三次元座標計測（第10回）
2005年度大学院講義
2006年1月24日

高増　潔
東京大学工学系研究科
精密機械工学専攻
E-mail: takamasu@pe.u-tokyo.ac.jp
HP: http://www.nano.pe.u-tokyo.ac.jp/

Estimation of uncertainty of measurements of 3D mechanisms after kinematic calibration

K Takamasu, O Sato, K Shimojima, S Takahashi and R Furutani

takamasu@pe.u-tokyo.ac.jp
Department of Precision Engineering, The University of Tokyo

Introduction (1)

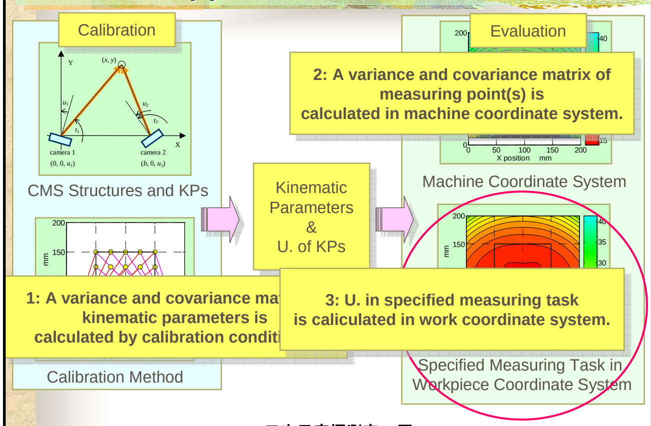
- Calibration methods of CMSs (Coordinate Measuring Systems) are essential to measure accurately and to evaluate uncertainty of measurements.
- We formulated the method to evaluate the uncertainty in coordinate metrology and proposed the error propagation method to estimate the uncertainty of kinematic parameters in the calibration of the CMSs.



Introduction (2)

- These evaluations of uncertainty of the parameters are calculated in the machine coordinate system.
- However, the specified measurement tasks are done in a workpiece coordinate system after the calibration.
- In this article, the estimation methods of uncertainties using the calibrated CMS after the calibration are formulated.

Introduction (3)



Uncertainty evaluation of a measuring point (1)

- 1st step: calculation of $\mathbf{S}_p$
- CMS Structure and Parameters
  - f　forward kinematics of CMS
  - p　kinematic parameters (KPs)
  - q　readings of encorders

$$\mathbf{x} = \mathbf{f}(\mathbf{p}, \mathbf{q}) = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \quad \mathbf{p} = \begin{pmatrix} p_1 \\ p_2 \\ \vdots \end{pmatrix}, \quad \mathbf{q} = \begin{pmatrix} q_1 \\ q_2 \\ \vdots \end{pmatrix}$$

Equation (1)

- Calibration Methods
  - A　Jacobian matrix in calibration
  - S　error matrix in calibration
- Uncertainties of kinematic parameters
  - $\mathbf{S}_p$　U. (variances) of KPs
  - $\mathbf{S}_r$　U. (variances) of coordinate conversion
  - $\mathbf{S}_{pr}$　covariance

$$\begin{pmatrix} \mathbf{S}_p & \mathbf{S}_{pr} \\ \mathbf{S}_{pr} & \mathbf{S}_r \end{pmatrix} = (\mathbf{A}^t\, \mathbf{S}^{-1}\, \mathbf{A})^{-1}$$

Equation (2)

Uncertainty evaluation of a measuring point (2)

- 2nd step: calculation of T_1, $T_{1\text{-}2}$ and T_n
- Uncertainty of a measuring point after the calibration
  - T_1 U. of a MP
  - *sx* U. of X coordinate
  - *sy* U. of Y coordinate
  - *sz* U. of Z coordinate
  - *sxy* covariance between X and Y coordinates
  - *syz* covariance between Y and Z coordinates
  - *sxz* covariance between X and Z coordinates

$$\mathbf{T}_1=\begin{pmatrix} sx^2 & sxy & sxz \\ sxy & sy^2 & syz \\ sxz & syz & sz^2 \end{pmatrix}=\mathbf{T}_p+\mathbf{T}_q+\mathbf{T}_m$$
$$=\mathbf{A}_p\,\mathbf{S}_p\,\mathbf{A}_p^t+s_q^2\mathbf{A}_q\,\mathbf{A}_q^t+s_m^2\,\mathbf{E}$$

Equation (3)

Uncertainty evaluation of a measuring point (3)

- T_p propagation from S_p and A_p (Jacobian of MP)
- T_q propagation from s_q (U. of encoder) and A_q (Jacobian of MP)
- T_m U. of probing is s_m (random)

source	uncertainty	different point	X, Y and Z coordinates
kinematic parameter	T_p	yes	yes
encoder	T_q	no	yes
probing	T_m	no	no

$$\mathbf{T}_1=\mathbf{T}_p+\mathbf{T}_q+\mathbf{T}_m$$
$$\mathbf{T}_p=\mathbf{A}_p\,\mathbf{S}_p\,\mathbf{A}_p^t \quad \mathbf{T}_q=s_q^2\mathbf{A}_q\,\mathbf{A}_q^t \quad \mathbf{T}_m=s_m^2\,\mathbf{E}$$

Equation (3)

Uncertainty evaluation of measuring points (4)

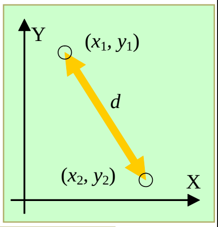
- Uncertainty of measuring points after the calibration in 2D
  - $T_{1\text{-}2}$ U. of two MPs
  - $T_{1\text{-}n}$ U. of *n*-MPs
  - (red box) measuring point 1
  - (pink box) measuring point 2
  - (green box) measuring points 1 & 2

$$\mathbf{T}_{1-2}=\begin{pmatrix} sx_1^2 & sx_1y_1 & sx_1x_2 & sx_1y_2 \\ sx_1y_1 & sy_1^2 & sy_1x_2 & sy_1y_2 \\ sx_1x_2 & sy_1x_2 & sx_2^2 & sx_2y_2 \\ sx_1y_2 & sy_1y_2 & sx_2y_2 & sy_2^2 \end{pmatrix}$$

$$\mathbf{T}_{1-n}=\begin{pmatrix} sx_1^2 & sx_1y_1 & sx_1x_2 & \cdots & sx_1y_n \\ sx_1y_1 & sy_1^2 & sy_1x_2 & & sy_1y_n \\ sx_1x_2 & sy_1x_2 & sx_2^2 & & \vdots \\ \vdots & & & \ddots & \\ sx_1y_n & sy_1y_n & \cdots & & sy_n^2 \end{pmatrix}$$

Uncertainty of a specified measuring task

- 3rd step: calculation U. s_d of a specified measuring task
  - Size *d* measurement (distance between point 1 and point 2) in XY coordinate plane (2D)
  - G_d definition of size measurement
  - A_d Jacobian matrix of Gd
  - s_d U. of size measurement from two points



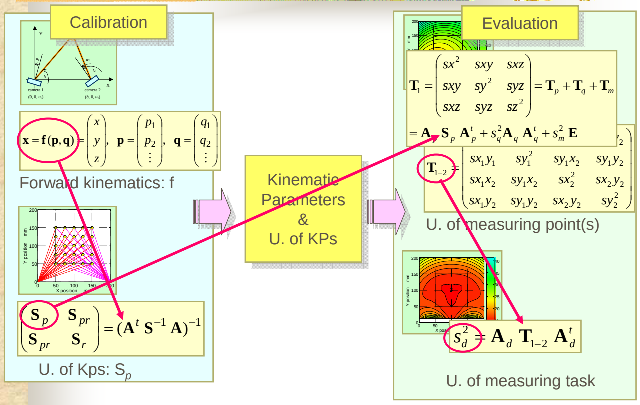
$$d=\mathbf{G}_d(\mathbf{x}_1,\mathbf{x}_2)=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$\mathbf{A}_d=\begin{pmatrix}\dfrac{\partial \mathbf{G}_d}{\partial \mathbf{x}_1} & \dfrac{\partial \mathbf{G}_d}{\partial \mathbf{x}_2}\end{pmatrix}=\frac{\begin{pmatrix}-x_1+x_2 & -y_1+y_2 & -x_1+x_2 & -y_1+y_2\end{pmatrix}}{\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}}$$

$$s_d^2=\mathbf{A}_d\,\mathbf{T}_{1-2}\,\mathbf{A}_d^t$$

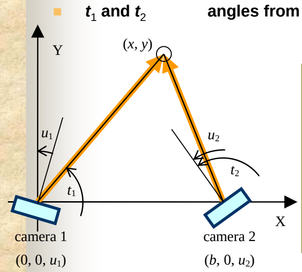
Equations (4), (5) and (6)

Summary of uncertainty evaluation



Example: 2D-CMS by two line cameras

- 2D-CMS: triangulation by two angles of line cameras
  - f forward kinematics
  - u_1 and u_2 offset angles of each camera from Y axis
  - *b* X coordinate of camera 2 of 200 mm
- Encoders q
  - t_1 and t_2 angles from images of each camera



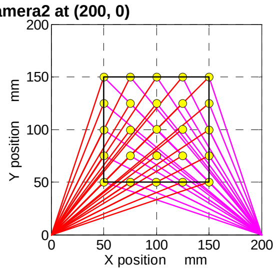
$$\begin{pmatrix}x\\y\end{pmatrix}=\mathbf{x}=\mathbf{f}(\mathbf{p},\mathbf{q})=\begin{pmatrix}\dfrac{b\tan(t_2-u_2)}{\tan(t_2-u_2)-\tan(t_1-u_1)}\\ \dfrac{b\tan(t_1-u_1)\tan(t_2-u_2)}{\tan(t_2-u_2)-\tan(t_1-u_1)}\end{pmatrix}$$

$$\mathbf{p}=\begin{pmatrix}u_1\\u_2\\b\end{pmatrix},\quad \mathbf{q}=\begin{pmatrix}t_1\\t_2\end{pmatrix}$$

Equation (7)

Calibration of KPs (1)

- Calibration conditions for 2-D CMS
  - calibrated by external 2-D CMS
  - no. of points 25
  - positions in X of 50-150 mm and Y of 50-150 mm at 25 mm intervals
  - cameras camera1 at (0, 0), camera2 at (200, 0)
  - U. of probing 10 μm
  - U. of external measuring system 5 μm
  - U. of line cameras 0.001 deg



Calibration of KPs (2)

- Calibration results of the two dimensional line camera
  - standard deviations of the three kinematic parameters
  - correlation coefficients between the parameters
  - standard deviations of u_1, u_2 and b are same levels of uncertainties in calibration
  - low correlation between u_1 and u_2
  - large negative correlation between u_1 and b
  - large positive correlation between u_2 and b

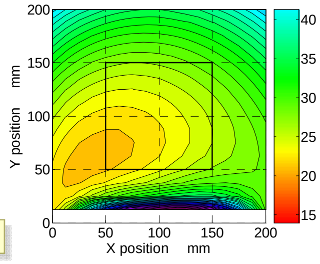
	standard deviation	correlation coefficient for	
		u_2	b
u_1	0.0033 deg	0.0077	–0.4744
u_2	0.0033 deg	-	0.4744
b	12.5 μm	-	-

Evaluation after calibration (1)

- Positioning uncertainty from T_1 in machine coordinate system
  - contour map of RSS of *sx* and *sy*
  - U. of probing s_m is 10 μm
  - U. of camera s_q is 0.001 deg
  - U. is 17.8 μm - 22.2 μm in measuring range

$$\mathbf{T}_1 = \begin{pmatrix} sx^2 & sxy \\ sxy & sy^2 \end{pmatrix} = \mathbf{T}_p + \mathbf{T}_q + \mathbf{T}_m$$
$$= \mathbf{A}_p \mathbf{S}_p \mathbf{A}_p^t + s_q^2 \mathbf{A}_q \mathbf{A}_q^t + s_m^2 \mathbf{E}$$

Equation (3)



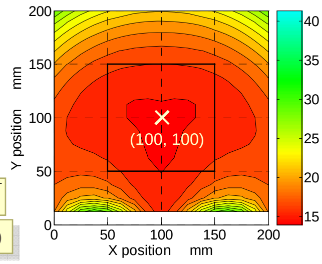
Evaluation after calibration (2)

- Size measurement uncertainties from a specified point (100, 100) by sd
  - contour map of sd
  - U. of probing s_m is 10 μm
  - U. of camera s_q is 0.001 deg
  - U. is 14.5 μm - 15.7 μm in measuring range

$$d = \mathbf{G}_d(\mathbf{x}_1, \mathbf{x}_2) = \sqrt{(x-100)^2 + (y-100)^2}$$

$$s_d^2 = \mathbf{A}_d \ \mathbf{T}_{1\text{-}2} \ \mathbf{A}_d^t$$

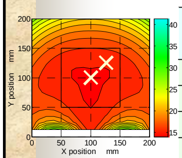
Equations (4) and (6)



Evaluation after calibration (3)

- Example: size measurement between (100, 100) and (150, 150)
  - same level standard deviations 11.5 μm - 14.6 μm for x_1, y_1, x_2 and y_2
  - large positive correlations between x_1 and x_2, y_1 and y_2
- U. of distance measurement
  - simple RSS: 26.1 μm (76% over estimate)
  - with correlations: 14.8 μm

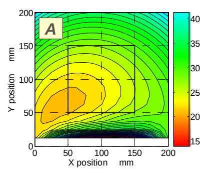
$$sd_{RSS} = \sqrt{14.6^2 + 12.0^2 + 13.9^2 + 11.5^2}$$
$$= 26.1$$
$$sd = 14.8$$

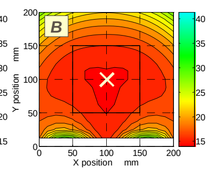


	standard deviation μm	correlation coefficient for		
		y_1	x_2	y_2
x_1	14.6	0.0266	0.4696	0.1091
y_1	12.0	-	0.0332	0.2171
x_2	13.9	-	-	0.1351
y_2	11.5	-	-	-

Evaluation after calibration (4)

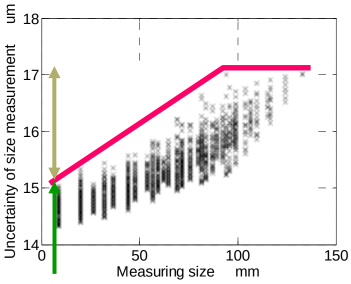
- Distribution-A: U. of a measuring point in machine coordinate system
  - evaluated by T1
  - effected by the selection method of the coordinate system
  - uncertainty of positioning is over estimated
- Distribution-B: U. of size measurement from (100, 100)
  - evaluated by s_d using $T_{1\text{-}2}$ and A_d
  - symmetrical distribution: selection method of the coordinate system and parameters does not influence





Evaluation after calibration (5)

- Relationship between measuring sizes and U. in measuring range after the calibration
  - random U.: 15 μm
  - correlation effects: 2 μm



Conclusions

- In this article, we formulate theoretically the evaluation (3 steps) method of uncertainty of measurements after the calibration of the coordinate measuring system.
- Using this method, we can evaluate the uncertainty in the specified measuring tasks such as size measurement in the workpiece coordinate system.
- Furthermore, we suggest that the uncertainties distribution of size measurements of the coordinate measuring system shows the performance of the coordinate measuring system.

講義の内容

- 01(10/18) :三次元座標測定機の基礎，ハードウェア，ビデオ
- 02(10/25) :トレーサビリティ，歴史，不確かさ(1)
- 03(11/8) :不確かさ(2)ビールジョッキ，電流，ノギス
- 04(11/15) :最小二乗法の入門
- 05(11/29) :線形最小二乗法，非線形，円
- 06(12/13) :信頼性の幅，CMMの不確かさの基礎，円筒の信頼性の幅
- 07(12/20) :未知の系統誤差，校正，形状の相関
- 08(1/10) :三次元メカのアーティファクト校正
- 09(1/17) :冗長三次元メカ
- 10(1/24) :校正後の評価

レポート

- 三次元計測に関連した英語論文を読み，その要約を書く(A4用紙3枚程度)，論文も添付すること.
- 講義の感想を書く
- 締め切り2006年2月20日
- レポートは事務室へ提出