

Spatial positioning measurements up to 150 m using temporal coherence of optical frequency comb

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ABSTRACT

A heterodyne interference system using an optical frequency comb has been developed for spatial positioning measurements and applied to long distances up to 152.85 m. The laser source is a Rb-stabilized optical-frequency comb, and temporal coherence interference occurs at discrete spatial positions, where two optical comb pulse trains in the different arms overlap. A piezoelectric transducer scans to find the peak of the interference fringe envelope corresponding to the position. The measurement of the absolute position is accomplished by shifting the frequencies of the optical comb by 100.1 MHz. The experimental results show that the measurement reproducibility is no more than 1.4 μm for distances up to 152.85 m and that the accuracy is 1.1 μm for a distance of 50.951 m.

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1. Introduction

Recently, optical frequency combs (optical comb) are considered as a useful tool in many measurement systems [1] because of their high frequency-stability and high frequency-accuracy, which is traceable to the definition of the second [2]. The optical comb can be used both as a frequency standard for other laser sources and as the light source of a long-path interferometer, which has prompted various efforts to investigate new absolute distance measurements that are not possible with conventional light sources [3,4]. The optical comb can work with other continuous wavelength laser sources [5,6], and the temporal coherence interference of the optical comb is also used directly for distance measurements [7–14]. The pulse interference of the optical comb is affected by air turbulence, but the heterodyne technique is useful for reducing this effect because the heterodyne frequency is usually outside the air turbulence frequency region, which extends to a few kilohertz [15,16].

In this paper, we propose a new heterodyne interferometer that utilizes an acoustic optical modulator (AOM) and an optical comb as the light source. Absolute position measurement using the temporal coherence interference for discrete distances is realized by scanning the heterodyne interference fringe with a phase-sensitive detector. Here, the experimental setup is used to measure distances of 50.951 m, 101.902 m, and 152.853 m. The stability, accuracy,

and resolution of the developed system are evaluated through long-duration measurement, short-duration measurement, and experiments in which the repetition frequency is shifted. The reproducibility is 1.4 μm and the accuracy is 1.1 μm for the distance of 50.951 m.

2. Optical frequency comb

Fig. 1 shows the frequency modes in an optical frequency comb [2], and all the mode lines are located at a certain frequency interval, which is the repetition frequency f_r . Therefore, the N th mode of an optical frequency comb can be described by f_r and the carrier envelop offset frequency f_{ceo} as

$$f_N = f_{\text{ceo}} + N \cdot f_r, \quad (1)$$

where N is an integer. The frequency of any mode in an optical comb can be stabilized by a high-accuracy frequency standard through stabilization of f_r and f_{ceo} .

In the time domain, an optical comb outputs an ultrashort pulses sequence, and the spatial distance between two pulses will be cf_r , where c is the speed of light in a vacuum. For an interference system using an optical comb as the laser source (Fig. 2), the laser beams of the reference arm and the measurement arm are combined by the beam combiner and temporal coherence interference occurs only when the pulses of the arms overlap.

Since the pulse length is usually less than 150 fs, the length of an envelope of interference fringes is tens of micrometers. According to Ref. [12], the temporal coherence peak appears at the same time

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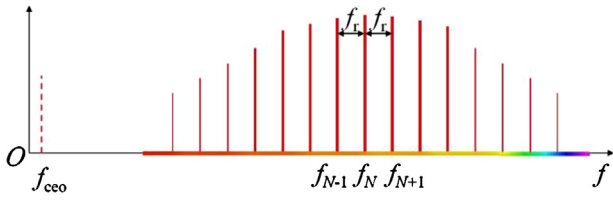


Fig. 1. Frequency modes of optical frequency comb.

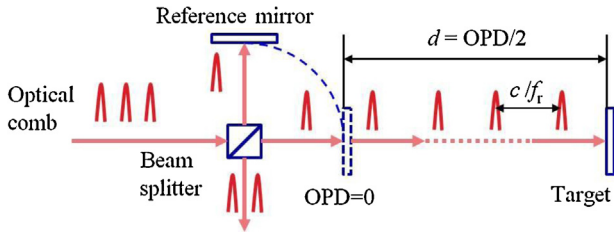


Fig. 2. Pulse interference occurs at discrete positions.

when the pulse train shows the high-intensity envelope peak with the time period of $1/f_r$. Therefore, the envelope peak of the interference fringes will be found when the optical path difference (OPD) of the two arms satisfies the following condition:

$$\text{OPD} = \frac{m \cdot c}{n f_r}, \quad (2)$$

where m is an integer and n is the group refractive index of air. This method is not influenced by the f_{ceo} .

The distance under measurement is $d = \text{OPD}/2$. When the value of m equals 0, 34, 68, or 102, the approximate value of d will be 0, 51 m, 102 m, or 153 m, respectively.

When the repetition frequency f_r is changed by Δf_r , for the same integer m , the position of the interference fringe peak will change by Δd , which can be measured directly. According to formula (2), the relationship between Δd and Δf_r is

$$\left| \frac{\Delta d}{d} \right| = \left| \frac{\Delta f_r}{f_r} \right|. \quad (3)$$

Therefore, the distance d can be determined by changing f_r and measuring Δd , which means that the absolute distance can be measured. The resolution and accuracy of the interference system can also be evaluated by this method [15].

3. Heterodyne interference system

Fig. 3 shows a schematic of the heterodyne interference system with an optical comb, which is based on an unbalanced optical-path Michelson interferometer. The laser source is a Menlo Systems C-Fiber Femtosecond Laser. The center wavelength is 1560 nm, and the output power is about 12 mW, and the repetition frequency f_r is

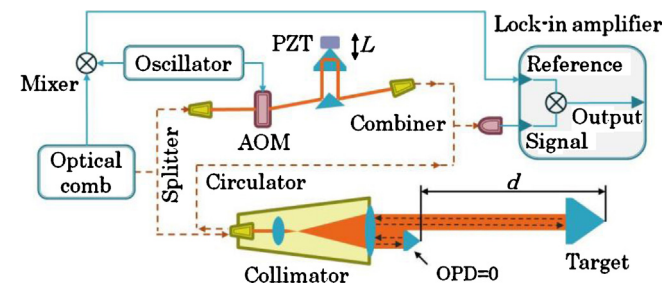


Fig. 3. Schematic of the heterodyne interference system using an optical comb.

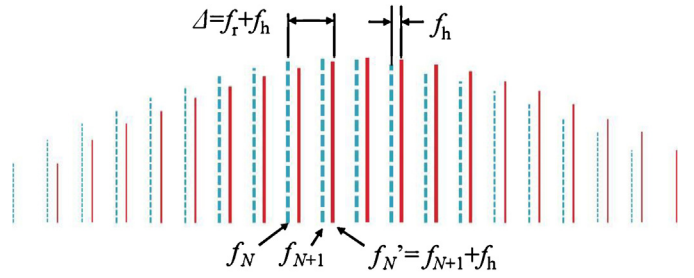


Fig. 4. Frequency shift of an optical comb and the heterodyne interference frequency f_h .

100.0000 MHz, which is stabilized to a Rb frequency standard, and the stability is on the order of 10^{-10} .

The light beam is divided by a splitter into two beams, which enter into the reference arm and the measurement arm separately. The two beams are recombined by a fiber combiner. A collimator in the measurement arm expands the beam diameter to more than 50 mm for the long-distance measurement. Two corner cubes are set in the optical path under measurement, and the distance between two corner cubes is d . The small cube is set at the position where the OPD of the two arms equals zero. The large cube with a diameter of 50 mm is the target, which is set at the position where the OPD satisfies formula (2). A circulator is installed in the measurement arm, so the OPD of the target is twice of the distance under measurement.

An AOM (AA Optoelectronic MGAS110-A1) in the reference arm generates a frequency shift of Δ , which is 100.100 MHz in this case. Thus the frequency shift can be written as $\Delta = f_r + f_h$, and the heterodyne frequency f_h is much smaller than f_r . According to formula (1), the frequency of the original N th mode is shifted to

$$f'_N = f_{\text{ceo}} + (N + 1) \cdot f_r + f_h = f_{N+1} + f_h \quad (4)$$

Accordingly, the shifted N th mode will interfere with the original $(N + 1)$ th mode of the optical comb (Fig. 4), and the heterodyne frequency will become 100 kHz.

A piezoelectric transducer (PZT) is set in the reference arm to move the corner cube backward and forward during the measurement. The scanning period is 50 s, and the scanning length L is about 230 μm . The scanning position is measured by a displacement sensor with a resolution of 10 nm. The relationship between the forward scanning position and the PZT driver signal is calibrated by a Michelson interference system using a stabilized laser diode as the laser source. Only the forward scanning period is used in the measurement, so the hysteric character of the PZT does not introduce error into the measurement result.

In one measurement period, the corner cube on the PZT moves far from the rectangular prism, and the interference fringes generated by the two corner cubes are showed in Fig. 5(a). The first peak is generated by the target, so the measured distance d can be calculated as

$$d = \frac{1}{2} m \times \frac{c}{n f_r} - d_m, \quad (5)$$

where d_m is the scanning displacement of the PZT between the appearance of two peaks of the interference fringe envelope. One fitted line is made to find the positions of the peaks of the two fringe envelope, showed in Fig. 5(b), and then the value of d_m can be calculated.

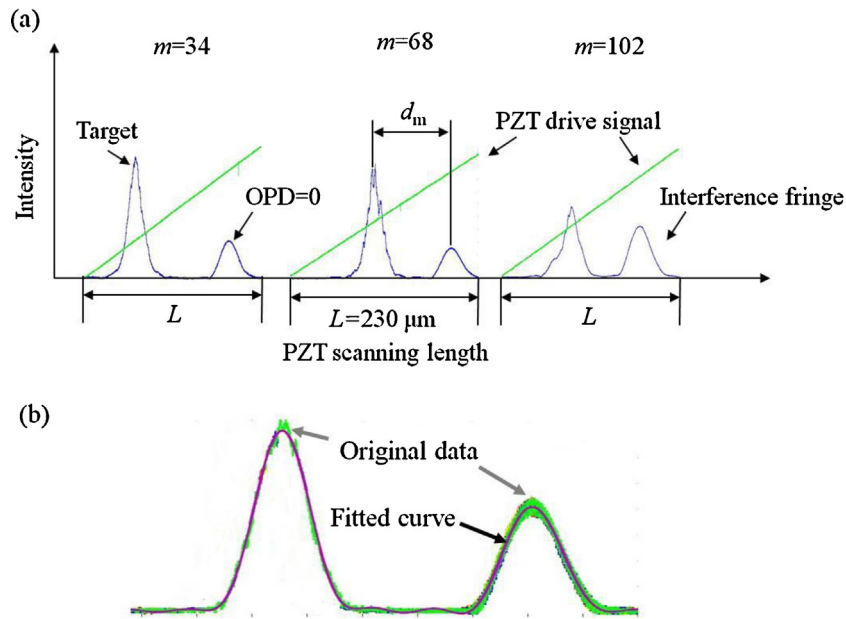


Fig. 5. (a) Interference fringes envelopes and PZT driver signal in one scanning period and (b) original fringe envelope and fitted curve.

4. Experiments and results

4.1. Positioning experiments

The positioning measurement experiments were conducted at the High Energy Accelerator Research Organization (KEK; Tsukuba, Japan). The temperature was controlled around 25 °C, and the environmental conditions of temperature, pressure, and relative humidity (RH) were recorded by air sensors. Fig. 6 shows the variations of the environmental conditions when the distance of 50.951 m was measured. The variations during 1 h were 0.2 °C, 0.5 hPa, and about 1% RH. These conditions are suitable for a general measurement environment.

Because the measurement distance is much longer than that of a conventional interference system, even though the variations in air conditions are small, the drift caused by the change in air refractive index is obvious. For a variation of 0.1 °C, the drift will be 5 μm for 51.951 m and 15 μm for 152.853 m, which cannot be ignored in a high-precision measurement. For the long distance measurements, if the air condition along the optical path is traced, the air refractive index distribution can be calculated by using empirical formulas, and then the measurement result can be corrected.

Fig. 7 shows the experimental results of positioning measurement for a distance of 50.951 m. The 16 data points were recorded over 50 min. The drift was about 10 μm in 1 h, and a comparison with the air condition variations recorded in Fig. 6 shows that

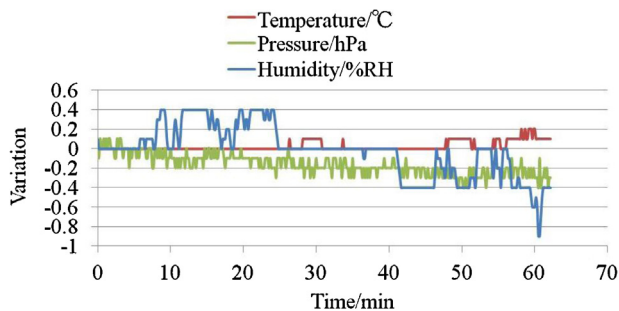


Fig. 6. Variations in environmental conditions when a distance of 50.951 m was measured.

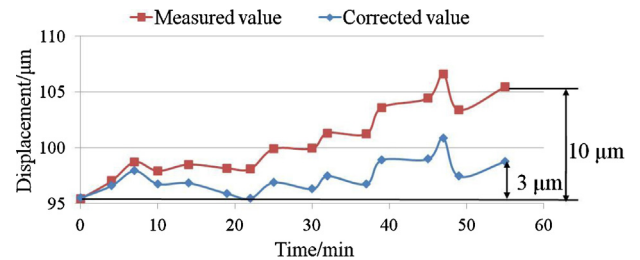


Fig. 7. Positioning experimental results at 50.951 m before and after the air refractive index correction.

the drift followed a similar trend as the environmental conditions. When the refractive index of the air was corrected by using the Edlén formula [17,18], the drift was reduced to 3.0 μm. This result demonstrates that the drift was mainly caused by changes in the environmental conditions, but that the noise was caused by vibration of the table and floor. There was only one air sensor that covered the 51 m distance, so the distribution of the air condition is not clear, and the correction is incomplete. More sensors are needed in order to record the air parameters along the entire optical path.

The distances of 101.902 m and 152.853 m were also measured with the optical comb interference system, and the measuring time was 3 min in each case. Table 1 shows that the reproducibility is better than 1.4 μm. The drifts and the deviations remained almost the same even the measured distance was longer, demonstrating that the heterodyne method is useful for reducing the effect of the air fluctuation. The uncertainty of the repetition frequency f_r is 10^{-10} , the uncertainty of the air refractive index correction is 3×10^{-8} , and the uncertainty of the relative PZT displacement measurement is 1×10^{-8} . So according to formula (2), the theoretical relative uncertainty of the system will be 2.2×10^{-8} , which is equivalent to 1.3 μm for 50.951 m. The average of the corrected value in Fig. 7 is

Table 1
Experimental results of positioning measurements for three distances.

Distance/m	50.951	101.902	152.853
Standard deviation/μm	1.4	0.2	1.1
Drift during 3 min/μm	1.0	0.8	1.0

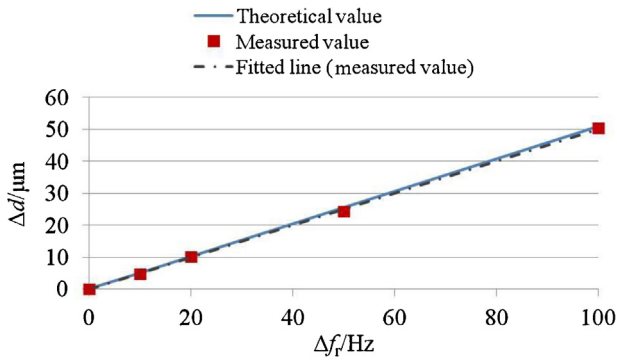


Fig. 8. Experimental results of f_r shift for 50.951 m.

97 μm , therefore, according to formula (5), the measured distance $d = 50.950961 \text{ m} - 97 \mu\text{m} = 50.950864 \text{ m} (\pm 1.3 \mu\text{m})$.

4.2. Frequency shift experiments

As discussed in Section 2, when the repetition frequency of the optical comb is shifted, the OPD of the temporal coherence interference will change and the position of the peak of the interference fringe envelope will be altered. For the target of the experimental system accurate displacement of several micrometers is difficult to achieve, so this method will be useful for evaluating the accuracy and resolution by shifting the repetition frequency and recording the change in the measured value.

Fig. 8 shows the experimental results of shifting the repetition frequency for a distance of 50.951 m. The red marks indicate measured values, and the blue line indicates the theoretical values. When the shifted value of the repetition frequency, Δf_r , is 10 Hz, 20 Hz, 50 Hz, or 100 Hz, the displacement change Δd should be 5.09 μm , 10.2 μm , 25.5 μm , or 50.9 μm , respectively. The theoretical linear equation is $\Delta d = 0.5095 \Delta f_r$, and the slope shows the distance being measured. The correlation coefficient of the measured data is 0.9998, and the maximum difference between the theoretical and measured values is 1.1 μm . The experimental results are consistent with theoretical calculations, and a resolution of 0.5 μm has been realized.

The method of shifting f_r can also be used to determine the value of the integer m in formula (5). To determine m uniquely, the measured value of Δd should be within the permissible error range, showed in Fig. 9.

The permissible error of the measurement result Δd is limited by the value of Δf_r . It can be calculated as $\pm 0.75 \text{ m} \times \Delta f_r / f_r$. For example, when $\Delta f_r = 100 \text{ Hz}$, the permissible error is $\pm 0.75 \mu\text{m}$.

Table 2 shows the experimental results for different Δf_r shifts. The repetition frequency was shifted by 20 Hz when the measured distance was about 51 m and 102 m. The permissible error was $\pm 0.15 \mu\text{m}$. The measured value of Δd was 10.04 μm , according

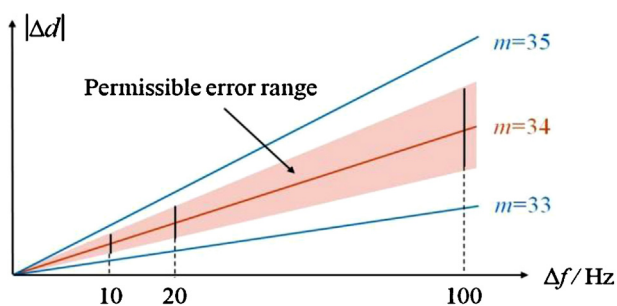


Fig. 9. Permissible error for 50.951 m ($m = 34$).

Table 2
Experimental results of shifting repetition frequency.

Measured distance/m	51.951	101.902
$\Delta f_r/\text{Hz}$	20	20
$\Delta f_r/f_r/\text{ppm}$	0.2	0.2
Permissible error/ μm	± 0.150	± 0.150
$\Delta d/\mu\text{m}$	10.042	20.00
Difference from theoretical value/ μm	0.148	0.38

to formula (3), the range of the measured distance d should be $(10.042 \pm 0.15)/0.2 \text{ m}$. Then according to formula (2), the range of $(2d \eta f_r / c)$ will be 33.005–34.006. Thus, the value of m could be determined as 34.

When Δd is out of the permissible error range, m will be incorrectly determined. For example, from the data for the 102 m distance in Table 2, the value of m can be determined as 67, but in fact the correct value is 68. Therefore, higher accuracy is required as $\Delta f_r / f_r$ becomes smaller. To determine m , especially for the longer distance, large Δf_r is better, because the measurement error will increase, and a larger permissible error is needed. That means that a longer PZT scanning range is required because when the distance is longer, the value of Δd will be larger at the same Δf_r .

5. Conclusion

Experiments on positioning measurements for distances of 50.951–152.853 m have been conducted using a new heterodyne interference system with an optical comb and an AOM. The reproducibility of the positioning measurement was 1.4 μm during the 3 min measurement period, and the deviation remained almost the same when the measured distance was longer, which shows that the deviation was mainly caused by vibration of the stage and floor. The drift was mainly caused by changes in environment conditions, for which a correction can be applied. The accuracy and resolution were evaluated through frequency-shifting experiments. The accuracy of the system was 1.1 μm for a distance of 50.951 m. A resolution of 0.5 μm was realized, and better results can be expected. This method can be used for absolute distance measurement.

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