

# ADVANCED ABSOLUTE LENGTH METROLOGY BASED ON PULSE TRAINS' CONSTRUCTIVE INTERFERENCE - TOWARDS MEASUREMENTS OF METER ORDER WITH AN ACCURACY OF NANO ORDER -

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## Abstract:

Measurements of lengths are strongly demanded for not only science purposes but also industry requirements. Because of the great potential for a technological revolution in high precision distance measurement and related areas, femtosecond mode-locked optical frequency combs (FOFCs) have attracted many attentions from researchers and engineers. Due to their improved frequency-stability and the very broad frequency-band, the FOFCs have led to new applications in several precision metrology areas such as high precision optical frequency metrology, spectroscopy, and distance measurements.

The interferometric measurement using the FOFC is in progress at present. We had analyzed the temporal coherence function of the FOFC since it is the fundamental description of interference phenomenon. As a result, it has been understood that the coherence peak exists during the time which is equal to the repetitions interval in the travelling direction of the FOFC.

In order to make the best use of the temporal coherence characteristic, we proposed a high precision length measurement method by the multi-interaction between different pulse train beams. As one demonstration, the concept of the use of a femtosecond optical frequency comb for high precision length measurement based on phase controlled pulse trains' constructive interference is proposed in this paper. The theoretical derivation shows that the present technique offers a significantly different approach to high precision length measurement. Preliminary experimental results of the constructive interference with different pairs of pulse trains show many applications possibility of the proposal method.

**Keywords:** Ultrafast phenomena, Length measurement, Coherency, Interferometry, Metrology

## 1. INTRODUCTION

A meter of the unit of length is defined in the International System of Units (SI) by the selection by General Conference of Weights and Measures (Conférence générale des poids et mesures:CGPM) in 1983 as follows.

"The meter is the length of the distance where light is transmitted in the vacuum at the time of  $1/299,792,458$  of one second."

The following is one method in the achievement of accurate length by using a frequency-well-known monochrome laser.

The wavelength of the laser in the vacuum can be decided by measuring the frequency of the laser because of the relation of  $\lambda = c_0 / f$  ( $c_0 = 299\,792\,458$  m/s). When the laser light is introduced into the Michelson interferometer, and one mirror is shifted with each half wavelength, the interference fringe repeats the light and darkness change. Accurate length can be achieved by comparing the wavelength of the laser and the desired length, and shifting the mirror to a constant amount.

In 2009, a national standard of length in Japan changed from the "iodine stabilized helium-neon (He-Ne) laser" to the "FOFC". For example, see reference [1]. In the future CGPM, the FOFC is expected as a new standard tool of the unit system of "length" and "frequency". Because the traceability of length is the infrastructure for not only science but also industry, and as seen later, a big difference exists between a He-Ne laser and an FOFC in the frequency domain, how to perform displacement metrology which directly linked to an FOFC frequency standard is an important and meaningful challenge.

The outline of this paper is as followings: Section 2 gives a summary of an FOFC light source, the TCF of the FOFC and interference fringes' formation between different pairs of pulse train, Section 3 and 4 investigates simulation and preliminary experiment. Discussion is given in Section 5. And lastly Section 6 presents a summary.

## 2. PRINCIPLES

### 2.1 An FOFC

For convenience of explanation, we start from the description of an FOFC in the time and frequency domain. For further details on the feature of an FOFC, we recommend the reference [2].

The feature of an FOFC can be summarized as follows:

1. In the time domain the "carrier" pulse moves with the center carrier frequency  $f_c$  of the FOFC. When the electric field packet repeats at the pulse repetition period  $T_r$ , the "carrier" phase slips by  $\Delta\phi_{cc}$  to the carrier-envelope phase.
2. In the frequency domain a mode-locked FOFC generates equidistant frequency comb lines with the pulse repetition frequency  $f_{rep} \propto 1/T_r$ , and due to

phase slip  $\Delta\varphi_{cc}$ , the whole equidistant-frequency comb is shifted by  $f_{CEO}$ .

In short, one only need two parameters,  $T_R$  and  $\Delta\varphi_{cc}$ , to describe an FOFC. Naturally, these two parameters are the key point how to stabilize an FOFC.

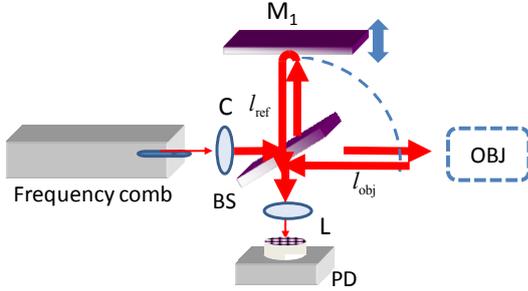


Fig. 1: Optical layout for a modified Michelson interferometer.

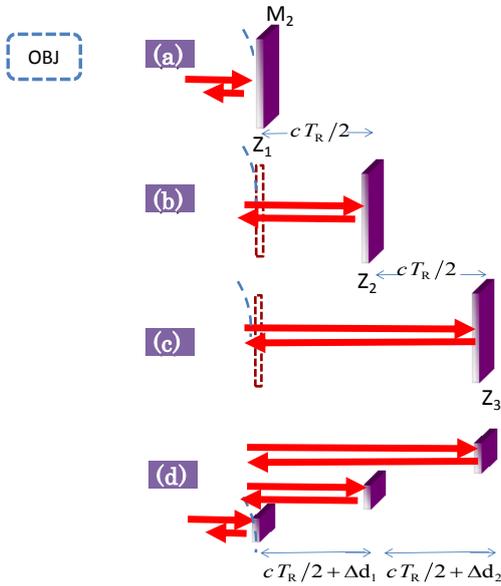


Fig. 2: The structure of the object mirrors. (a) Relative positions at  $Z_1$ . (b) Relative positions at  $Z_2$ . (c) Relative positions at  $Z_3$ , (d) Multiple object-mirrors arrangement.

## 2.2 TCF of an FOFC

We herein briefly review and summarize the most important characteristic of the temporal coherence function of an FOFC, which can be founded in details in Ref [3-4].

Essentially, the temporal coherence function of an FOFC can be derived as follows. The power spectrum of an FOFC light source can be expressed as the multiplication of a comb function and a Gaussian function. Since, the Fourier transform of a comb function is a comb function, and the Fourier transform of a Gaussian function is also a Gaussian function. Based on the Wiener-Khintchine theorem, the interferometric signal of the autocorrelation function is the convolution integral of a comb function and a Gaussian function.

As a result, it is understood that the temporal coherence function periodically displays a high temporal coherence

peak with the pulse repetition period where the pulse trains display a high-intensity peak.

## 2.3 Formation of interference fringes

Next, let us consider the interference fringe pattern formed by a modified Michelson interferometer shown in Fig. 1-2. When the object pulse train relatively delayed to the reference pulse train, as shown in Fig. 2(a-c), and finally overlap in space, as shown is Fig. 3(a-c), as analyzed ahead, we can obtain the interference fringes.

If the object mirrors are arranged as shown in Fig. 3(d), the total interference fringes are then expressed as superposition of the different interference fringes which overlapped in space. We obtain the total interference fringes as

$$I(OPD) \propto \sum_n \sqrt{I_{ref} \times I_{obj_n}} |\gamma(OPD_n)| \cos(2\pi \times OPD_n / \lambda_c), \quad (1)$$

where  $I_{ref}$ ,  $I_{obj_n}$  mainly depend on the reflectivity of the reference mirror and the (n)th object mirror, respectively,  $\lambda_c$  is the central wavelength, and  $OPD_n$ s are the relative optical delays of the (n)th object pulse trains to the reference pulse train.

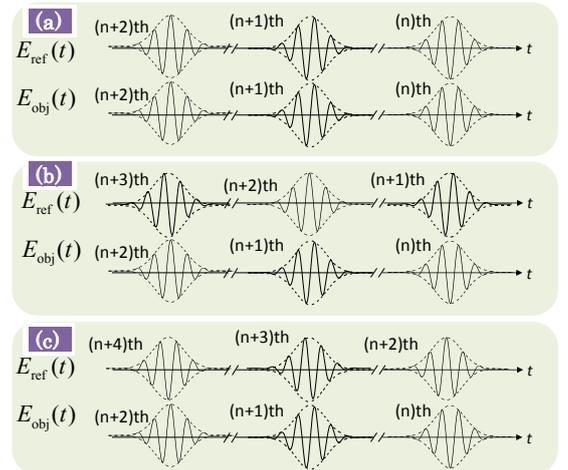


Fig. 3: Relative delays between two pairs of pulse trains. (a) Relative positions at  $Z_1$ . (b) Relative positions at  $Z_2$ . (c) Relative positions at  $Z_3$ .

## 3. SIMULATION

The simulation is carried out with a Gaussian-profile polarization mode locked FOFC. The pulse duration of the FOFC is 180 femtoseconds. The output wavelength of the pulse is centered at  $\lambda = 1550$  nm with a spectral width of  $\Delta\lambda = 20$  nm. The noise was set as random noise with 10% S/N rate.

Figure 4-6 illustrates the result of the simulation.

Figure 4 illustrate the pairs of interference fringe pattern, which are respectively formed between the reference mirror and two different reference mirrors, have the same value of amplitude and move in the same direction, and the slippage of the interference fringe phase to the carrier-envelope between them is same (namely, 0). The result is that the two interference fringes (the red one and the blue one) completely constructive interference when there is complete

overlap (see Fig. 4(b)). Figure 5 illustrate, at the state of destructive interference, almost no interference fringes can be observed (see Fig. 5(b)).

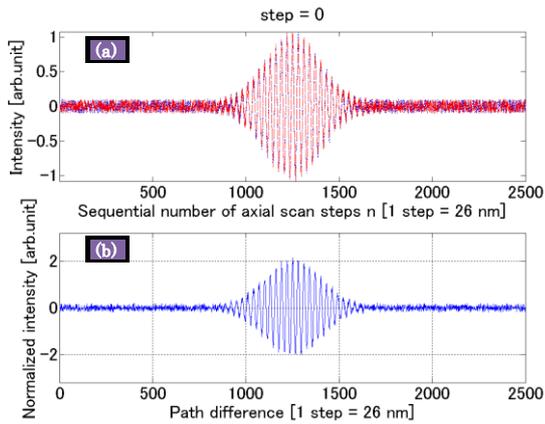


Fig. 4: Constructive interference between two pairs of pulse trains.

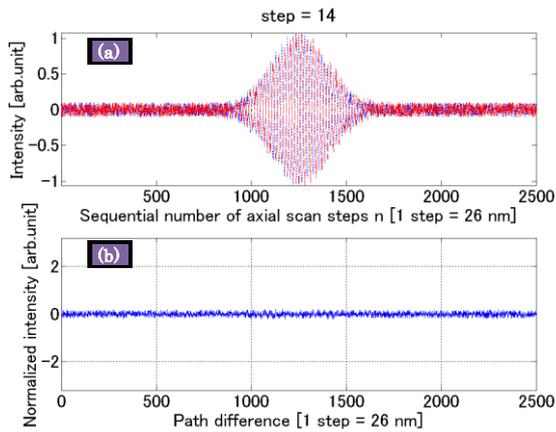


Fig. 5: Destructive interference between two pairs of pulse trains.

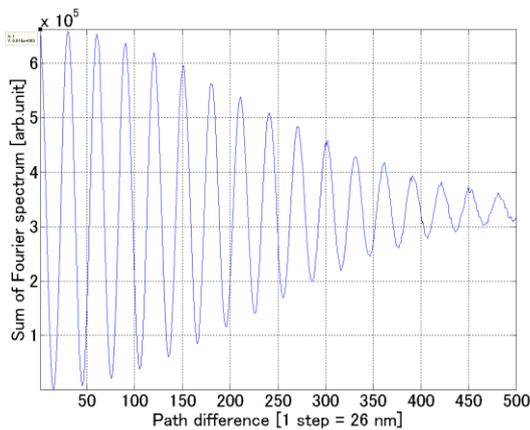


Fig. 6: Interference between two pairs of pulse trains with the different displacements.

Figure 6 illustrate the obtained Fourier spectrum data between two pairs of pulse trains with the different displacements. Table 1 listed the related data at the constructive interference and destructive interference. One can confirm that at the when there is complete overlap of the

constructive interference the sum of the Fourier spectrum of the data's frequency component is the largest.

To obtain the Fourier spectrum of the data frequency component, the following basic steps of the analytical process are used.

1. The obtained interference fringes (as shown in Fig. 4(b). and Fig. 5(b).) are Fourier transformed. The Fourier spectrum of the data frequency component is obtained.
2. The unwanted noise has been filtered out by a band pass filter, and the peak at  $f = +f_{Scan}$  is summed.

Table. 1: Obtained sum of Fourier spectrum at different Path differences

PDiff	0	14	29	44	59	74
SFSpe	66180	1108	65870	6085	65390	21780

PDiff means path difference [1 step = 26 nm].

SFSpe means sum of Fourier spectrum [arb.unit].

#### 4. PRELIMINARY EXPERIMENT

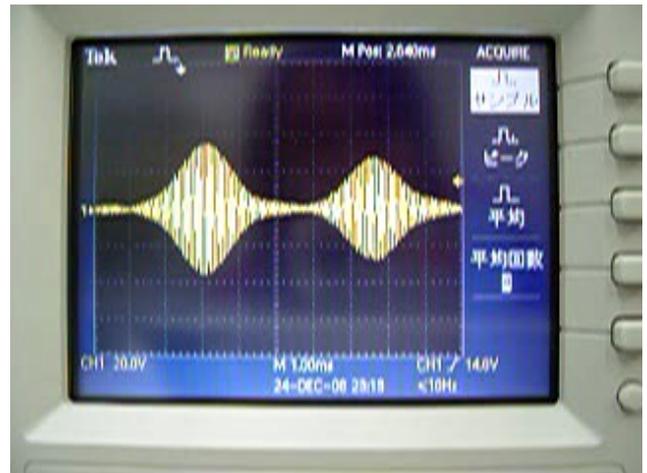


Fig. 7: Observed two peaks of interference fringes when two pairs of pulse trains were separated.



Fig. 8: Observed constructive interference when the two pairs of pulse trains were overlapped.

The experimental setup was modified from Ref. [5], and can be founded in details in Ref. [5].

Figure 7 illustrates the obtained interference fringes when two pairs of pulse trains separated. From the peak values as shown in Fig. 7-8, we can confirm that figure 8 demonstrated the result of the constructive interference when the two pairs of pulse trains were overlapped.

The procedure of data processing is investigating.

## 5. DISCUSSION

Recently, we proposed the concept of advanced optical metrology based on pulse trains' destructive interference, and verified it by the computer simulation [6] and the preliminary experiment [7]. To clarify the same and difference of measurement based on destructive interference and constructive interference, they are compared here.

First, they share the same basic idea. The key point is one can obtain the length information between two separated points by observing the acquired overlapped interference fringes' value.

Second, the control of the carrier phase slippage is required. As understood from Eq. (1), the total interference fringes is a function of the relative optical delay between the two object mirrors and the carrier phase slippage. By fixing the carrier phase slippage to a definite value the relative optical delay, means length information, can be measured according to the value of the acquired interference fringes. In case of destructive interference and constructive interference, the carrier phase slippage is fixed to  $\pi$  and 0, respectively. When the distance between two points is required, it seems that the calculation is the simplest for the both case.

Third, the two methods have the advantage and disadvantage. In case of destructive interference, the pairs of pulse trains cancel each other out. At the state of complete overlap, no interference fringes can be observed. This means the measurement is simple when the noise was very small. On the other hand, for constructive interference, the peak of the interference fringes can be determined by the general envelope detection algorithm of the conventional white-light interferometry. For a well-known envelope detection algorithm, see Ref. [8]. Compared with destructive interference method, constructive interference method requires more time for data analysis. But it is thought that constructive interference method is stronger to the noise.

## 6. SUMMARY

In summary, we have presented the concept of high precision length measurement based on phase-controlled pulse trains' constructive interference from an FOFC. It is important to note that because it was possible to simultaneously observe overlapped high temporal coherence between different pairs of phase-controlled pulse trains, the relative positions of two separated points could be measured with ultra-high precision. The results of this investigation show that, with an appropriate optical system, the proposed

method can be used to achieve our final goal, measurements of meter order with an accuracy of nanometer order. Finally, it is anticipated that the presented method will be a powerful metrological tool for ultra-high precision length measurement.

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