulated measurement n times
point sets of complete measurements (i points)
simulation of measurement
(n • i) points
statistical evaluation: compare n times
estimated uncertainty of result
measuring task, strategy
real measurement
i measured points
regular evaluation by CMM software
result

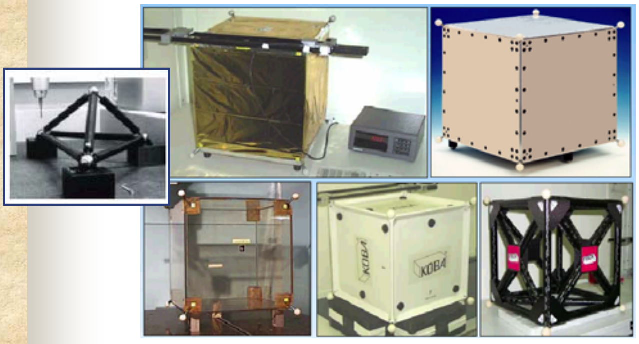
校正用アーティファクト 1次元



校正用アーティファクト 2次元

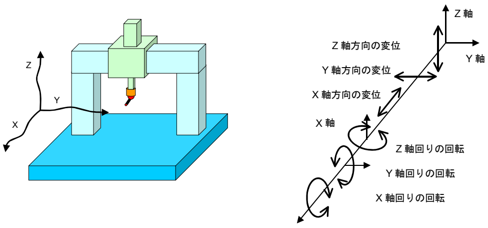

3D Metal-Check

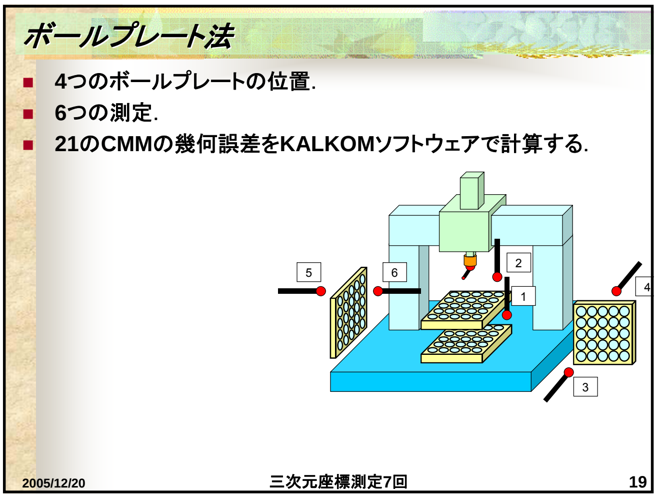
校正用アーティファクト 3次元



21の幾何学的な誤差を推定

- 各軸の並進誤差3方向
- 各軸の回転誤差3方向
- 各軸間の直角誤差3つ
- それぞれを10から20の位置で推定する
- トータル200くらいのパラメータとなる


Z
Y
X
Z 軸
Y 軸
Z 軸方向の変位
Y 軸方向の変位
X 軸方向の変位
X 軸
Z 軸回りの回転
Y 軸回りの回転
X 軸回りの回転



ボールプレート法

- 4つのボールプレートの位置.
- 6つの測定.
- 21のCMMの幾何誤差をKALKOMソフトウェアで計算する.

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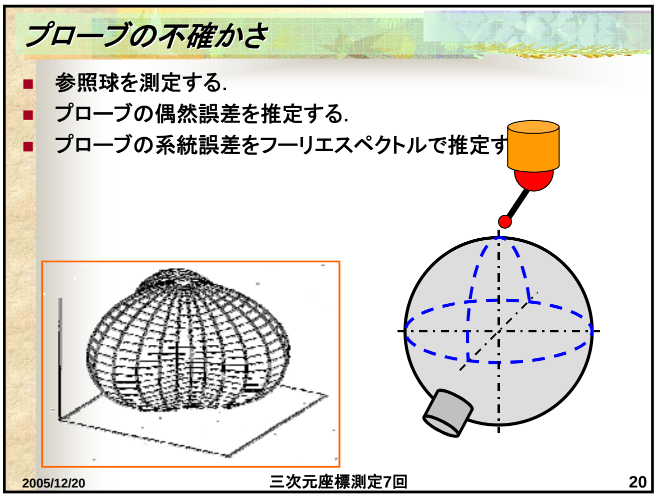

プローブの不確かさ

- 参照球を測定する.
- プローブの偶然誤差を推定する.
- プローブの系統誤差をフーリエスペクトルで推定す

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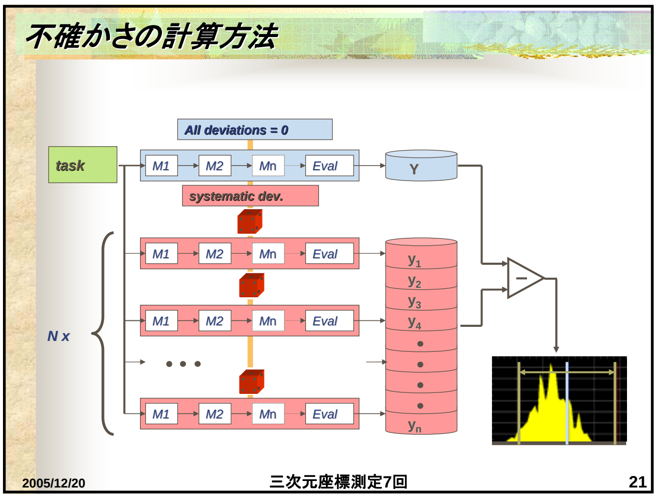

不確かさの計算方法

All deviations = 0
task
M1
M2
Mn
Eval
Y
systematic dev.
N x
y1
y2
y3
y4
yn

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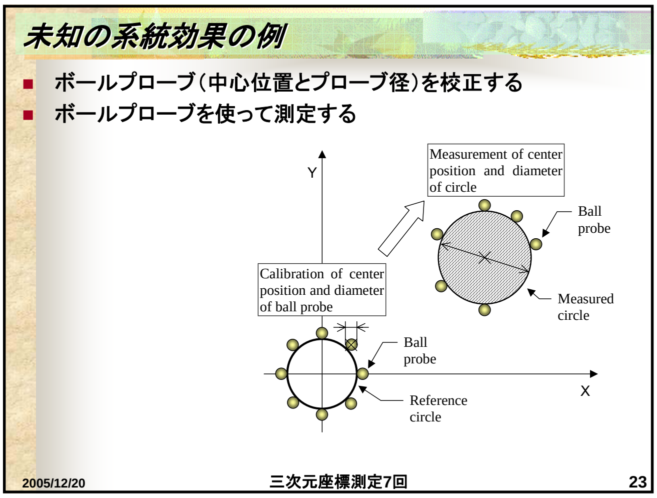
理論的な検討

プローブ校正の影響
形状偏差の影響



未知の系統効果の例

- ボールプローブ(中心位置とプローブ径)を校正する
- ボールプローブを使って測定する

Y
X
Measurement of center position and diameter of circle
Ball probe
Measured circle
Calibration of center position and diameter of ball probe
Ball probe
Reference circle

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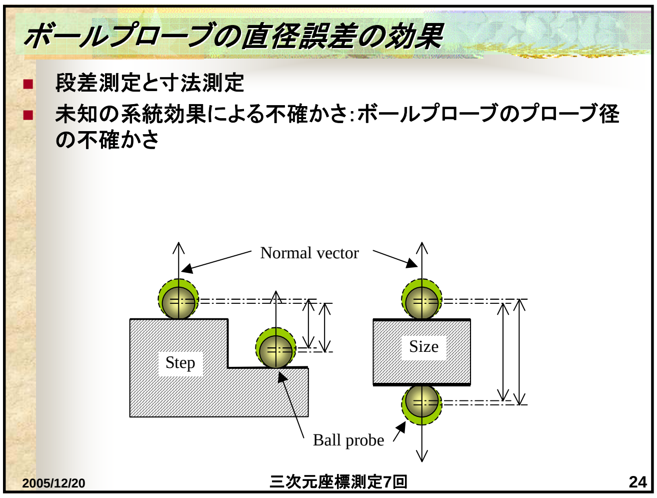

ボールプローブの直径誤差の効果

- 段差測定と寸法測定
- 未知の系統効果による不確かさ:ボールプローブのプローブ径の不確かさ

Normal vector
Step
Size
Ball probe

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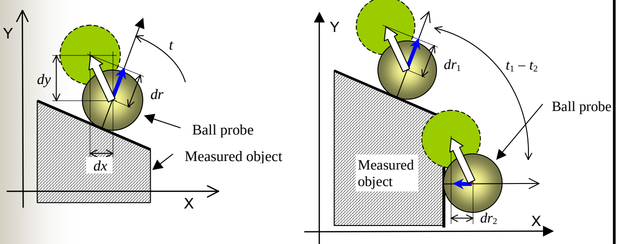
ボールプローブの中心位置の誤差の効果

- 2つの測定点の誤差に相関が生じる



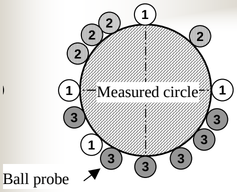
共分散を含む誤差行列の計算

$$s_i^2 = s_p^2 + c_x^2 + \frac{c_d^2}{4}$$

$$s_{ij}^2 = c_x^2 \cos(t_i - t_j) + \frac{c_d^2}{4}$$

$$\mathbf{S}=\left(\begin{array}{ccc|ccc} & & & \frac{s_c^2}{4} & \cdots & \frac{s_c^2}{4}\\ & \mathbf{S}_1 & & \vdots & \ddots & \vdots \\ & & & \frac{s_c^2}{4} & \cdots & \frac{s_c^2}{4}\\ \hline \frac{s_c^2}{4} & \cdots & \frac{s_c^2}{4} & & & \\ \vdots & \ddots & \vdots & & \mathbf{S}_2 & \\ \frac{s_c^2}{4} & \cdots & \frac{s_c^2}{4} & & & \end{array}\right)$$

計算例:3種類のプローブによる円の測定



Center position error of probe c_x, c_y	×	○	×	○	○
Diameter error of probe c_d (s_c not included)	×	×	○	○	○
Certificate error of reference circle s_c	×	×	×	×	○
X position error of measured circle s_x	1.928	2.441	1.933	2.444	2.444
Y position error of measured circle s_y	1.956	2.473	2.266	2.720	2.720
Diameter error of measured circle s_c	2.733	3.197	3.419	3.805	4.845

Evaluation of Uncertainty by Form Deviations of Measured Workpieces in Specified Measuring Strategies

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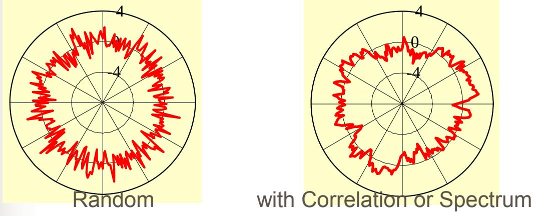
Effect of Form Deviation of Circle

- Two circle features have same form deviation (standard deviation) of 1 μm.
- Question: When the same measuring strategy (number and position of measured points) is obtained, the uncertainty of diameter and center position of each circle feature is same?

Answer: In this paper, we will formulate the calculation methods of the uncertainties effected by the form deviation of the circle feature.



Random with Correlation or Spectrum

Methods to Estimate Uncertainty in CM

- Least Squares Method + Error Propagation
- Simulation with Constraints
  - Using statistically characteristics (parameters: 21 geometrical errors, temperature effects…) of average of target CMMs.
  - Statistical parameters as variance and covariance and/or correlations between the parameters.
  - Cannot handle biases of the parameters.
- Virtual CMM (PTB)
  - Using characteristics of the target CMM.
  - Can handle biases of the parameters.
  - Need to measure the parameters of the target CMM.

The least squares method + error propagation is theoretically estimation;
Good to find theoretical characteristics of uncertainties in coordinate metrology.

Calculation of Uncertainty of Feature

- **From the uncertainty of measured point, the uncertainty of measured feature can be calculated statistically by least squares method + error propagation.**
- **A : Jacobian matrix: Positions of measured points**
- **S : Uncertainty matrix of measure points**
- **W : Weighting function = Inverse matrix of S**

⇩

- **P : Uncertainty matrix of parameters of feature Uncertainties of diameter, coordinate of center ...**

$$\mathbf{C} = (\mathbf{A}^t\mathbf{S}^{-1}\mathbf{A})^{-1}\mathbf{A}^t\mathbf{S}^{-1} = (\mathbf{A}^t\mathbf{W}\mathbf{A})^{-1}\mathbf{A}^t\mathbf{W}$$

$$\mathbf{P} = \mathbf{C}\mathbf{S}\mathbf{C}^t = (\mathbf{A}^t\mathbf{S}^{-1}\mathbf{A})^{-1} = (\mathbf{A}^t\mathbf{W}\mathbf{A})^{-1}$$

P: Uncertainty Matrix of Circle Feature

- **In this paper, we examine the method of estimation of measured parameters for circle feature measurement.**
- **Uncertainty matrix P of the circle features;**
  - **U. (variance) of X and Y coordinates of center s_x^2 and s_y^2**
  - **U. (variance) of diameter s_d^2**
  - **covariance of these parameters s_{xy}, s_{xd} and s_{yd}**
- **Jacobian matrix A and Uncertainty matrix of measure points S are need to calculate P**

$$\mathbf{P} = \begin{pmatrix} s_x^2 & s_{xy} & s_{xd} \\ s_{xy} & s_y^2 & s_{yd} \\ s_{xd} & s_{yd} & s_d^2 \end{pmatrix}$$

S: Uncertainty Matrix of Measured Points

- **S_{ran} : random error**

When the form deviation has the specified correlation function, S_{cov} is defined by the autocorrelation matrix R_{cov} and s_f^2.

R_{cov}: correlation coefficients between the form deviations of measured points.

- **S_{cov} : error with specified autocorrelation function**

When the form deviation is random error, S_{ran} is the unit matrix multiplied by the variance s_f^2 of form deviation.

$$\mathbf{S}_{\mathrm{ran}} = \begin{pmatrix} s_f^2 & & 0 \\ & \ddots & \\ 0 & & s_f^2 \end{pmatrix} = s_f^2 \begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix} = s_f^2\mathbf{E}$$

$$\mathbf{S}_{\mathrm{cov}} = \begin{pmatrix} s_f^2 & s_{12} & \cdots & s_{1n} \\ s_{12} & \ddots & s_{ij} & \vdots \\ \vdots & s_{ij} & \ddots & s_{n-1n} \\ s_{1n} & \cdots & s_{n-1n} & s_f^2 \end{pmatrix} = s_f^2 \begin{pmatrix} 1 & r_{12} & \cdots & r_{1n} \\ r_{12} & \ddots & r_{ij} & \vdots \\ \vdots & r_{ij} & \ddots & r_{n-1n} \\ r_{1n} & \cdots & r_{n-1n} & 1 \end{pmatrix} = s_f^2\mathbf{R}_{\mathrm{cov}}$$

Estimation of Uncertainty Matrix P

- **From two types of error matrices S_{ran} and S_{cov}, there are three types of uncertainties of measured parameters P_{ran}, P_{cov} and P_{r+c}.**
- **P_{ran}: Form: Random LSM: Random**
- **P_{cov}: Form: Not Random LSM: with Autocorrelation**
- **P_{r+c}: Form: Not Random LSM: Random**

Usually the calculating program in CMM can not handle the autocorrelation function. Therefore, the uncertainties of the normal calculating situation in measuring by CMM are defined by P_{r+c}. We Use This Method

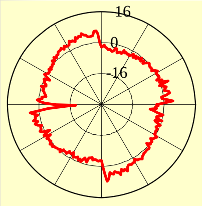
P_{ran} is the uncertainty matrix of parameters when the form deviation is assumed as the random function.

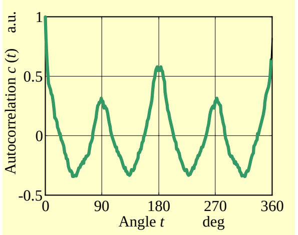
P_{r+c} is the uncertainty matrix of parameters when the form deviation has the specified autocorrelation function and calculated using the normal least squares method without the autocorrelation function.

P_{cov} is the uncertainty matrix of parameters when the form deviation has the specified autocorrelation function and calculated using the autocorrelation function. Best Method

Examples of Uncertainty Estimation

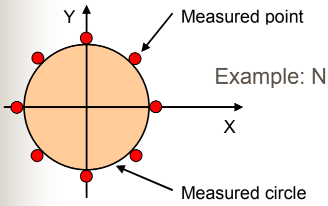
- **Example: a hole with circularity of 28.8 μm, diameter of 20 mm, s_f = 3.1 μm.**
- **Autocorrelation function of the hole:**
  The autocorrelation function and power spectrum of the hole has large 2nd and 4th order frequency values.



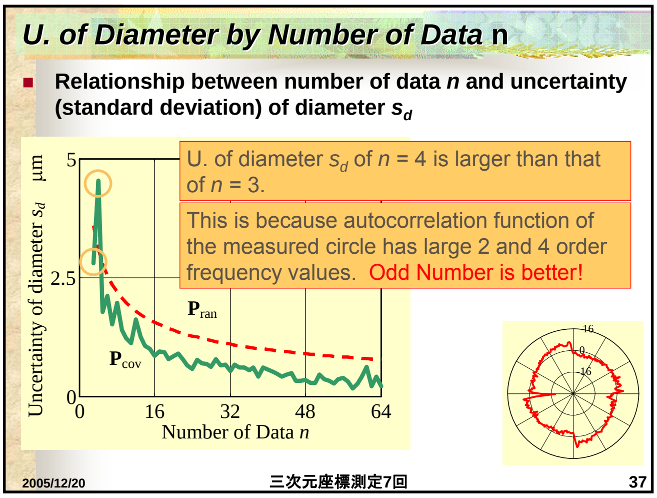


Measured Points Uniformly on Circle

- **When measuring points are set uniformly on the measured circle,**
  P_{cov} and P_{r+c} are completely same values, because of S_{cov} and S_{ran} are the same weighting function for all measured points.



Example: Number of Data n = 8

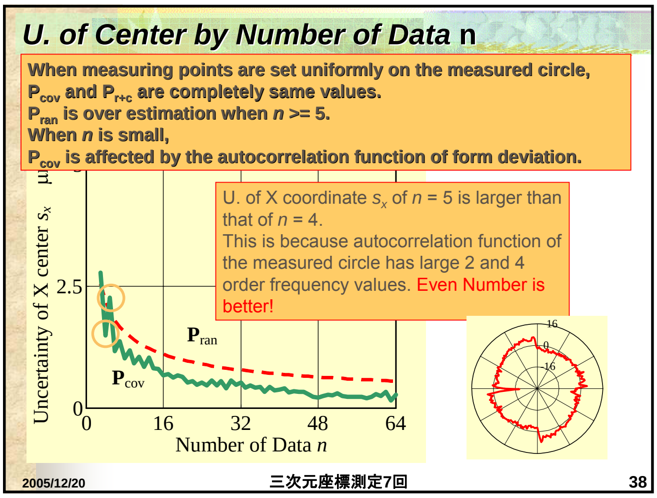


U. of Diameter by Number of Data n

- Relationship between number of data *n* and uncertainty (standard deviation) of diameter s_d

U. of diameter s_d of n = 4 is larger than that of n = 3.

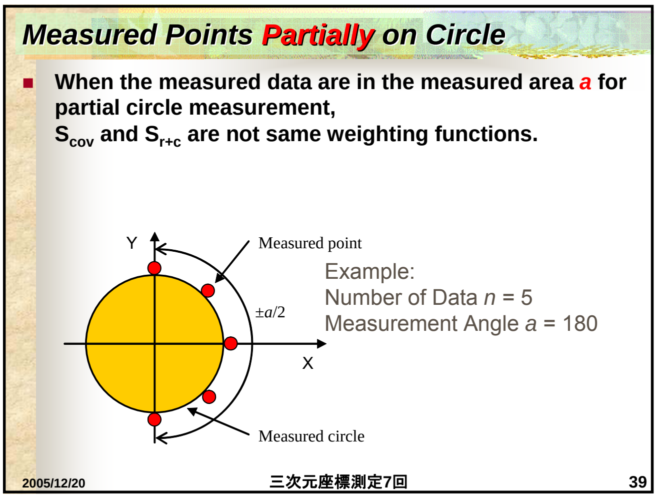
This is because autocorrelation function of the measured circle has large 2 and 4 order frequency values. Odd Number is better!



U. of Center by Number of Data n

When measuring points are set uniformly on the measured circle,
P_{cov} and P_{r+c} are completely same values.
P_{ran} is over estimation when n >= 5.
When n is small,
P_{cov} is affected by the autocorrelation function of form deviation.

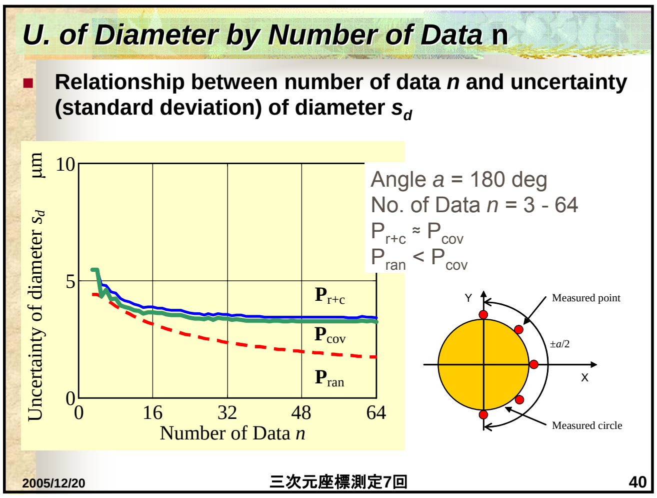
U. of X coordinate s_x of n = 5 is larger than that of n = 4.
This is because autocorrelation function of the measured circle has large 2 and 4 order frequency values. Even Number is better!



Measured Points Partially on Circle

- When the measured data are in the measured area *a* for partial circle measurement, S_{cov} and S_{r+c} are not same weighting functions.

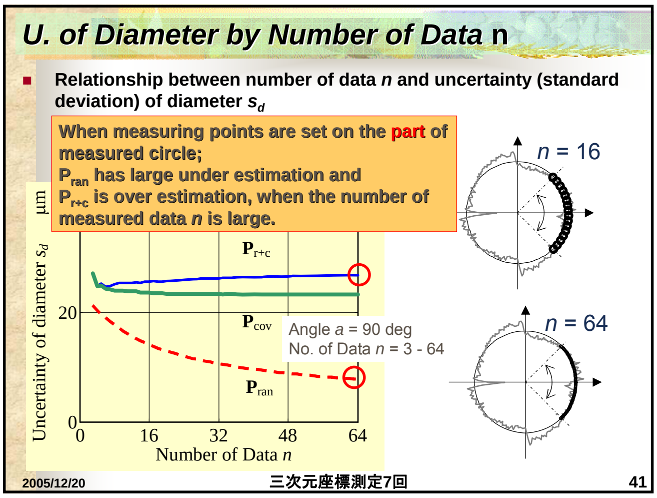
Example:
Number of Data n = 5
Measurement Angle a = 180



U. of Diameter by Number of Data n

- Relationship between number of data *n* and uncertainty (standard deviation) of diameter s_d

Angle a = 180 deg
No. of Data n = 3 - 64
$P_{r+c} \approx P_{cov}$
$P_{ran} < P_{cov}$

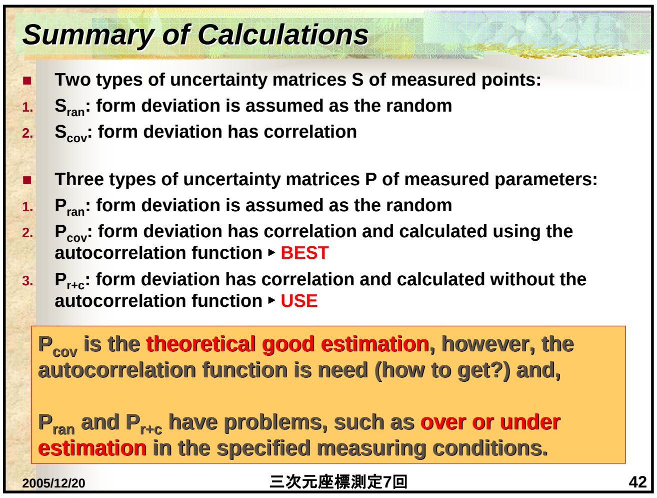


U. of Diameter by Number of Data n

- Relationship between number of data *n* and uncertainty (standard deviation) of diameter s_d

When measuring points are set on the part of measured circle;
P_{ran} has large under estimation and
P_{r+c} is over estimation, when the number of measured data n is large.

Angle a = 90 deg
No. of Data n = 3 - 64

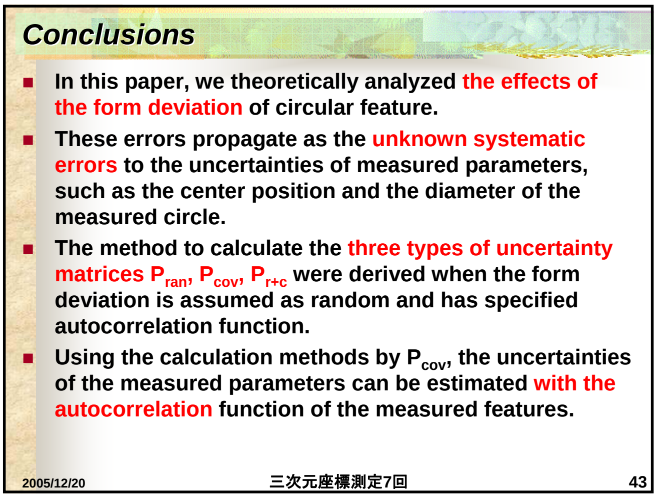


Summary of Calculations

- Two types of uncertainty matrices S of measured points:
  1. S_{ran}: form deviation is assumed as the random
  2. S_{cov}: form deviation has correlation
- Three types of uncertainty matrices P of measured parameters:
  1. P_{ran}: form deviation is assumed as the random
  2. P_{cov}: form deviation has correlation and calculated using the autocorrelation function ▸ BEST
  3. P_{r+c}: form deviation has correlation and calculated without the autocorrelation function ▸ USE

P_{cov} is the theoretical good estimation, however, the autocorrelation function is need (how to get?) and,

P_{ran} and P_{r+c} have problems, such as over or under estimation in the specified measuring conditions.



Conclusions

- In this paper, we theoretically analyzed the effects of the form deviation of circular feature.
- These errors propagate as the unknown systematic errors to the uncertainties of measured parameters, such as the center position and the diameter of the measured circle.
- The method to calculate the three types of uncertainty matrices P_{ran}, P_{cov}, P_{r+c} were derived when the form deviation is assumed as random and has specified autocorrelation function.
- Using the calculation methods by P_{cov}, the uncertainties of the measured parameters can be estimated with the autocorrelation function of the measured features.

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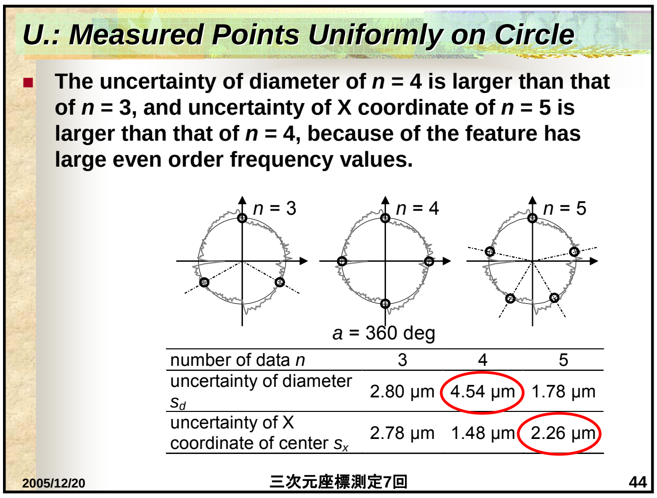

U.: Measured Points Uniformly on Circle

- The uncertainty of diameter of n = 4 is larger than that of n = 3, and uncertainty of X coordinate of n = 5 is larger than that of n = 4, because of the feature has large even order frequency values.

number of data n	3	4	5
uncertainty of diameter s_d	2.80 μm	4.54 μm	1.78 μm
uncertainty of X coordinate of center s_x	2.78 μm	1.48 μm	2.26 μm

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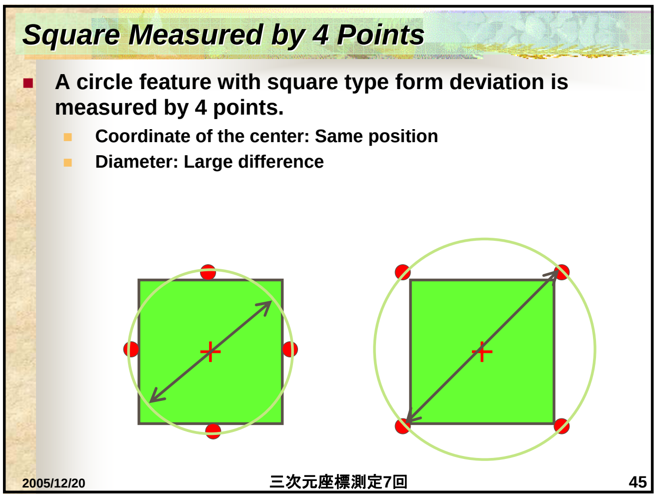

Square Measured by 4 Points

- A circle feature with square type form deviation is measured by 4 points.
  - Coordinate of the center: Same position
  - Diameter: Large difference

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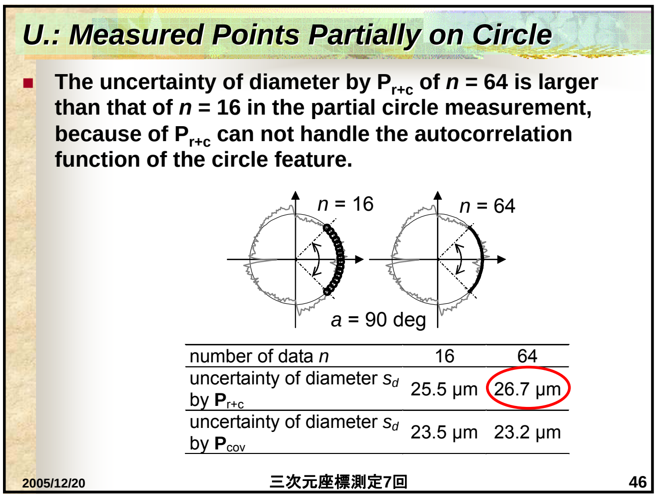

U.: Measured Points Partially on Circle

- The uncertainty of diameter by P_{r+c} of n = 64 is larger than that of n = 16 in the partial circle measurement, because of P_{r+c} can not handle the autocorrelation function of the circle feature.

number of data n	16	64
uncertainty of diameter s_d by P_{r+c}	25.5 μm	26.7 μm
uncertainty of diameter s_d by P_{cov}	23.5 μm	23.2 μm

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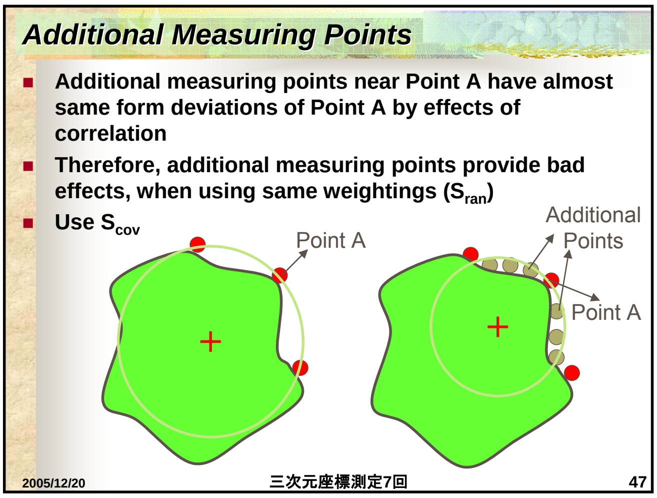

Additional Measuring Points

- Additional measuring points near Point A have almost same form deviations of Point A by effects of correlation
- Therefore, additional measuring points provide bad effects, when using same weightings (S_{ran})
- Use S_{cov}

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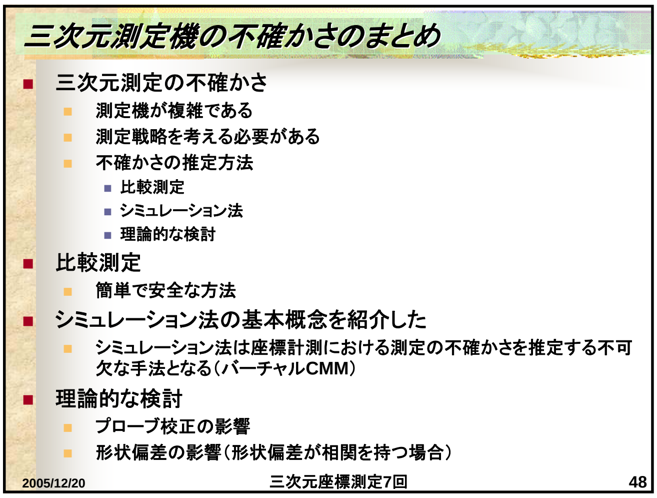

三次元測定機の不確かさのまとめ

- 三次元測定の不確かさ
  - 測定機が複雑である
  - 測定戦略を考える必要がある
  - 不確かさの推定方法
    - 比較測定
    - シミュレーション法
    - 理論的な検討
- 比較測定
  - 簡単で安全な方法
- シミュレーション法の基本概念を紹介した
  - シミュレーション法は座標計測における測定の不確かさを推定する不可欠な手法となる（バーチャルCMM）
- 理論的な検討
  - プローブ校正の影響
  - 形状偏差の影響（形状偏差が相関を持つ場合）

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