

# Space position measurement using long-path heterodyne interferometer with optical frequency comb

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**Abstract:** A heterodyne interference system was developed for position measurement. A stabilized optical-frequency comb is used as the laser source. The preliminary experiment to measure a distance of 22.478 m shows a drift of 1.6  $\mu\text{m}$  in 20 minutes after the temperature compensation. Comparison and frequency shift experiments have been done for a distance of about 7.493 m. The experimental results show that the drift is mainly caused by environmental condition changes and the vibration of the table and floor also has some effects. It was verified that the absolute distance measurement can be realized by fringe scanning and frequency-shifting methods.

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## 1. Introduction

Various interferometers have been proposed to perform long-distance measurement, which is required in many areas of industry for the production and safety evaluation of large, high-accuracy equipment [1]. Recently, optical frequency combs are used in many measurement systems because of its high frequency-stability and high accuracy, which is traceable to the definition of second [2].

An optical comb has tens of thousands of modes within its frequency spectra, and any mode can be expressed by two parameters: the repetition frequency rate,  $f_r$ , and the carrier envelope offset frequency,  $f_{ceo}$ . The frequency uncertainty of these two parameters can be traced with high precision to the frequency standard in use, so the optical comb is often used as a frequency standard for other laser sources [3,4]. On the other hand, the optical-frequency comb can also be used directly as the light source of long-path interferometers, which has prompted various efforts to investigate new absolute distance measurement possibilities that were not possible with conventional light sources [5–15]. For a position measurement system, the availability of the optical comb laser with its highly stabilized repetition period can realize high accuracy at the sub-micrometer level in the measurement of several tens of meters. Since the temporal coherence interferometry of the optical comb is also affected by air turbulence, the heterodyne technique is often considered useful for reducing this effect, because the heterodyne frequency is usually out of the frequency region of air turbulence, which is within a few kilohertz.

In this paper, we propose a new heterodyne interferometer using an optical comb as the light source. Absolute distance position measurement is realized by scanning measurement and shifting the repetition frequency of the optical comb. The experimental setup is used to measure a distance of 22.478 m and is compared with a commercial length measurement interferometer at the distance of 7.493 m. Moreover, the method of repetition frequency shifting is verified by experiment.

## 2. Principle

### 2.1 Temporal coherence interference of optical comb

An optical comb outputs a train of ultra-short pulses, and the time interval between adjacent pulses is  $1/f_r$ , where  $f_r$  is the repetition frequency. An interference measurement system is shown in Fig. 1, and the temporal coherence interference occurs at discrete positions, where two optical comb pulse trains overlap with each other. Since the duration of a pulse is about 100 fs, the coherence length of an optical comb is several tens of micrometers. The envelope peak of the interference fringe appears when the optical path difference (OPD) between the two arms of the interferometer satisfies

$$\text{OPD} = m \cdot c / n f_r, \quad (1)$$

where  $m$  is an integer,  $c$  is the light velocity in a vacuum and  $n$  is the refractive index of air.

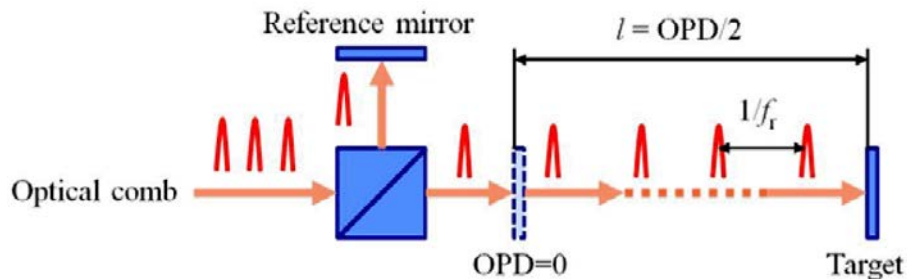


Fig. 1. Unbalanced-arm interferometer; temporal coherence interference occurs at discrete positions

The distance  $l$  between the target and the position where  $OPD = 0$  is  $l = OPD / 2$ . When  $m$  is 1, 2, 3, ..., and  $f_r$  is 100 MHz, the approximate value of  $l$  will be 1.5 m, 3 m, 4.5 m, ..., respectively.

The value of  $m$  can be determined by shifting the repetition frequency  $f_r$ . Since  $l = m \cdot c / 2nf_r$ , the relation between the displacement of the positions where the envelope peaks of interference fringes appear,  $\Delta l$ , and the shifted frequency,  $\Delta f_r$ , will be

$$\frac{\Delta l}{l} = \frac{\Delta f_r}{f_r}. \quad (2)$$

For example, when  $m = 2$ , and  $\Delta f_r / f_r = 10^{-6}$ ,  $\Delta l$  will be 3  $\mu\text{m}$ . Correspondingly, when  $\Delta l$  and  $\Delta f_r$  are known, the value of  $m$  can be determined, and then the absolute distance of the positions being measured will be known.

## 2.2 Heterodyne interference system with optical comb

Figure 2 shows a schematic of the heterodyne interference system with an optical comb, which is based on an unbalanced optical-path Michelson interferometer. The light beam is separated by a splitter, and one of the beams passes through an acoustic-optical modulator (AOM), which is set in the reference arm. The frequency shift of AOM is  $\Delta$ , which can be written as  $\Delta = f_r + f_h$ . Here, the repetition frequency  $f_r$  is 100 MHz, and  $f_h$  is 100 kHz. Since any mode of an optical comb can be expressed as  $f = f_{\text{ceo}} + N \cdot f_r$ , the frequency of the original  $N$ -th mode is shifted to be

$$f'_N = f_{\text{ceo}} + N \cdot f_r + \Delta = f_{\text{ceo}} + (N+1) \cdot f_r + f_h = f_{N+1} + f_h. \quad (3)$$

So the shifted  $N$ -th mode will interfere with the original  $(N+1)$ -th mode of the optical comb, and the heterodyne frequency is  $f_h$  (Fig. 3). A collimator is used to expand the beam diameter for the long distance measurement. Two corner cubes are set in the measurement arm, after the collimator. One is placed on the position where the OPD between two arms equals

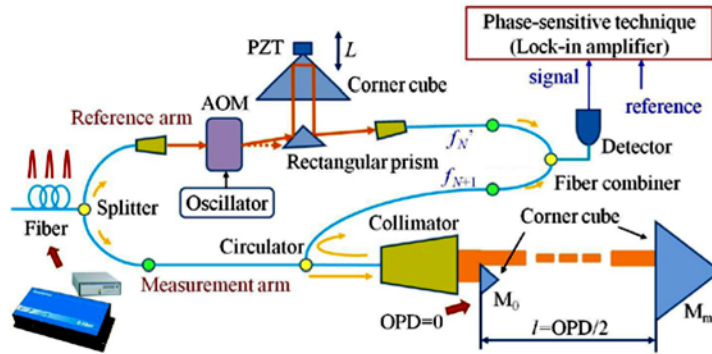


Fig. 2. Schematic of the heterodyne interference system.

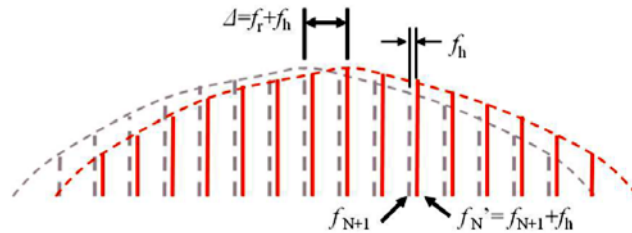


Fig. 3. The frequency shift of the optical comb and the heterodyne frequency  $f_h$ .

zero ( $M_0$ ), and the other is the target ( $M_m$ ). The distance between two corner cubes is  $l$ . The light beams are combined by a fiber combiner, and the interference signal is detected by a photo detector and then sent to a lock-in amplifier. The reference signal of the lock-in amplifier is generated from the difference between the signals of  $f_r$  and  $\Delta$ .

A piezoelectric transducer is set in the reference arm to realize the interference fringe scanning. The scanning period is  $T$ , and the scanning distance is  $L$ . In one period, the corner cube with PZT will be moved far away from the rectangular prism. When  $L$  is several tens of micrometers, the interference fringe generated by the two corner cubes will be detected (Fig. 4). When two peaks of the interference fringe envelopes are found, the time interval between the two peaks,  $t$ , will be got. The measured distance  $l$  can be calculated as

$$l = l_1 + l_2 = \frac{1}{2} m \cdot \frac{c}{nf_r} \pm L \cdot \frac{t}{T}. \quad (4)$$

The sign before  $L$  is determined by the scanning direction of the PZT and the sequence of the two peaks. The former part,  $l_1$ , is the value calculated according to Eq. (1), and the latter part,  $l_2$ , is the measured value.

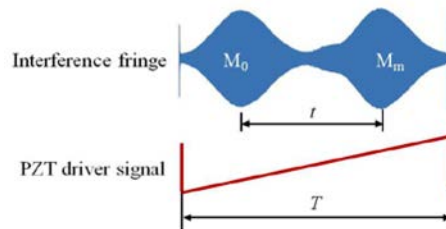


Fig. 4. The driver signal of PZT and the interference fringe envelope.

### 3. Experiments and results

Figure 5 shows the photographs of the heterodyne interference system. Since the interferometer is composed of optical fiber, this system is very useful in practice.

The laser source is a Menlosystems C-Fiber Femtosecond Laser. The center wavelength is 1560 nm, the repetition frequency  $f_r$  is 100.0000 MHz, which is stabilized to a rubidium frequency standard, and the stability is on the order of  $10^{-10}$ . The lock-in amplifier in use is NF LI5630 (frequency < 100 kHz, phase resolution is  $0.01^\circ$ ) and the time constant of 100  $\mu$ s is adjusted for scanning the interference fringe over about 200  $\mu$ m.

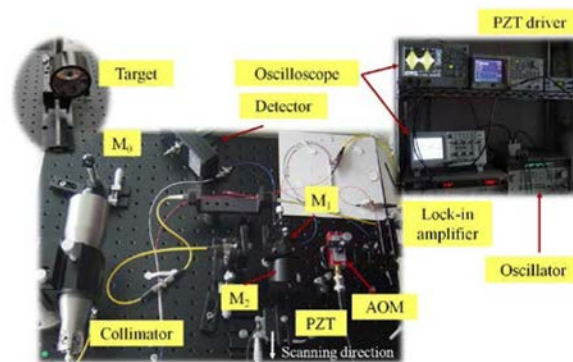


Fig. 5. Photograph of the interference system.

A long-distance-positioning experiment was done to measure a distance of 22.478 m (Section 3.1). A comparison experiment with a Renishaw He-Ne length interferometer involving the measurement of a distance of 7.493 m has also been done (Section 3.2). Then

the method mentioned in Section 2.1 to determine the value of  $m$  was preliminarily proved (Section 3.3).

### 3.1 Positioning measurement experiment of 22.478 m

Two groups of positioning experiments for measuring a distance of 22.478 m ( $m = 15$ ) were done. The scanning distance  $L$  is about 230  $\mu\text{m}$ , and the scanning period is 50 s, so the scanning speed is 4.6  $\mu\text{m/s}$ . The center wavelength of the optical comb is 1560 nm, so the frequency of the interference fringe is about 5.9 Hz, and the half period of one fringe is about 85 ms. Therefore the maximum time constant of the lock-in amplifier is 10 ms. If the time constant is too long, the signal will be weakened. The typical interference signal from the lock-in amplifier is shown in Fig. 6. X-data output and R-data output are used. Since the time constant is selected as 100  $\mu\text{s}$ , the original data include some noise due to the mechanical vibration and air turbulence, and the fitted method is adopted to find the envelope function of the interference fringe, then the locations of the peaks can be determined. The PZT is calibrated by a Michelson interferometer using a stabilized laser diode. A quadratic function is found to describe the relation between the scanning distance, and the driver voltage  $U$ , which is  $L = -0.064U^3 + 1.012U^2 + 18.915U - 6.541$ . When the voltage of the two peaks of the interference fringe envelope,  $U_0$  and  $U_m$ , are known, the distance  $l_2$  can be calculated as

$$l_2 = L(U_m) - L(U_0) \approx L \cdot \frac{t}{T} \approx L \cdot \frac{U_m - U_0}{U}, \quad (5)$$

where  $t$  is the time interval between the two peaks' appearance,  $T$  is the scanning period, and  $U$  is the amplitude of the driver voltage of PZT. The measured distance  $l$  can be several hundreds of micrometers, and the accuracy can be several micrometers or sub-micrometers, which equals the accuracy of  $l_2$ .

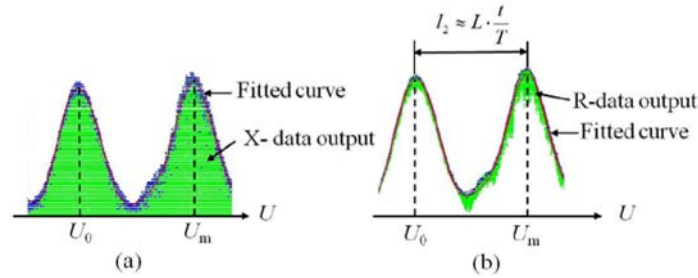
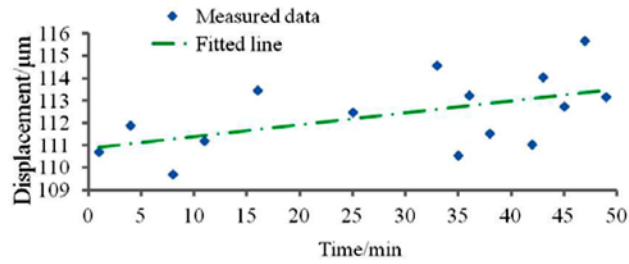


Fig. 6. Interference fringe recorded by lock-in amplifier and the fitted curve. (a) X-data output; (b) R-data output.

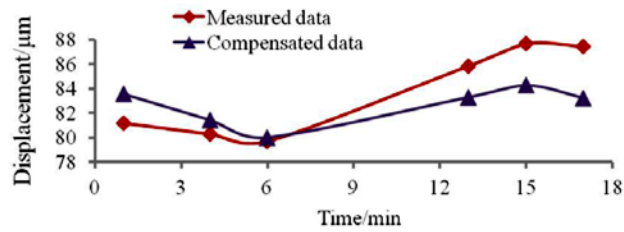
Figure 7(a) shows 15 values measured without air-temperature compensation. The average value of  $l_2$  is 112.40  $\mu\text{m}$ , and the standard deviation is 1.66  $\mu\text{m}$ . The dash-dot line shows the drift in an hour, which is 2.68  $\mu\text{m}$ . To find the cause of the drift, the temperature of environment is recorded and the refractive index of the air is compensated based on the Edlén formula. Figure 7(b) shows the experimental results before and after the temperature compensation. The 6 values were recorded over 20 min. The standard deviation is reduced from 3.7  $\mu\text{m}$  to 1.6  $\mu\text{m}$ , and the drift is reduced from 8.0  $\mu\text{m}$  to 1.8  $\mu\text{m}$  after temperature compensation. This result proves that the drift is mainly caused by changes of the environmental condition, but the deviation is caused by the vibration of stage and floor.

### 3.2 Comparison experiment with Renishaw interferometer

The heterodyne interference system with an optical comb is compared with a Renishaw length-measuring He-Ne interferometer, the wavelength of which is 633 nm. The comparison experiment was done at the distance of 7.493 m ( $m = 5$ ). Since the He-Ne interferometer is an incremental length-measuring system, the experimental result can only express the drift of the measurement.

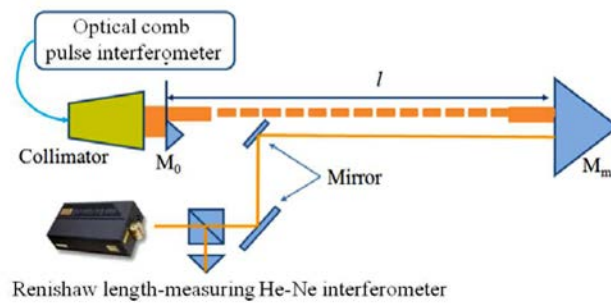


(a)

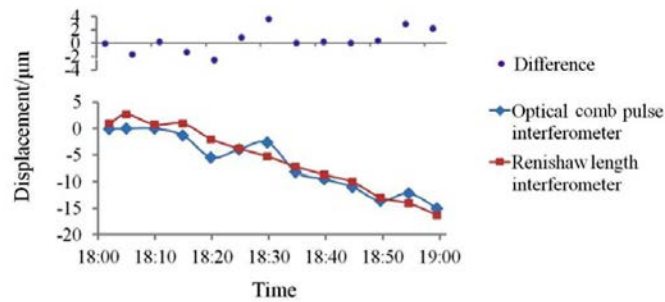


(b)

Fig. 7. Experimental results of 22.478 m measurement. (a) Results in an hour without the air-condition compensation; (b) Results in 20 min with air-condition compensation



(a)



(b)

Fig. 8. Comparison experiment. (a) The schematic of the two interference systems; (b) Experimental results over an hour.

Figure 8(a) shows a schematic of the two interference systems and Fig. 8(b) shows the experimental result. The two systems share the same target, the corner cube prism,  $M_m$ , and almost the same optical path. Thirteen values were recorded in an hour without temperature compensation. The result shows that the measurement value variations of the two length

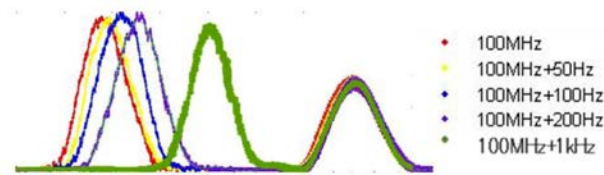
measurement systems have the same drift tendency, and the drift values of the two systems are both 15  $\mu\text{m}$ . The average measured values of the optical comb interference system and the Renishaw system are  $-6.4 \mu\text{m}$  and  $-5.8 \mu\text{m}$ , respectively. However, the standard deviations of these two systems are  $5.3 \mu\text{m}$  and  $6.4 \mu\text{m}$ . The maximum difference between the two curves is  $3.65 \mu\text{m}$ . This result proved that the drift was caused by changes in the experimental condition, as the experimental room is a general office on the 10<sup>th</sup> floor of a building. The data in the Renishaw system are recorded by hand, and there was a time interval of tens of seconds in the recording of the data between the two systems, which may have caused the differences in the results.

### 3.3 Repetition frequency shifted experiment

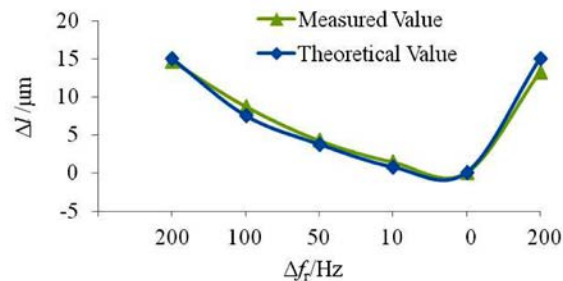
According to Section 2.1, when the repetition frequency  $f_r$  is changed a certain amount  $\Delta f_r$ , the integer  $m$  can be determined by measuring the displacement change  $\Delta l$ . To verify this method, the repetition frequency  $f_r$  is shifted by different values when the integer  $m$  is 5 and the displacement change is measured. Table 1 shows the values of  $\Delta f_r$  and the theoretical value of  $\Delta l$ .

**Table 1. The value of  $\Delta f_r$  and the theoretical value of  $\Delta l$**

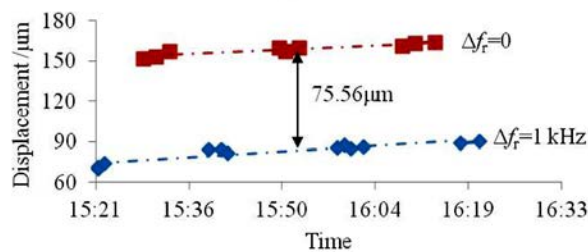
Shifted frequency $\Delta f_r$ / Hz	Theoretical Value $\Delta l$ / $\mu\text{m}$
0	0
10	0.75
50	3.75
100	7.49
200	14.99
1000	74.93



(a)



(b)



(c)

Fig. 9. (a) The interference fringe; (b) The experimental result when  $\Delta f_r$  is changed from 0 to 200 Hz; (c) The experimental results when  $\Delta f_r$  is 0 and 1 kHz.

Figure 9(a) shows the interference fringes for different values of  $\Delta f_r$ . Figure 9(b) shows the measurement result of  $\Delta l$  when  $\Delta f_r$  is changed from 0 to 200 Hz. The green curve is the measured value. The maximum difference between the measured value and the theoretical value is 1.24  $\mu\text{m}$ . Figure 9(c) shows the measured value of  $l_2$  over one hour when  $\Delta f_r$  is 0 and when it is 1 kHz. Although there was a drift during the measurement, the displacement change  $\Delta l$  remained about 75.56  $\mu\text{m}$ , which showed a difference of 0.63  $\mu\text{m}$  from the theoretical value. The experiment proved that the value of  $m$  can be determined by shifting the repetition frequency and measuring the displacement change. The absolute positioning measurement can be realized using this optical comb heterodyne interference system. This method is very useful because  $m$  is uniquely determined by using different  $\Delta f_r$  values according to the measurement error.

#### 4. Conclusions

A new heterodyne interferometer based on the optical comb has been developed using an AOM and a PZT. The experiment to measure a distance of 22.478 m showed a reproducibility of 1.6  $\mu\text{m}$  over an hour after air-temperature compensation, which was  $7 \times 10^{-8}$  of 22.478 m. Since the drift of the measured result of the distance between the two corner cubes was smaller than the drift of the measured result for the position of the target, the effect of the vibration of the table and optical fiber change was reduced by measuring the position where  $\text{OPD} = 0$ . A comparison experiment with the Renishaw length measurement interferometer at a distance of 7.493 m showed the consistency of the two systems, and proved that the drift of the measured data was caused by the experimental environmental condition changes, and the deviation is caused by the vibration of the table and floor. The reproducibility of the system can be improved if these conditions are detailed recorded along the optical path and then the measured results can be compensated more precisely. The method to determine the value of  $m$  was verified by experiment. It was proved that absolute measurement can be realized by this system.

At present, experiments involving the measurement of longer distance of several hundred meters at very stable locations are being planned, and multiple air sensors will be used to record the environmental conditions.

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