Self-Calibration of CMM with Redundant Degree of Freedom

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Abstract
It is described how to calibrate the kinematic parameters of a parallel CMMs, in this paper. The artefact which consists of spheres is selected as a physical constraints for self-calibration. The number of spheres and the capability of calibration are discussed. The kinematic calibration of the parallel CMM is performed using the artefact of just one sphere. The parallel CMM can measure a sphere in a lot of different orientations so that a lot of sensor information can be got at the identical location. If the number of sensor information is larger than that of whole parameters, e.g. the kinematic parameters and the orientation and location of stylus, the whole parameters can be self-calibrated without any information of artefacts. As an example, the 2D planar parallel CMM was developed and the kinematic parameters of it was self-calibrated.

Introduction
Different type of CMMs from Cartesian CMM have been developed. As the non-Cartesian CMMs, an Articulating CMM and a Parallel CMM, have different structure and sensor inputs, when the parameters of non-Cartesian CMM is calibrated, the method and the artefact are also different from those of Cartesian CMM. Parameter calibration methods were classified into following two methods in (2).
1. The method using redundant information from additional sensors.
2. The method using physical constraints.
The parameter calibration is performed by either method.
In (3), the system has the redundant sensors, the kinematic calibration is performed by first method.
In (4-6), the spheres are used as the artefact, then the kinematic calibration is performed by second method. In (1), the physical constraints are also adapted and the artefact type was classified and considered. As a result in (1), the kinematic parameters of some structure of CMM can be self-calibrated without any calibrated artefact.
In this paper, 2D planar parallel CMM was developed, which has three translational sensors and three degree of freedom. As a result, the kinematic parameters were self-calibrated.

Principle
We considered the parameter calibration of the Cartesian CMM, the articulating CMM and the parallel CMM. The articulating CMM and the parallel CMM have more than three degree of freedom of movement. As a result, those CMMs could
measure the same point in different orientation of stylus. When the artefact, which constrains a position, is used, the orientation of stylus is redundant freedom. Therefore, when a point is measured, only one information is taken in the case of Cartesian CMM. A lot of information is taken in the case of the articulating and the parallel CMM in proportion to the number of orientations of stylus.

In this paper, the physical constrains consist of some spheres, i.e. points. The number of spheres, measurements of each sphere and dispositions of the physical constraints are $n_s$, $n_m$ and $n_d$ respectively. The freedom of position is $n_c$. The number of the parameter is $n_p$. The number of parameter of transformation from coordinate system of physical constraints to world coordinate system is $n_t$.

The number of equations is

$$n_c n_s n_m n_d$$

The number of parameters is

$$n_p + n_d n_t,$$

when the coordinates of spheres are calibrated.

The parameter could be estimated in the case of

$$n_c n_s n_m n_d \geq n_p + n_d n_t$$

![Fig.1 Model of 2D Planar CMM](image-url)
Model of 2D Planar Parallel CMM
The model of 2D planar parallel CMM is shown in Fig.1. The platform, which is a small triangle, is supported by three rod and the stylus is fixed on the platform. The platform has three degree of freedom, translational movement in x- and y-direction and rotational movement around stylus. In Fig.1, Machine coordinate system (X-Y) and Platform coordinate system (U-V) are introduced. The coordinates of stylus is (0,0) in the platform coordinate system. Each supporting rod is connected with a slider with a linear encoder. When the platform moves and rotates, the rod also moves and finally the output from a linear encoder is changed. The location of stylus can be calculated using the output from three linear encoders.

Calibration of Parameter of 2D Planar Parallel CMM
The Parallel CMM is generally modelled as
\[ g(x, p, q, t) = 0 \]  
(4)
x, p, q and t are a generalized coordinate of stylus, parameters, sensor inputs and parameter of coordinate transformation. The parameters p are divided to \( p_r \) and \( p_t \), which are the rotational parameters and the translational parameters. The sensor inputs are also divided to \( q_r \) and \( q_t \), which are the rotational sensor inputs and the translational sensor inputs. The generalized coordinates of spheres are divided to position \( x_r \) and orientation \( x_i \).

\[ g(x_r, x_i, p_r, p_t, q_r, q_t, t) = 0 \]  
(5)
Here, the parameter calibration using just one sphere artefact can be considered. In this case, the sphere has no coordinate information. Therefore Eq.(5) becomes Eq.(6)

\[ kg(x_r, x_i, p_r, p_t, q_r, q_t, t) \neq g(x_r, kx_i, p_r, kp_t, q_r, kq_t, t) \]  
(6)
From Eq.(6), the parameters can be calibrated using just one sphere artefact. In the case of 2D planar Parallel CMM shown in Fig.1, as the CMM does not have any rotational sensor inputs, \( q_r \) is removed. The translational sensor inputs \( q_t \) can not be multiplied by k. As a sphere does not have any orientation information, \( x_r \) is removed.

\[ kg(x_i, p_r, p_t, q_t, t) = g(kx_i, p_r, kp_t, kq_t, t) \neq g(x_i, p_r, p_t, q_t, t) \]  
(7)

Experiment
Fig.2 shows the 2D planar parallel CMM.
The kinematic parameters are\( x_1, x_2, y_2, B_1, B_2, B_3, u_1, v_1, v_2, u_3, v_3, R_1, R_2, R_3 \) as shown in Fig.1. The number of kinematic parameters is 14.
In Experiment, the stylus was fixed at 9 locations of which the coordinates are unknown. In each location, the sensor inputs were detected in 10 different
orientations of platform. The number of whole parameters is $14 + 2 \times 9 + 9 \times 10 = 122$. Meanwhile, the number of sensor inputs are $3 \times 9 \times 10 = 270$. As the number of sensor inputs is larger than the number of whole parameters, these parameters could be estimated.

![Fig.1 Model of 2D Planar CMM](image)

**Summary**
The kinematic parameters of the parallel CMM with translational input sensor are self-calibrated with just one sphere artefact.
It is experimentally proved the self-calibration is possible in the parallel CMM with translational input sensors.

**References**
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