

Femtosecond optical frequency comb's temporal coherence characteristic-based high-accuracy distance measurement

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Abstract—In July 2009, the national standard tool for measuring length in Japan changed from an iodine-stabilized helium-neon (He-Ne) laser to a femtosecond optical frequency comb (FOFC). Owing to the outstanding frequency stability of the FOFC, in the near-future General Conference of Weights and Measures (Conférence générale des poids et mesures: CGPM), the FOFC is expected as a new standard tool for the International System of Units (SI) of the meter. Because the traceability of the meter is the infrastructure for both scientific and industrial uses, how to practically perform a distant metrology that is directly linked to an FOFC length standard tool is the most urgent challenge.

The basic idea of applying an FOFC's temporal coherence characteristic to an absolute length measurement is simple. Motivated by the analogy between monochromatic light source and phase-related multi-wavelength light source, in this paper, a novel interferometric technique is proposed to measure arbitrary length. In the case of a monochromatic light source, using the interference, measuring length based on the wavelength of light is a general method. In the case of an FOFC, using the interference, we propose a method of measuring length based on, instead of the wavelength of light, the repetition interval between the pulse and the pulse from an FOFC.

I. INTRODUCTION

High precision distant measurement based on a femtosecond optical frequency comb (FOFC) was first performed by Minoshima and Matsumoto [1] and has been examined by various authors using different methods [2-4]. As the simplest method, interference method, it has been studied by several groups [5-7]. But those approaches were not so favorable. This research program aims to develop an arbitrary and absolute length measurement method using an FOFC for not only scientific inquiry but also manufacturing necessity.

The combination method (for example, combination of time of flight method and time domain interference method [5]) and the interference method for a discrete length measurement [7] were therefore given priority. More suitable method for an arbitrary and absolute length measurement has now been investigated in this paper, which are based on the analysis of the formation of the interference fringes of different pulse trains, which proves to be more general to those of the other interference methods mentioned.

Compared with other approaches to single FOFC-based interference length measurement, this method being mentioned here has a number of novel features. First, as we shall see in the experiment chapter, this method owns its simplicity. Second, as we shall see in the coming principles chapter, by knowing the phases of the different interference fringes, this method is possible to measure an arbitrary and absolute distance. Finally, one of the great advantages of this method results the measurements which are traceable to national standards of length, because in July 2009 an FOFC was adopted as the national standard tool for length in Japan.

The remains of this paper are arranged as follows. In section 2, we explain the principles, including the temporal coherence function (TCF), the formation of the interference fringes between different pulse trains, etc. And, we describe the concept and implementation of the proposed method. In addition 3, we present the preliminary result of the proposed method and the hurdle in the current optical system. Finally, we give a summary and comment on future works in section 4.

II. PRINCIPLES

In our previous research works [8-17], we partially understood and solved some problems concerning how to measure length using an FOFC. In this chapter we first introduce the necessity and minimum description of an FOFC. In what follows we review and summarize our previously reported studies on the TCF of an FOFC [8], and the formation of the interference fringes between different pulse trains [18]. During this part, we also clarify the relationship between the proposed method and the conventional length measurement methods, such as, frequency scanning method, low coherence interference method, etc. Finally, we mention the key point which is how to express an arbitrary length by the repetition interval between the pulses of an FOFC light source instead of the wavelength of an ordinary light source, for example, a helium-neon (He-Ne) laser.

A. FOFC

For convenience of explanation, we herein briefly give the necessity and minimum description of an FOFC, which can be found in details in Ref. [19].

An FOFC basically is a pulse laser. In addition, it is an

extremely stabilized femtosecond mode-locked pulse laser [20]. An FOFC has several points which are different from an ordinary pulse laser. The feature of an FOFC can be summarized as follows in the time and frequency domain.

In the time domain, as mentioned above, because of the characteristic of a pulse laser, the electric field packet repeats at the pulse repetition period T_R . When the packet repeats, the carrier pulse moves with the center carrier frequency f_c of the FOFC, and the carrier phase slips by $\Delta\phi_{ce}$ to the carrier envelope phase. This stable carrier phase slippage is one feature of the FOFC in comparison with an ordinary pulse laser.

In the frequency domain, a mode-locked FOFC generates almost one million frequency comb lines which are equidistant with the pulse repetition frequency $f_{rep} \approx 1/T_R$. And due to carrier phase slippage, the whole frequency comb lines is shifted by the offset shift frequency f_{CEO} . The second feature of the FOFC in comparison with an ordinary pulse laser is many of the number of the mode. The third feature of the FOFC is the equidistant arrangement between the whole separated modes.

B. TCF of an FOFC

In this part our focus is on the TCF of an FOFC. The deriving of expression of the TCF of an FOFC can be founded in details in Ref. [8].

Herein, we show a simplified explanation. In general, when thinking about the TCF, the Wiener–Khintchine theorem is the basic. According to the Wiener–Khintchine theorem, we have to consider the power spectrum of the light source used. The power spectrum of an FOFC can be expressed as the multiplication of a comb function and a Gaussian function, which can be expressed as

$$P(f) \propto A(f - f_c) \times \text{comb}(f_{rep}), \text{comb}(f_{rep}) \equiv \sum_{m=-\infty}^{\infty} \delta(f - mf_{rep}), \quad (1)$$

where $A(f - f_c)$ is the envelope function of the FOFC power spectrum. Since, the Fourier transform of a comb function is a comb function, and the Fourier transform of a Gaussian function is also a Gaussian function. Based on the Wiener–Khintchine theorem, the interferometric signal of the correlation function is the convolution integral of the comb function and the Gaussian function, and we have

$$\gamma(\tau) \propto F^{-1} [A(f - f_c)] \otimes \text{comb}(T_R), \text{comb}(T_R) \equiv \sum_{m=-\infty}^{\infty} \delta(\tau - mT_R). \quad (2)$$

From Eq. (2), as a result, it is understood that the TCF periodically displays a high temporal coherence peak with the pulse repetition period where the pulse trains display a high-intensity peak.

C. Formation of single interference fringes

The analysis of the formation of the interference fringes between different pulse trains has been shown in Ref. [18]. In the following two parts we summarize the essential expression of the formation of the interference fringes by multiple pulse trains. And we also clarify the relationship between the proposed method and the conventional length measurement methods.

Generally, in order to confirm the TCF of a light source, we simply introduce the light beam from the source into a Michelson interferometer. Firstly we split the incoming light beam into two parts at the beam splitter. We delay one of the dividing two beams relative to the other. And then we recombine both splitting beam at the beam splitter to observe interference fringes. After performing the time integration, in a general case, we obtain the intensity of the recombination optical field at the beam splitter as

$$I(x, y; k, l) = S(x, y; k) \times \{a(x, y) + b(x, y) |\gamma[l(x, y)]| \cos[(k \times l(x, y))]\}, \quad (3)$$

where $S(x, y; k)$ is the spectral intensity of the light source, $a(x, y)$ and $b(x, y)$ are the spatial intensity distributions resulting from the swings of incident light intensity and heterogeneity of mirror reflectance, k is the wave number of the incident light, $l(x, y)$ is the optical path difference between the points of the measured object and reference mirrors, and $|\gamma[l(x, y)]|$ is the correlation function at each point of the measured object.

It is instructive to clarify the relationship concerning the conventional length measurement method, such as, frequency scanning method, low coherence interference method, etc. In the case of the frequency scanning method, we scan the k parameter and obtain $l(x, y)$ height information. In the case of the low coherence interference method, because of the short coherence length of the low coherence source, a one-to-one relationship arises between each height of the object surface and each peak of the fringe-visibility curve. One can obtain the height of the surface $l(x, y)$ corresponding to the location of the peak of the fringe-visibility curve $|\gamma[l(x, y)]|$ along the scanning axis.

The spatial distribution of the light source is also a controllable parameter for length measurement indicated by Takeda *et al.* [21] and applied to measurement by Duan *et al.* [22, 23].

When introducing an FOFC with a Gaussian spectral distribution into the Michelson interferometer, we obtain the intensity of the recombination optical field at the beam splitter as

$$I(l) = a + b \times \exp \left[- \left(2\sqrt{\ln 2} l / L_{\text{coh}} \right)^2 \right] \otimes \sum_{m=-\infty}^{\infty} \delta(l - m \times c \times T_R) \times \cos(k \times l). \quad (4)$$

where $a = I_{\text{ref}} + I_{\text{obj}}$ and $b = 2\sqrt{I_{\text{ref}}I_{\text{obj}}}$; I_{ref} and I_{obj} are the intensities reflected by the reference and object mirrors, respectively. And L_{coh} is the temporal coherence length of one pulse.

Many instances have been reported [2, 5-7, 24-30] which dealt with the single interference fringes. It is instructive to summarize these reported FOFC-based interference instances. The basic idea of those reports is that one can measure the repetition interval of the pulse train from the peaks of two adjacent interference fringes, as mentioned earlier in the TCF part, because different pulse trains can interfere with each other. This technique is particularly useful in measuring discrete length. And the observed interference fringes can be indicated by Eq. (4).

D. Formation of multiple interference fringes

We shall see below that there are two parameters to control an FOFC. The possibility of the length measurement changes by the stability of the two parameters. First, we think what kind of length measurement is possible when both two parameters are constant.

As expressed in Eq. (4), different interference fringes occur between different pulse trains. Thus, the total interference fringes are then expressed as the superposition of the different interference fringes [18]. Since we discuss the length measurement in this work, we only consider the case which has been shown in Fig. 1.

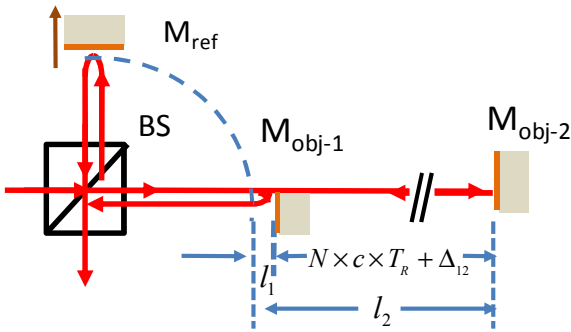


Fig. 1. Multiple-pulse train interference for length measurement.

As shown in Fig. 1, for a given pulse repetition period T_R , the relationship of the distances between the reference mirror and the two object mirrors l_1 and l_2 and the absolute distance between two object mirrors L to be measured can be expressed as

$$\begin{aligned} l_2 &= l_1 + L \\ &= l_1 + (c \times T_R)(N + \varepsilon) \\ &= l_1 + N \times c \times T_R + \Delta_{12}, \end{aligned} \quad (5)$$

where N and ε denote an integer ($N = 1, 2, 3, \dots$) and an

excess fraction ($1 \geq \varepsilon \geq 0$), respectively. As expressed in Eq. (5), to measure the absolute distance L , N and ε should be determined. In the what follows we consider how to determine N and ε .

Note that $\Delta\varphi_{\text{ce}}$ is the phase slippage from pulse to pulse per round-trip length $c \times T_R$. By considering the interference fringes which are resulted in Fig.1, we obtain

$$\begin{aligned} I(l_2) &= a_1 + b_1 \times \exp\left[-\left(2\sqrt{\ln 2} l_1 / L_{\text{coh}}\right)^2\right] \times \cos(k \times l_1) \\ &+ a_2 + b_2 \times \exp\left[-\left(2\sqrt{\ln 2} (l_1 + \Delta_{12}) / L_{\text{coh}}\right)^2\right] \\ &\times \cos(k \times (l_1 + \Delta_{12}) - N \times \Delta\varphi_{\text{ce}}). \end{aligned} \quad (6)$$

Eq. (6) indicates that by moving the common reference mirror M_{ref} , we can observe the multiple pulse trains' interference (MPTI) fringes. In the case when the MPTI fringes are separated, the distance between the peaks of the separated MPTI fringes Δ_{12} and the measured length-related carrier phase slippage of the separated MPTI fringes $N \times \Delta\varphi_{\text{ce}}$ can be presumed from the observed interference fringes. The relationships of the distance between the peaks of interference fringes Δ_{12} , the integer N , and absolute distance between object mirrors L are expressed as Eq. (5). Note that in the frequency domain, the offset shift frequency f_{ceo} and the pulse repetition frequency f_{rep} are the only two key parameters used to stabilize an FOFC. The conclusion drawn from the result of the theoretical analysis, that is, an FOFC can be used for not only an extreme-precision frequency metrology but also a high accuracy distant evaluation, is supported by the well-connected time- and frequency-domain expressions $\Delta\varphi_{\text{ce}} = 2\pi f_{\text{ceo}} / f_{\text{rep}}$ and $T_R = 1 / f_{\text{rep}}$, which can be found in the Ref. [19, 31]. In other words, with the knowledge of the stable values of f_{ceo} and f_{rep} , in the frequency domain, we can perform an extreme-precision frequency metrology, as described in Ref. [19, 31], and in the time domain, we can perform an arbitrary and absolute distant evaluation by calculating N and Δ_{12} from the separated MPTI fringes.

As for the above-mentioned discussion, the assumption is that both parameters, f_{ceo} and f_{rep} , are constant. However, in some cases, only parameter f_{rep} might be stabilized in result. For example, we correspond in the case when FOFC will be sent to each factory through the optical fiber in the near future. Because the length of the optical fiber connected to each factory is different even if the parameter f_{ceo} is stabilized by the dispatching side, the parameter f_{ceo} reaches a value different at each factory. In addition, the parameter f_{ceo} changes by the influences such as the temperature changes, earthquakes, etc. When paraphrasing, the measurement will

be executed when the parameter f_{ceo} is not constant. As indicated in Eq. (6), the displacement measurement is possible; even only one parameter f_{rep} is constant. And the measurement range without ambiguity is $c \times T_{\text{r}} / 2$.

In summary, the main advantages of the proposed technique are its possibility to measure an arbitrary and absolute length.

III. EXPERIMENT

To shed some light on the question of the reliability of proposal method, we perform the following experiment. Owing to restricted equipment availability, we cannot observe the carrier phase slippage $\Delta\phi_{\text{ec}}$ for the measurement of an arbitrary length by stabilizing and changing the offset shift frequency parameter f_{ceo} . Regarding the separated MPTI fringes, only the measurements of relative distance changes are carried out in this experiment. In this chapter we describe the set-up of the experimental framework, the basic steps of the data process and the result of the performed experiment.

A. Set-up

The experimental set-up is simple, and the optical layout is provided in Fig. 2.

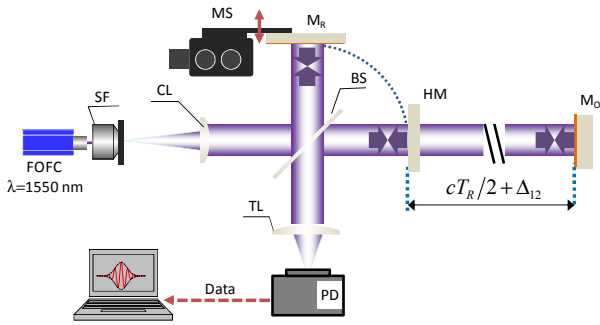


Fig. 2. Optical layout. FOFC: femtosecond optical frequency comb, CL: collimator lens, TL: Tube Lens, BS: beam splitter, PD: photo detector, MS: motorized stage, HM: half mirror, $M_{\text{R-O}}$: mirrors.

The optical system is modified from Ref. [10]. In brief, the proposed executable optical schematic consists of a mode-locked FOFC, a modified Michelson interferometer, and system controls. The pulse trains from the FOFC are expanded and collimated by a collimator lens and introduced into the modified Michelson interferometer. The modified Michelson interferometer is a combination of an ordinary Michelson interferometer and an unbalanced optical-path Michelson interferometer. The ordinary Michelson interferometer is composed of a beam splitter BS, a reference mirror M_{R} , and an object mirror HM. The unbalanced Michelson interferometer is composed of the same BS and M_{R} , and a different object mirror M_{O} to vary the relative delay between the pulse trains, which are reflected by the half mirror HM. During the measurement, by moving the common reference mirror M_{R} of the two interferometers, we could

observe the following two interference fringes, as shown in Fig. 3(a), which are formed for the ordinary and unbalanced Michelson interferometer.

The results for the obtained interference fringes between the different pairs of pulses train are plotted in Fig. 3(a) with normalized amplitude.

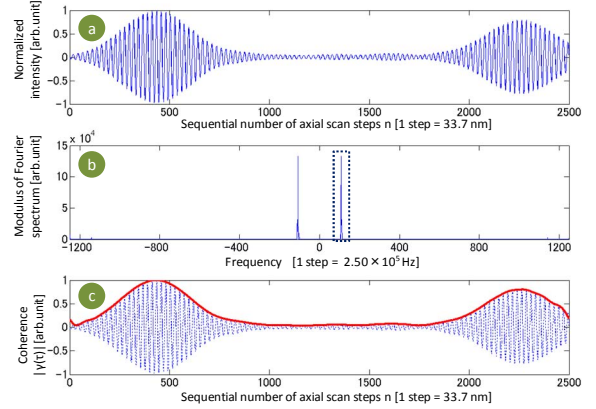


Fig. 3. Procedure of data processing

B. Data process

Figure 3(a)-(c) illustrate the procedure of data processing. To reconstruct TCF from the obtained interference fringes and obtain the related optical distance, in summary, the computation of the analytical process comprises the following steps:

1. The obtained interference fringe is Fourier transformed, as shown in Fig. 3(b).
2. The unwanted noise has been filtered out by a band pass filter, and the peak at $f = +f_{\text{Scan}}$ is inverse Fourier-transformed into the time domain, as shown in Fig. 3(c).
3. The TCF $|\gamma(\tau)|$ can be obtained as the 2 times absolute value of the inverse Fourier-transformed value, as shown in Fig. 3(c).
4. Finally, the distance of related TCF s' peaks is found as performed in the conventional white light interference method.

C. Results

In this section we present the results that have been achieved to date on the length measurement.

The results of the measured lengths and the obtained standard deviations are illustrated in Table 1. There is no relativity between the data set acquired on the different days.

As a result, we confirmed that in the current experimental environment we have achieved the measurement about 1.5 meter with tens of micro meters' standard deviation. In another word, we have identified the level of the environmental noise and turbulence in the current experimental environment. This is the hurdle of the current optical system. Compared to the target measurement

uncertainty, from the above optical experiment, results show that the future effort is necessary to make more stability of the optical experiment system.

TABLE I
LENGTHS AND THE RELATED STANDARD DEVIATIONS MEASURED
AT DIFFERENT TIMES.

	Data set				
	1	2	3	4	5
Length (1.5 [m] +) [μ m]	129	100	-57	-63	-63
Standard deviation [μ m]	5	7	2	4	9

IV. SUMMARY

In closing, this paper describes a single FOFC based method that was developed in order to perform an arbitrary and absolute length measurement.

In the principles chapter, we have introduced the necessity and minimum description of an FOFC. In what follows we have reviewed and summarized the TCF of an FOFC, the formation of the interference fringes between the different pulse trains. During this part, we also have clarified the relationship between the proposed method and the conventional length measurement methods.

In the experiment chapter, we have introduced the proposed experiment set-up and the procedure of data processing. And we have achieved measurement results with a several tens of micro-meter over about 1.5 meter.

The proposed experiment set-up is still in development. From a point of view of the proposed method, we believe our method which offers a lot of potential for arbitrary and absolute length measurement. The clear advantages of the multiple pulse trains' interference based length measurement over the frames in these length measurement methods is its simplicity and possibility to measure an arbitrary and absolute length.

Much work still needs to be done to improve our experiment. Future work will be focused on how to decrease the environmental noise and turbulence in the current experimental environment. We are also currently working on another approach to verify the proposal technique experimentally with an appropriate design of the system to perform an arbitrary and absolute length measurement.

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