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# Nanometer profile measurement of large aspheric optical surface by scanning deflectometry with rotatable devices: Uncertainty propagation analysis and experiments

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ABSTRACT

High-accuracy mirrors and lenses with large dimensions are widely used in huge telescopes and other industrial fields. Interferometers are widely used to measure near flat surfaces and spherical optical surfaces because of their high accuracy and high efficiency. Scanning deflectometry is also used for measuring optical near flat surfaces with sub-nanometer uncertainty. However, for measuring an aspheric surface with a large departure from a perfect spherical surface, both of these methods are difficult to use. The key problem for scanning deflectometry is that high-accuracy autocollimators usually have a limited measuring range less than 1000″, so it cannot be used for measuring surfaces having a large slope. We have proposed a new method for measuring large aspheric surfaces with large slopes based on a scanning deflectometry method in which rotatable devices are used to enlarge the measuring range of the autocollimator. We also proposed a method to connect the angle data which is cut by the rotation of the rotatable devices. An analysis of uncertainty propagation in our proposed method was done. The result showed that when measuring a large aspheric surface with a diameter over 300 mm and a slope of 10 arc-deg, the uncertainty was less than 10 nm. For the verification of our proposed method, experimental devices were set up. A spherical optical mirror with a diameter of 35 mm and curvature radius of 5000 mm was measured. The measuring range of the autocollimator was successfully enlarged by our proposed method. Experimental results showed that the average standard deviation of 10 times measurement was about 20 nm.



## 1. Introduction

Large aspheric mirrors and lenses with high precision of several hundred nanometers are frequently used in modern industry, such as in the mirrors for reflecting telescopes and the brightest X-ray beam-reflecting mirrors. However, the shape measurement of such large aspheric mirrors still faces many problems.

Interferometric methods are widely used when measuring optical flat and spherical surfaces because of their high efficiency and high accuracy [1]. And when the departure from the fitting sphere is too large, accessories such as null lenses and computer-generated holograms (CGH) are applied to make an aspheric wavefront [2,3]. However, if the designed surface is not known before the measurement or the departure from perfect spherical surface is too large, this kind of method does not work. Irregularly shaped surfaces such as a cylinder's surface also cannot be measured by this kind of method.

A scanning deflectometry method with a pentagon prism is proposed to measure near flat optical surfaces as shown in Fig. 1 [4–7]. Angle changes in the surface normal are measured by an autocollimator. Because the pitching error of the linear stage affects the angle result very much, the pentaprism is fixed on the linear stage to eliminate the pitching error of the stage while scanning. As a result, the angle change of the optical surface could be measured accurately. Experiments showed that the uncertainty of this method was less than sub-nanometer when measuring large flats with diameter of 500 mm [6]. However, this method cannot be used to measure non-flat mirrors because the high-accuracy measuring range of the autocollimator is less than 1000″.

We proposed a new method based on scanning deflectometry for measuring a large aspheric optical surface. The method we proposed also uses a high-precision autocollimator and linear stage to scan the angle changes of the optical surface normal. Instead of a pentaprism, two flat mirrors are fixed on the linear stage. By reflecting the laser from the autocollimator twice, those two mirrors have the same effect as the pentaprism of eliminating the pitching error of the linear stage [8]. And one of the mirrors is a rotatable mirror driven by a motorized rotation stage. Therefore, the measuring

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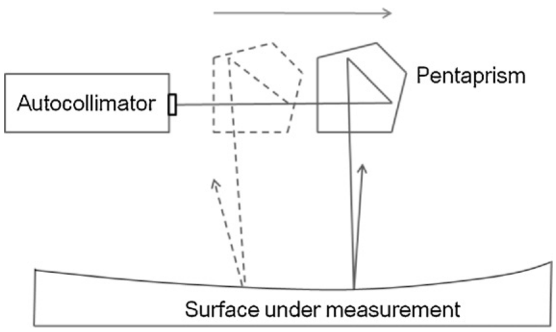


**Fig. 1.** Principle of scanning deflectometry method with pentaprism.

range of the precision autocollimator is enlarged and the new scanning deflectometry method can be used to measure a large surface with a large slope including a large aspheric surface.

In this paper, the principle of the proposed method is introduced, and uncertainty propagation analysis is performed. Furthermore, experimental devices are set up and a spherical mirror with diameter of 50 mm and concave radius of 5000 mm are measured. The average standard deviation of 10 repeated measurements was about 20 nm.

## 2. Principle

The basic principle is shown in Fig. 2. For this method three modules are used: the autocollimator module, the mirror under measurement module and the linear stage module. The autocollimator module and the surface under measurement module are immobile during the measurement. A motorized rotation stage is fixed to the linear stage, along with two mirrors. One mirror is fixed directly to the linear stage and the other to the motorized rotation stage. A laser beam from the autocollimator head is reflected twice by the two mirrors and then by the mirror under measurement. After two more reflections, the laser beam is directed back to the autocollimator head. The angle of the mirror on this light spot is measured by the autocollimator. Then the linear stage moves a certain distance and obtains another angle. When the detected angle comes close to exceeding the measuring range of the autocollimator, the motorized rotation stage turns a certain angle to make the displayed value return. Finally, the angle changes of the surface normal are detected. By taking the integral of the angle data, the profile of the aspheric mirror becomes known.

### 2.1. Connection of angle data

Because of the rotation of the mirror, laser beam turns a certain angle $\alpha$. So that the measured point also moved by $S$ as shown in Fig. 3. As a result, the angle detected by autocollimator is interrupted by rotation as shown in Fig. 4(a). To connect the angle data

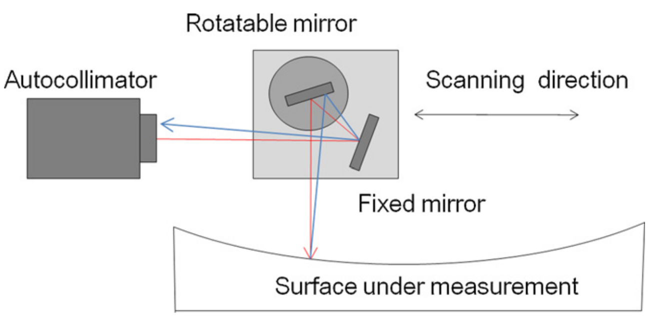


**Fig. 2.** Principle of scanning deflectometry method with rotatable mirror.

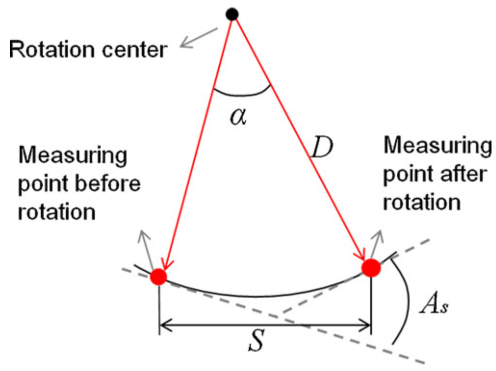


**Fig. 3.** Distance change $S$, according angle change $A$ and rotatable stage turned angle $\alpha$.

we have to know moved distance $S$ caused by rotation and the angle change $A_s$ caused by the distance change as shown in Fig. 4(b).

Because the turned angle $\alpha$ is an angle on the order of several hundreds of micro-radians, the distance $S$ is able to be approximate as the arc length. As a result, if the turned angle $\alpha$ and the distance between the rotation mirror and the mirror under measurement $D$ are known, the missed distance $S$ can be calculated as $\alpha D$. The slope of these two points $k$ is assumed as slope of the least-square line of angle data before the rotation is done as shown in Fig. 4(a). The according angle change $A_s$ is approximated as $kS$. If the accuracy of the motorized rotation stage is sufficiently high, the turned angle $\alpha$ can be known easily. However, the repeatability of a motorized rotation stage with a large range of movement, over 10 arc-degrees, is usually more than 10″. So the turned angle need to be measured by another sensor or calculated indirectly.

We have proposed a method to calculate the rotated angle $\alpha$ of the rotation stage from the data of the autocollimator as follows.

The change of angle data $\beta$ between the data before the rotation and the data after the rotation is measured by autocollimator. And the relationship between $\beta$ and $\alpha$ is shown in Fig. 5.

Because $\theta$ is supposed to be an angle on the order of several hundreds of micro-radians, we can assume the circle arc length of $R\theta$ is the same as the circle arc length of $D\alpha$. As a result, the relationship between $\alpha$ and $\beta$ is deduced as Eq. (1).

$$\alpha = \theta + \beta, \qquad R\theta = D\alpha, \quad \alpha = \frac{R}{R-D}\beta \tag{1}$$

Here $R$ is the curvature radius of the surface in the rotated part.

$\beta$ is read from the angle data shown in Fig. 4(a). $D$ is measured by the caliper with an accuracy of tens of micrometers. And $R$ is the reciprocal of curvature, which is in turn defined by Eq. (2).

$$R = \frac{[1+f'^2]^2}{[1+f'^2]^{1/2} f''} \tag{2}$$

Here $f$ is the profile of the curve. Because of the small scale of the angle data, $f'$ approaches angle $A$ and $f''$ approaches the slope $k$ of the least square line.

$$f' = \tan(A) \approx A \tag{3}$$

From Eqs. (2) and (3), $R$ is calculated as Eq. (4).

$$R \approx \frac{1}{f''} \approx \frac{1}{k} \tag{4}$$

Finally, the angle $\alpha$ that the rotatable mirror turned is calculated from the angle data measured by autocollimator.

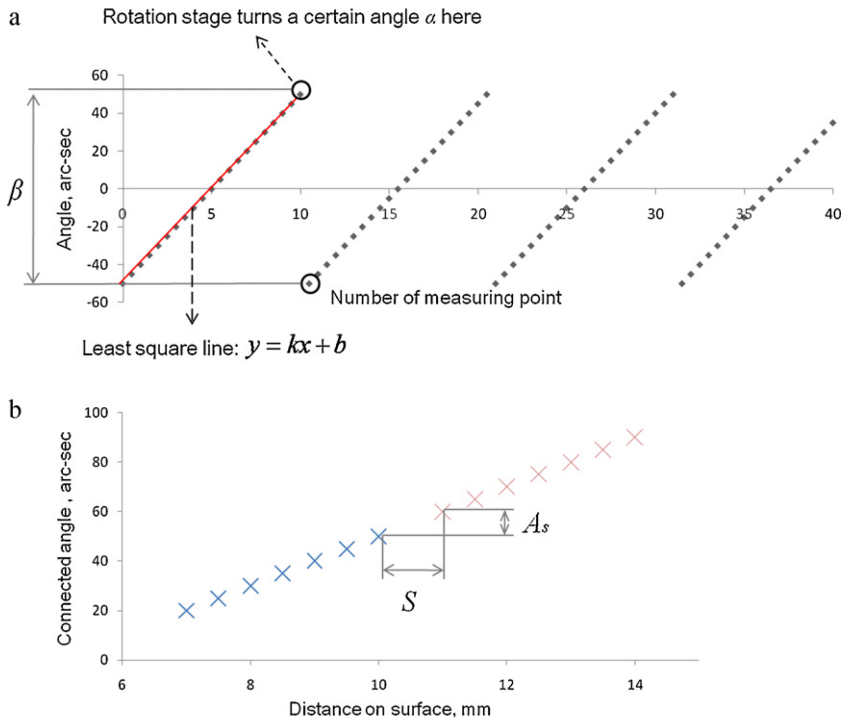


**Fig. 4.** (a) Supposed angle data measured by autocollimator. (b) Connection of two angle data interrupted by rotation.

From Eqs. (1) to (4), we calculate the displacement $S$ and according angle change $A_s$ via Eq. (5).

$$S = \alpha D = \frac{R\beta D}{R-D} = \frac{\beta D}{1-kD}$$
$$A_s = kS = \frac{k\beta D}{1-kD} \quad (5)$$

## 2.2. Integral of angle data

By taking a numerical integration of the connected angle data, we can calculate the profile data $f_i$ of the surface approximately by Trapezoidal rule as shown in Eq. (6).

$$f_i \approx f_{i-1} + h\left(\frac{f'_{i-1} + f'_i}{2}\right) \quad (i = 1, 2, \ldots, n)$$
$$f'_i = \tan(A_i) \quad (6)$$

where $h$ is the scanning interval, $f_i$ is the displacement data of point $i$, and $f'_i$ is the derivative of position $f_i$, which equals the tangent of angle data $A_i$.

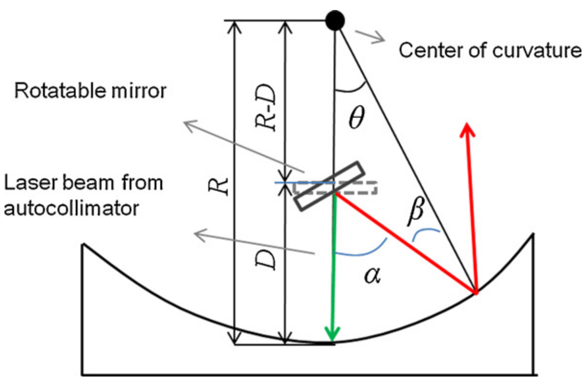


**Fig. 5.** The relationship between $\alpha$ and $\beta$.

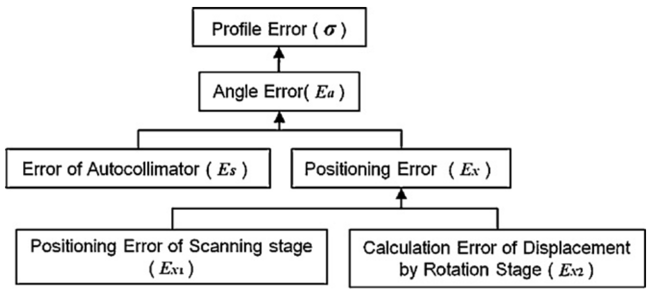


**Fig. 6.** Error propagation of surface profile error.

**Table 1**
Parameters that affect profile error.

| | Parameters | | |
|---|---|---|---|
| | $L$ (mm) | $R$ (mm) | $E_{x1}$ (μm) |
| Value | 300 | 5000 | 5 |

| | Parameters | | |
|---|---|---|---|
| | $E_{x2}$ (μm) | $E_s$ (μrad) | $h$ (mm) |
| Value | 0.024 | 0.5 | 0.1 |

We define the start point of the profile $f_0$ to be zero so that the following profile can be calculated one by one. When the whore profile data is acquired, the slope of the profile also needs a standard to be defined. There are several methods to define the standard of a curve. One is to make the first point $f_0$ and the last point $f_n$ zero, so that the line defined by $f_0$ and $f_n$ is subtracted from the curve. Another method is to make the least square line zero, so that the curve is subtracted from it. In this paper, we use the latter method to define the slope standard.

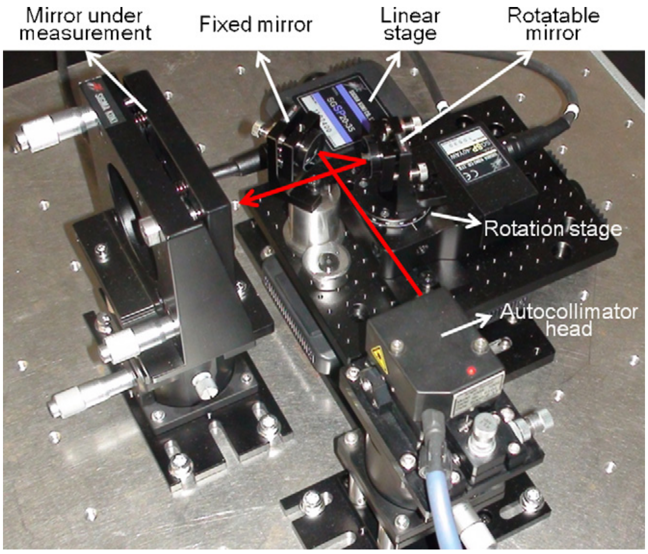


**Fig. 7.** Experimental setup.

## 3. Error analysis

We performed error analysis to check what kinds of factors are sensitive to the measurement result. This is necessary before setting up the experiment devices.

### 3.1. Angle error

Fig. 6 shows the error propagation from the error factors to the final profile error. Profile error $\sigma$ comes from the angle error $E_a$ measured by the autocollimator. The angle error is partly caused by the error of autocollimator $E_s$. The positioning error of the measuring point also cause angle error, which is defined as $E_x$. The positioning error of the linear stage $E_{x1}$ and the calculation error of the displacement of measuring point caused by rotation $E_{x2}$ together determine the positioning error $E_x$ as shown in Eq. (7).

$$E_x = \sqrt{E_{x1}^2 + E_{x2}^2} \tag{7}$$

If the curvature radius of the measuring point is $R$, the derivative of angle $\mathrm{d}A/\mathrm{d}x$ is set equal to $1/R$, which is also considered to be the angle change. Then, if the positioning error is $E_x$, the angle measured will be different by $E_x/R$.

The error of the autocollimator $E_s$ and positioning error $E_x$ lead to the angle error $E_a$ as shown in Eq. (8).

$$E_a = \sqrt{{E_s}^2 + \left(\frac{E_x}{R}\right)^2} = \sqrt{{E_s}^2 + \frac{({E_{x1}}^2 + {E_{x2}}^2)}{R^2}} \tag{8}$$

### 3.2. Propagation from angle error to profile error

The profile data are calculated by numerically integrating the angle data with the method introduced in Section 2.2. At the same time, the angle error $E_a$ is propagated to the profile error $\sigma$ as shown in Eq. (9) [9].

$$\sigma = \sqrt{N}hE_a = \sqrt{hL}E_a \tag{9}$$

where $h$ is the sampling interval; $N$ is the number of sampling points; $L$ is the length of the sample. An example is calculated to show how much the profile error will be when we measure the profile of a concave mirror with 300 mm length and 5000 mm curvature radius. We set the parameters in Eqs. (8) and (9) as shown in Table 1 according to the ordinary experimental devices. The expected uncertainty of the profile measurement is 6.08 nm.

## 4. Experiments

### 4.1. Experimental setup

To verify the proposed method, an experimental setup was built as shown in Fig. 7.

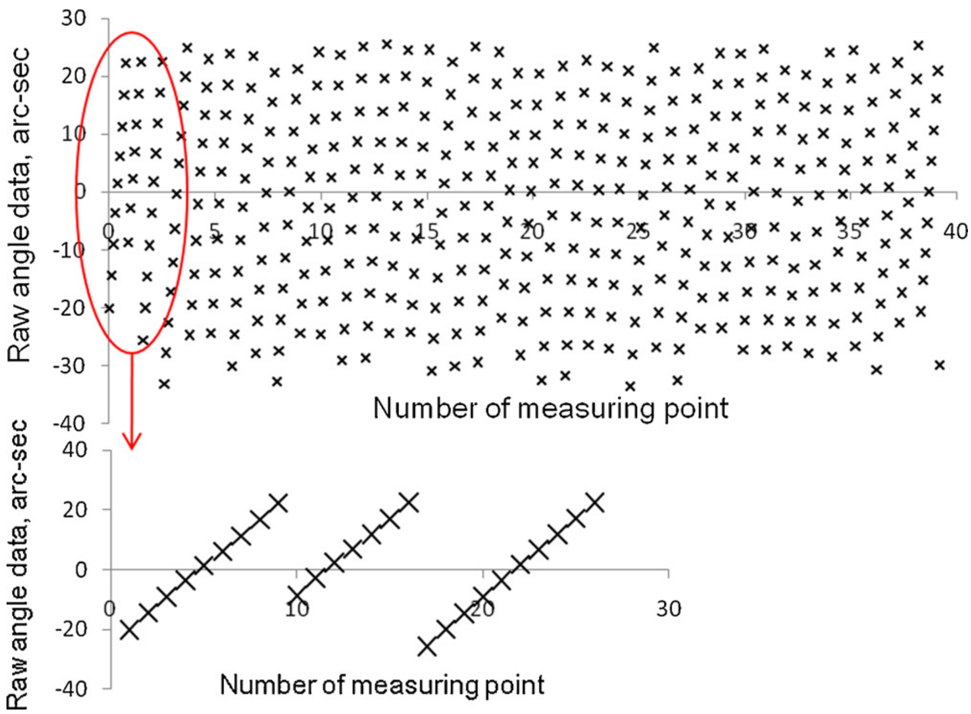


**Fig. 8.** Raw angle data.

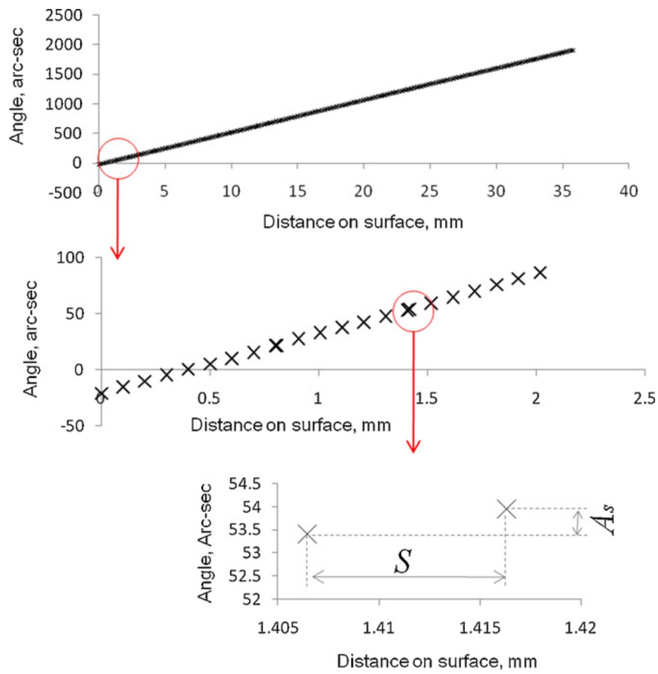


**Fig. 9.** Connected angle data.

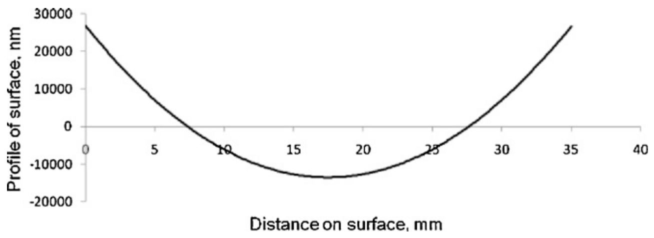


**Fig. 10.** Profile of concave mirror.

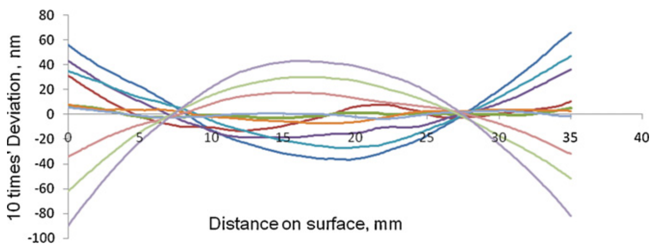


**Fig. 11.** Difference between profile data of 10 times measurement and average data.

The mirror under measurement is fixed vertically in the mirror holder. The linear stage is parallel to the base of the mirror under measurement. A motorized rotation stage with a flat mirror is fixed on the linear stage and another flat mirror is fixed directly on the linear stage. A laser autocollimator is fixed on a vertical manual stage and the direction of the laser from the autocollimator is the same as the moving direction of the linear stage. The vertical manual stage is under the autocollimator so that the height of the laser is adjustable. The laser coming from autocollimator head is reflected twice by the mirrors on the linear stage and then reflected by the mirror under measurement. After being reflected again by the mirrors on the linear stage, the laser beam goes back into the autocollimator.

The measuring range of the autocollimator is ±100″ and the resolution is 0.1″. The moving range of the linear stage is 35 mm and the repeatability of positioning is 3 μm. The moving range of the motorized rotation stage is 360° and the resolution is 0.36″.

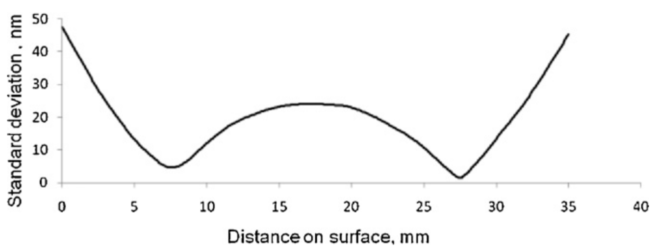


**Fig. 12.** Standard deviation of profile data measured 10 times.

### 4.2. Experiments

An aluminum concave mirror with a diameter of 50 mm and the curvature radius of 5000 mm (±2%) was measured. The mirror under measurement was not aspheric but spherical so that comparison with another method could be done easily in future. Because of the limitation of the moving range of linear stage, 35 mm length of the mirror was measured. The scanning interval $h$ is set as 0.1 mm. And when the detected angle is over 20″, the rotation mirror is turned 20″ back. Because the twice reflection on rotation mirror, the detected angle then is supposed to return 40″.

Fig. 8 shows the raw angle data measured by the autocollimator. The angle was then connected with the method introduced in Section 2.1. Fig. 9 shows the connected angle data. In the first connection position, position change $S$ is 9.8 μm and according angle change $A_s$ is 0.53″. And the angle range of the mirror under measurement is 1927.73″. As the measuring range of the autocollimator is 200″, the measuring range of our method is nearly 10 times of that of the autocollimator.

Then the profile data is calculated by making numerical integral of the connected angle data as Eq. (6) in Section 2.2. The profile data in which the least square line is set as the slope standard are shown in Fig. 10.

The measurement was repeated 10 times, and the profile deviation of 10 times measurement is shown in Fig. 11. The standard deviation of 10 times measurement is shown in Fig. 12. The average of the standard deviation is 20.2 nm.

If we calculate the error with the method introduced in Sections 3.1 and 3.2, the standard deviation of the profile data $\sigma$ should be 4.8 nm. So the experiments repeatability is larger than the error analysis value. From the deviation result, we found that the system error is much larger than the random error. Further research need to be done to find the main key factor that affects the experiment result.

## 5. Conclusion

We proposed a new method based on the scanning deflectometry method with an autocollimator to measure a large aspheric optical surface with rotatable devices. An angle connection method is introduced to connect the discontinuous angle data caused by the rotation. Error analysis is done to estimate the measurement uncertainty. The experiment devices were set up and the profile of a concave mirror with the largest angle change around 2000″ is successfully measured. Repeatability (standard deviation) of 10 times measurement was around 20 nm.

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