

A study of the possibility of using an adjacent pulse repetition interval length as a scale using a Helium–Neon interferometer

Dong Wei^{a,*}, Kiyoshi Takamasu^b, Hirokazu Matsumoto^b

^a Global Center of Excellence Program – Mechanical Systems Innovation, School of Engineering, The University of Tokyo, Japan

^b Department of Precision Engineering, School of Engineering, the University of Tokyo, Japan

ARTICLE INFO

Article history:

Received 14 June 2012

Accepted 29 January 2013

Available online 12 February 2013

Keywords:

Length measurement

Adjacent pulse repetition interval length

Length standard

Metrology

Femtosecond optical frequency comb

Interferometry

ABSTRACT

The possibility of using an adjacent pulse repetition interval length (APRIL) as a scale is investigated. Theoretical analysis showed that an APRIL can be used as a standard for a high-accuracy distant evaluation. In an experiment, an APRIL was measured by using a Helium–Neon interferometer, and the measurement was compared with the result of a direct frequency count. The difference was a few hundred nanometers, and thus the APRIL's effectiveness as a length scale was confirmed. The present concept and analysis pave the way for the development of the remote transfer of APRIL as a length standard via fiber networks.

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1. Introduction

In recent years, the development of femtosecond optical frequency comb-based (FOFC-based) length measurement methods has become of great interest because it has brought a fundamental change in traceability and measurement of length. In 2009, an FOFC was adopted as the national standard tool for measuring length in Japan. This fact increased the importance of the development of FOFC-based length measurement technology and application.

Many FOFC-based length measurement methods have been extensively investigated over the last few years. Mainly, depending on what is used as a length scale, these methods can be divided into two groups. One is the same as the traditional heterodyne method, which is to treat a distance as a function of wavelength [1,2], and the other is to measure a length based on the adjacent pulse repetition interval length (APRIL) [3–5]. The present investigation was an exploration of the latter idea.

As already reported in Ref. [6], we are now developing an easy-to-use method for FOFC-based length measurement. An FOFC can be considered as a coherent combination of thousands of ultra-stable continuous wave lasers. A monochromatic wavelength or an APRIL of a single FOFC can be used as a length reference. In the former case, since the monochromatic wavelength signal of an FOFC is low (usually, nanowatt order), the FOFC can be only used as a wavelength reference light source [1,7,8]. In the latter case, length

measurements can be directly linked to an FOFC [9–11]. Measurement methods by considering the phase shift of stable intermode beats from two FOFCs were reported. However, in this intermode scheme, one needs to synchronize two FOFCs to obtain stable intermode beats. In addition, since there is only one national standard (namely, a single FOFC), how to link this intermode scheme to the national standards is still unknown. Based on these considerations, we prefer to investigate the possibility of an APRIL from a single FOFC light source to be used as a length reference.

Considerable work has been done on long distance measurement using FOFC-based method, but there are few reports on the stability of an APRIL, which is basic for length traceability and measurement when considering APRIL as a scale. The purpose of this study was to evaluate the stability of the APRIL, and to confirm its validity as a length scale. We measured an APRIL by using a Helium–Neon (He–Ne) laser and compared the result with the result of the direct frequency count to confirm the effectiveness of the APRIL as a length scale for the first time.

This paper is organized as follows. First, a theoretical derivation of stability of an APRIL is given in Section 2. Next, an experimental evaluation of stability of an APRIL in air is described in Section 3. Finally, the main conclusions and future work are summarized in Section 4.

2. Principle

For convenience of explanation, let us briefly review the essence of an FOFC. The features of an FOFC can be summarized as follows [12,13]: In the frequency domain, a mode-locked laser generates

* Corresponding author. Tel.: +81 3 5841 6472; fax: +81 3 5841 6472.
E-mail address: weidong@nanolab.t.u-tokyo.ac.jp (D. Wei).

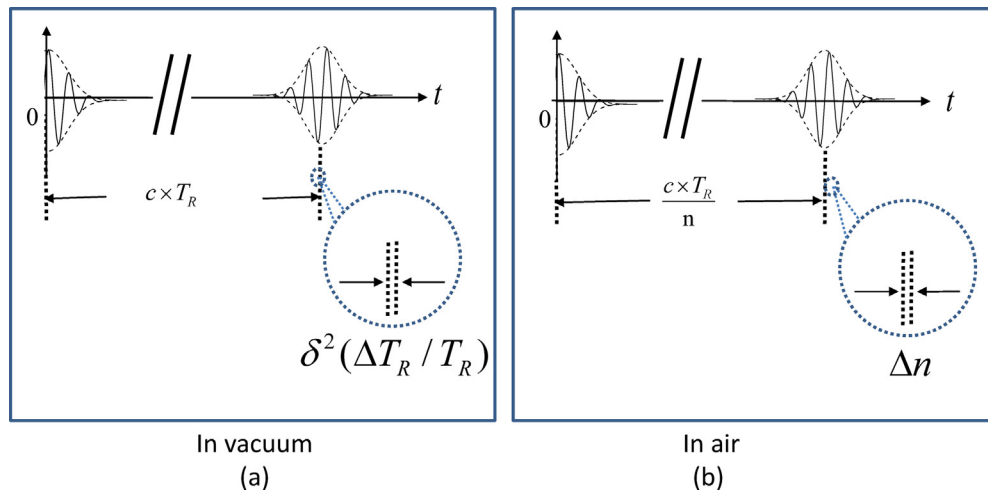


Fig. 1. The stability of an APRIL. (a) In vacuum and (b) in air.

equidistant frequency comb lines with the pulse repetition frequency f_{rep} , and the whole equidistant frequency comb is shifted by offset frequency f_{CEO} from zero frequency. In the time domain, when the electric field packet repeats at the pulse repetition period $T_R = 1/f_{rep}$, due to offset frequency f_{CEO} , the carrier phase slips by $\Delta\phi_{ce} = 2\pi f_{CEO}/f_{rep}$ to the carrier-envelope phase.

APRIL is calculated by $(c \times T_R)/n$, where c is a light velocity in the vacuum and n is the refractive index of light propagated medium. The pulse repetition period, T_R , and the pulse repetition frequency, f_{rep} , are connected by $T_R = 1/f_{rep}$. Based on the theory of uncertainty propagation, we get $\sigma^2(\Delta T_R/T_R) = \sigma^2(\Delta f_{rep}/f_{rep})$. Then, we turn our attention to the frequency domain to obtain an estimate for $\sigma^2(\Delta f_{rep}/f_{rep})$.

For convenience of explanation, we summarize the essence of the stability of the pulse repetition frequency [14]. Currently, the highest absolute frequency stability that can be achieved by an FOFC is about 10^{-18} order [15]. In general, higher harmonics of the pulse repetition frequency are phase locked to a radio frequency (RF) standard [16]. An RF standard can already provide a frequency stability at the 10^{-13} and 10^{-14} levels [17,18]. The pulse repetition frequency can be simply detected by a photodiode. We can conclude that the measurement uncertainty of the pulse repetition frequency $\sigma^2(\Delta f_{rep}/f_{rep})$ is better than 10^{-13} levels. This fact means

the measurement uncertainty of the pulse repetition period can be easily achieved at better than 10^{-13} levels. (Fig. 1(a))

Next, we consider the refractive index of light propagated medium. Because the length measurement in air is general, the discussion is limited to air. In general, Edlén equation [19,20] and Ciddor equation [21] are used to obtain the refractive index of air n . Then the speed of light in air can be calculated as $c_n = c/n$. It is well known that the uncertainty of the Edlén equation (or Ciddor equation) Δn is about 10^{-8} order. (Fig. 1(b))

From the above-mentioned analysis, we can conclude that the APRIL of an FOFC is very steady, and it is suitable as a ruler in vacuum and air.

3. Experiments

3.1. Experimental set up

In this experiment, we compared the value of an APRIL calculated by frequency measurement with that measured using an optical interferometer. The experiment was carried out with a system consisting of a frequency counter system and an optical length measurement system.

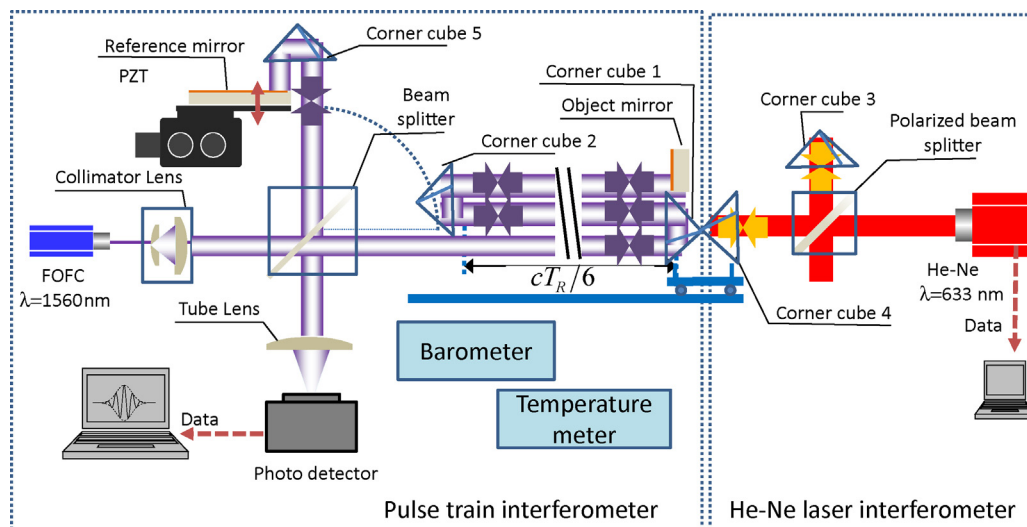


Fig. 2. The experimental set up.

The frequency counter system consisted of a fiber laser, a rubidium frequency standard, a frequency synthesizer, a set of repetition rate synchronization electronics, and a frequency counter. The rubidium frequency standard (Stanford Research Systems, FS725) was used as a frequency standard. The signal from the frequency counter was sent to a frequency synthesizer, and a 10-MHz signal was generated. The 10-MHz signal was sent to repetition rate synchronization electronics (Menlo Systems, RRE100), which controlled the repetition frequency of a polarization-mode-locked femtosecond fiber laser (Menlo Systems, FC1500). The repetition frequency of the fiber laser was observed with the frequency counter (Iwatsu, SC-7206), and it was steady at 100,000,000.0 MHz for several days. (The integration time of the frequency counter was 10 s.)

With the knowledge of the repetition frequency, we can calculate the APRIL as c/f_{rep} , which is 2.99792458 m.

The optical length measurement system consisted of a pulse train interferometer and a He-Ne interferometer. The optical experimental setup was similar to the system described in [4]. Its optical schematic is illustrated in Fig. 2.

The pulse train interferometer consisted of the fiber laser, a Michelson interferometer, and the system control. The pulse duration, repetition rate, and total output power of the fiber laser were 180 fs, 100 MHz, and 20 mW, respectively. The output wavelength of the pulse was centered at 1560 nm with a bandwidth of 20 nm.

The pulse train from the fiber laser was expanded and collimated by a collimator lens and introduced into an ordinary Michelson interferometer, which is composed of a beam splitter, a reference mirror (via corner cube 5), and an object mirror (via corner cube 1 and corner cube 2). The object mirror and the corner cube 1 were fixed on a 600-mm-long translation stage (IKO Nippon Thompson, LT100CDMF-600/5DE127). When the translation stage was at the end near the beam splitter, we could observe pulse train interference with its own signals. When the translation stage moved to the far end of the interferometer, we could observe the interaction of multiple pulse train interferences. The Michelson interferometer thus became an unbalanced optical-path Michelson interferometer.

After traveling different arms of the Michelson interferometer, two pairs of pulse trains sequentially overlapped at the beam splitter. A tube lens imaged the interference fringes onto a photo detector (PD; New Focus, Inc., Front-end optical receivers Model 2011). The intensity of the interference fringes signal through the PD was measured with a digital oscilloscope (Tektronix, Inc., TDS2002B,) and sent to a computer.

4. Experiments

The displacement of the translation stage was measured by a He-Ne laser interferometer (Renishaw, ML10 Laser). The specification of the vacuum wavelength of the He-Ne laser was $0.6329905770 \mu\text{m} \pm 0.1 \text{ ppm}$. The calibration (by Renishaw plc) shows that the stability of the wavelength is better than $\pm 0.01 \text{ ppm}$. The He-Ne laser interferometer is composed of a polarized beam splitter and two corner cubes, labeled corner cube 3 and corner cube 4. The corner cube 4 and the corner cube 1 are connected, and their axes are arranged in a straight line.

Before obtaining the data, we examined the influence from the vibration by the servomechanism of the stage and scanning of the reference mirror using the He-Ne laser interferometer. A typical data set is shown in Fig. 3. The capture rate of data was 500 Hz, and the acquisition time was 341 s. The following three facts can be understood from this data. First, the stage always vibrated by about $\pm 50 \text{ nm}$ due to the experimental condition.

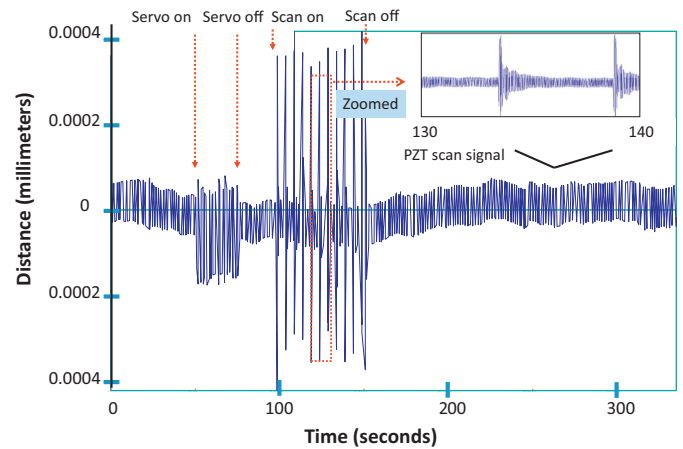


Fig. 3. The stability of the stage and vibration introduced by the servomechanism of the stage and scanning of the reference mirror.

Second, the vibration introduced by the servomechanism of the stage and scanning of the reference mirror was about $\pm 100 \text{ nm}$ and about $\pm 400 \text{ nm}$, respectively. Third, no large shift of the stage due to these two kinds of vibration has been observed. In other words, after receiving the vibration, the stage returns to its former location.

In the experiment, the measurement was done in two steps. In the first step, the object mirror was positioned at the far end of the translation stage so that cross correlation could be observed. We scanned the reference mirror of the pulse train interferometer and recorded the interference fringes (3 times) (see Fig. 4(c-1)). At this time, the measurement value of the He-Ne laser interferometer was set to 0 and the record started. In the second step, the stage was moved to the position in which the autocorrelation could be observed. We scanned the reference mirror again to record the interference fringes (3 times) (see Fig. 4(c-2)). The record by the He-Ne laser interferometer ended after a few minutes.

Fig. 4 shows a typical data set recorded by the laser interferometers. One measurement took about 6 min. We can confirm the vibration introduced by scanning of the reference mirror before (see Fig. 4(a)) and after (see Fig. 4(b)) the operation of the stage. We obtained the corresponding temperature data and atmospheric pressure data by knowing the time of this vibration.

To calculate the refractive index of air, we recorded the temperature and the atmospheric pressure while measuring interference data. The vibration of the stage was about $\pm 50 \text{ nm}$, and in comparison to the APRIL, it is $\pm 0.03 \text{ ppm}$ ($= \pm 5 \times 10^{-8}/1.5$). We did not need to measure the humidity of air or the concentration of carbon dioxide, because the calculated refractive index would only change -0.07 ppm and 0.015 ppm by a 10% change of the humidity and a 100-ppm change of the carbon dioxide density. The influence by both is about the same as the order of the influence due to the vibration of the stage.

We analyzed the obtained interference fringes by the Fourier transform method to get the envelope, as described previously [22]. To find the peak of the envelope we used the differentiation of the obtained envelope curve. The length of the APRIL measured in air was obtained by subtracting the displacement of the gap between two peaks (autocorrelation and cross correlation) from the displacement by the He-Ne laser interferometer (see Fig. 5). The refractive index of air was calculated by using the temperature and the atmospheric pressure. The length of the APRIL in the vacuum was calculated by using the calculated refractive index of air and the measured length of the APRIL in air.

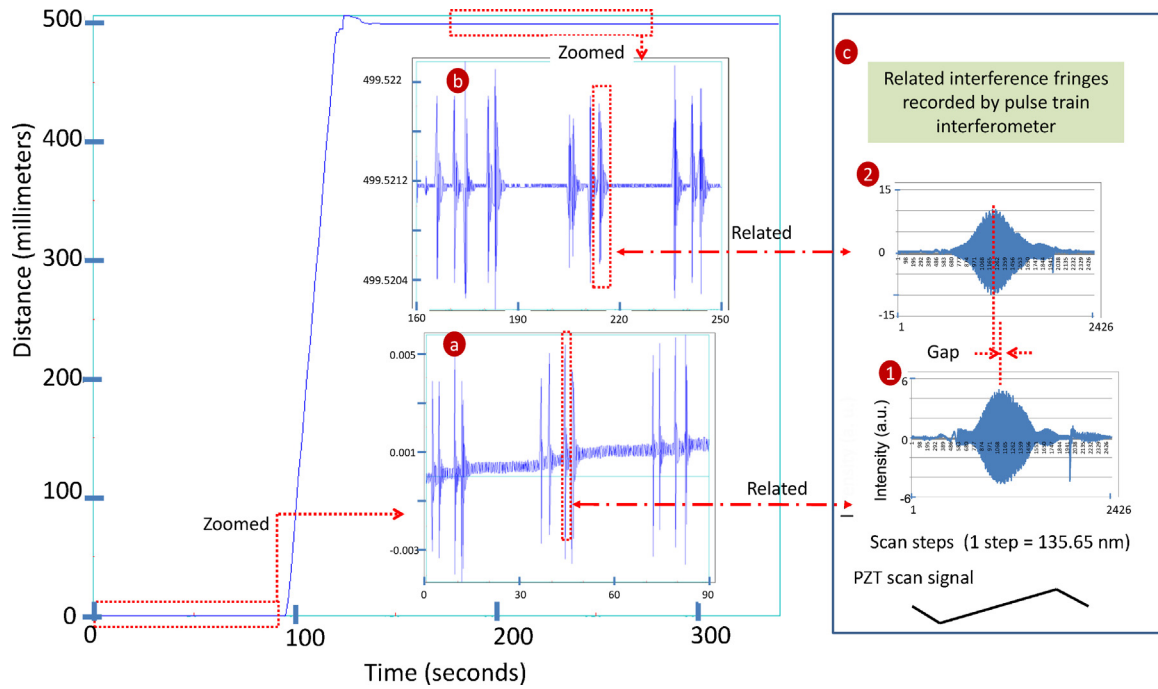


Fig. 4. Displacement recorded by the He-Ne laser interferometer and related interference fringes recorded by a pulse train interferometer.

5. Measurement result

The measurement result of the length of the APRIL in the vacuum is shown in Fig. 6. The measured average length of the APRIL in vacuum was 2.99792471 ± 0.0000040 m. The difference between the result measured by optical interferometer and the calculation by a frequency counter was a few hundred nanometers. Environmental conditions such as non-uniform temperature limited the experimental accuracy. A detailed discussion about complete quantitative error estimation for the experiment will be reported in another paper. Theoretically, this laser system was capable of measuring an APRIL up to ± 0.03 ppm. Further improvements on the experiment

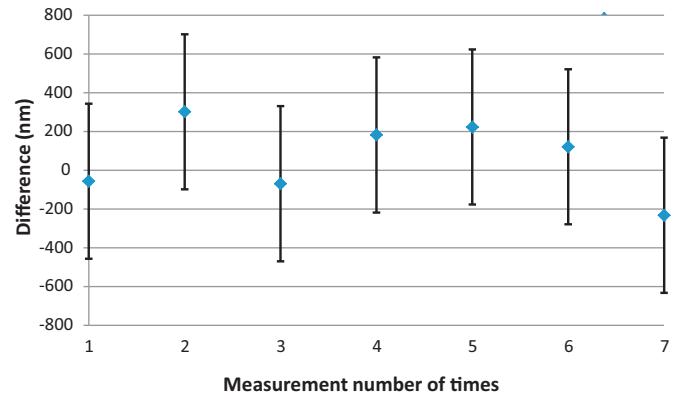
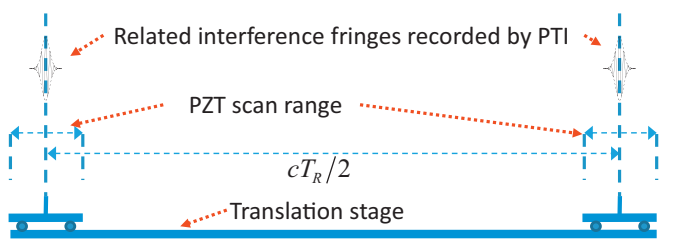
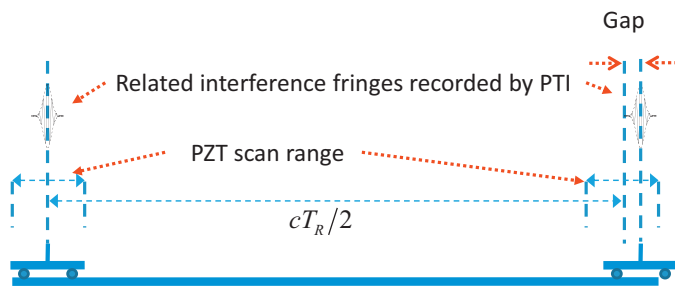


Fig. 6. Measuring results.



(a) A measured displacement is equal to an APRIL.



(b) A measured displacement is the sum of an APRIL and a gap.

Fig. 5. Measuring method.

environment should be promising, enabling the measurement of a 3-m APRIL over 100 nm.

6. Conclusion

We studied the stability of adjacent pulse repetition interval length (APRIL). The results show that APRIL has the same degree of stability as wavelength in air. The theoretical derivation of the expected stability of the APRIL agrees with the results of the simple proof-of-the-principle experiment using a He-Ne interferometer. To our knowledge, this is the first report regarding the stability of the APRIL. We showed the evidence of why an APRIL is suitable as a length scale for the first time. Based on this new understanding, new APRIL-based applications can be proposed fairly readily. For example, compared with the wavelength, APRIL does not receive the influence of nonlinear effect, and the delivery of length standard and traceability of length via fiber networks are considerable.

In future work, by making the best use of the unique characteristic of APRIL, we plan to test the idea of remote transfer of APRIL as a length standard over an optical fiber network.

Acknowledgments

This research work was financially supported by a Grant-in-Aid for Young Scientists (Start-up) (23860013), the “Development of System and Technology for Advanced Measurement and Analysis” Program at the Japan Science and Technology Agency (to H. M.) and the Global Center of Excellence Program on “Global Center of Excellence for Mechanical Systems Innovation” granted to the University of Tokyo, from the Japanese Government, respectively.

References

- [1] Minoshima K, Matsumoto H. High-accuracy measurement of 240-m distance in an optical tunnel by use of a compact femtosecond laser. *Applied Optics* 2000;39:5512–7.
- [2] Yokoyama S, Yokoyama T, Hagihara Y, Araki T, Yasui T. A distance meter using a terahertz intermode beat in an optical frequency comb. *Optics Express* 2009;17:17324–37.
- [3] Ye J. Absolute measurement of a long, arbitrary distance to less than an optical fringe. *Optics Letters* 2004;29:1153–5.
- [4] Cui M, Zeitouny MG, Bhattacharya N, van den Berg SA, Urbach HP, Braat JJM. High-accuracy long-distance measurements in air with a frequency comb laser. *Optics Letters* 2009;34:1982–4.
- [5] Wei D, Takahashi S, Takamasu K, Matsumoto H. Time-of-flight method using multiple pulse train interference as a time recorder. *Optics Express* 2011;19:4881–9.
- [6] Wei D, Takamasu K, Matsumoto H. Pulse repetition interval-based excess fraction method for an arbitrary and absolute distance measurement using a femtosecond optical frequency comb. *Frontiers in Ultrafast Optics: Biomedical, Scientific, and Industrial Applications* 2012;XII (SPIE):82470K–11.
- [7] Coddington I, Swann WC, Nenadovic L, Newbury NR. Rapid and precise absolute distance measurements at long range. *Nature Photonics* 2009;3:351–6.
- [8] Yasui T, Kabetani Y, Ohgi Y, Yokoyama S, Araki T. Absolute distance measurement of optically rough objects using asynchronous-optical-sampling terahertz impulse ranging. *Applied Optics* 2010;49:5262–70.
- [9] Matsumoto H, Wang X, Takamasu K, Aoto T. Absolute measurement of baselines up to 403 m using heterodyne temporal coherence interferometer with optical frequency comb. *Applied Physics Express* 2012;5:046601.
- [10] Narin C, Satoru T, Kiyoshi T, Hirokazu M. A new method for high-accuracy gauge block measurement using 2 GHz repetition mode of a mode-locked fiber laser. *Measurement Science and Technology* 2012;23:054003.
- [11] Wang X, Takahashi S, Takamasu K, Matsumoto H. Space position measurement using long-path heterodyne interferometer with optical frequency comb. *Optics Express* 2012;20:2725–32.
- [12] Helbing F, Steinmeyer G, Keller U. Carrier-envelope offset phase-locking with attosecond timing jitter. *IEEE Journal of Selected Topics in Quantum Electronics* 2003;9:1030–40.
- [13] Ye J, Cundiff ST. *Femtosecond optical frequency comb: principle, operation and applications*. New York, NY: Springer; 2005. pp. xii, 361 p.
- [14] Wei D, Matsumoto H. Measurement accuracy of the pulse repetition interval-based excess fraction (PRIEF) method: an analogy-based theoretical analysis. *Journal of the European Optical Society: Rapid Publications* 2012;7:12050.
- [15] Nakajima Y, Inaba H, Hosaka K, Minoshima K, Onae A, Yasuda M, et al. A multi-branch, fiber-based frequency comb with millihertz-level relative linewidths using an intra-cavity electro-optic modulator. *Optics Express* 2010;18:1667–76.
- [16] Ye J, Jones R, Chen L, Holman K, Jones D. In: Karshenboim S, Peik E, editors. *Applications of femtosecond laser comb to nonlinear molecular spectroscopy astrophysics, clocks and fundamental constants*. Heidelberg: Springer Berlin; 2004. p. 275–95.
- [17] Nelson LM, Levine J, Hetzel P. Comparing primary frequency standards at NIST and PTB. *Frequency Control Symposium and Exhibition. Proceedings of the 2000 IEEE/EIA International*; 2000. p. 622–8.
- [18] Ye J, Peng J-L, Jones RJ, Holman KW, Hall JL, Jones DJ, et al. Delivery of high-stability optical and microwave frequency standards over an optical fiber network. *Journal of the Optical Society of America* 2003;B 20:1459–67.
- [19] Bengt E. The refractive index of air. *Metrologia* 1966;2:71.
- [20] Birch KP, Downs MJ. An updated Edlén equation for the refractive index of air. *Metrologia* 1993;30:155.
- [21] Ciddor PE. Refractive index of air: new equations for the visible and near infrared. *Applied Optics* 1996;35:1566–73.
- [22] Wei D, Takahashi S, Takamasu K, Matsumoto H. Analysis of the temporal coherence function of a femtosecond optical frequency comb. *Optics Express* 2009;17:7011–8.