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Abstract. A unique absolute length measurement method is proposed and demonstrated for the first time. Since it takes advantage of both the high-accuracy measurement capability of a pulse train interference method and the ability of a two-color method to compensate for environmental changes, the present method is expected to be useful for high-precision length measurement for not only the purposes of laboratory science but also for satisfying the requirements of industry. A length measurement was performed to demonstrate the feasibility of the proposed method. [DOI: 10.1117/1.OE.53.12.122413]

Keywords: refractive index; two-color method; laser optics; length measurement; metrology.

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1 Introduction

The two-color method is a well-known approach to making interferometers less sensitive to changes in the refractive index of air. The two-color technique was first studied by Bender and Owens.[1] Since then, many contributions have been made to the correction of the atmospheric index of refraction by different groups.[2–13] The main point with the two-color method was how to record a length difference (phase difference) between two colors (i.e., two optical frequencies).

As an alternative to the wavelength-based two-color method, a technique[4] based on a pulsed laser was also proposed, in which the length difference between two-color pulses was found by using the peaks detected in the pulse intensities. However, the length difference is measured using an incoherent method. In this sense, the temporal coherence property[14] that exists between pulses is not fully utilized.

Recently, two-color methods using a femtosecond optical frequency comb (FOFC)[9,12,13] have been carried out. The coherence between the pulses was used to generate beat signals to record a length difference in the form of phase information. It is also possible to use this coherence property to observe pulse train interference fringes in order to record a length difference in the form of intensity information.[15]

As an alternative to the conventional two-color (i.e., wavelength-based) length measurement technique, we propose a two-color method based on the adjacent pulse repetition interval length (APRIL), the physical length associated with the pulse repetition period. The length information can be corrected with a rough measurement of environmental parameters (namely, the temperature, the atmospheric pressure, and the humidity) and a precise measurement of the relative distance between interference fringes formed by two pulses of different colors. This two-color method uses an APRIL as a ruler for measuring length. The proposed technique enables an absolute length measurement that is free from both 2π ambiguity and the need for a long-time continuous line of sight to the target, which limits the measurement performance of the wavelength-based two-color method. The experimental results presented demonstrate the validity of the proposed principle.

We next clarify the relation of the proposed APRIL-based two-color method to existing length measurement techniques. Homodyne interferometers and heterodyne interferometers are widely used for length measurement.[16] Both of these use wavelength as a practical standard for measuring length and suffer from 2π ambiguity.

Instead of using wavelength, we have employed the APRIL as a standard and proposed an APRIL-based homodyne interferometer.[17] A dual-comb–based heterodyne interferometer, which uses two synchronized combs to generate a beat frequency, has also been proposed.[18–20] To our knowledge, there is still no known method for generating an APRIL based on beat techniques.

As mentioned above, wavelength has been used as a practical standard for measuring lengths in meters. The wavelength-based two-color method has been used to eliminate the inhomogeneous disturbances of optical effects that can be caused by variation of the phase refractive index. In both of the preceding two sentences, wavelength is the key element; it is the connection between (1) the link from the properties of light to the definition of the meter and (2) the correction of the phase refractive index. Since the APRIL is also connected to the stabilized frequency parameters, the APRIL can be used as a practical standard for measuring lengths in meters. Because (1) the speed of the pulse envelope is affected by the group refractive index and (2) the length information measured using the peaks of the envelope of the interference fringes in the proposed method, the APRIL-based two-color method can be used to compensate for the inhomogeneous disturbances of optical effects that can be caused by variation of the group refractive index. Finally, based on the analogy between wavelength and the APRIL, the APRIL will serve as the connection between (1) the link from the properties of light to the

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definition of the meter and (2) the correction of the group refractive index.

2 Principle

Before presenting the principle of the APRIL-based two-color method, we first give a brief introduction to the FOFC. An FOFC is the output of a laser producing a train of ultrashort pulses. In the frequency domain, an FOFC generates equidistant frequency comb lines separated by the pulse repetition frequency f_{rep}. In the time domain, the electric field packet repeats at the pulse repetition period $T_R = 1/f_{\text{rep}}$. Details about the FOFC can be found in Ref. 21.

An APRIL in vacuum, Λ_{vac}, is calculated as

$$\Lambda_{\text{vac}} = c/f_{\text{rep}}. \tag{1}$$

Here, c is the speed of light in vacuum.

The stability of an APRIL is equal to the stability of the repetition frequency. At present, the repetition frequency can be stabilized to the order of 10^{-14}.[22] From this, it is clear that the advent of the FOFC has provided a new ruler for precision length measurements that allows us to measure distance in a more stable manner.

The APRIL in vacuum, Λ_{vac}, and the distance between pulses in air $\Lambda_{\text{air}}(\lambda)$ have the following relationship

$$\Lambda_{\text{air}}(\lambda) = \Lambda_{\text{vac}}/n_{\text{g}}(\lambda, T, P, H), \tag{2}$$

where T, P, and H are temperature, barometric pressure, and humidity, respectively. λ is the center frequency of the FOFC, and $n_{\text{g}}(\lambda)$ is the group refractive index, which can be calculated as[23]

$$n_{\text{g}}(\lambda_i) = n_{\text{p}}(\lambda_i) - \lambda_i\{d[n_{\text{p}}(\lambda)]/d\lambda\}_{\lambda_i}. \tag{3}$$

Here, $n_{\text{p}}(\lambda)$ is the phase refractive index, and in this report, values of $n_{\text{p}}(\lambda)$ are calculated using the Edlén equation.

In the following, in order to distinguish the two quantities, we call the distance between two adjacent pulses in vacuum "the APRIL," and we refer to the corresponding distance in air as the distance between the pulses.

Armed with an understanding of the FOFC and the APRIL, we now turn to the question of measuring the absolute length. A schematic diagram of the proposed method is illustrated in Fig. 1. The interferometer is irradiated by two-color pulses which have different distances between the pulses, $\Lambda_{\text{vac}} \times n_{\text{g}}(\lambda_1)$ (e.g., $\lambda_1 = 780$ nm) and $\Lambda_{\text{vac}} \times n_{\text{g}}(\lambda_2)$ (e.g., $\lambda_2 = 1560$ nm), respectively.

An absolute length L measured in air using pulses with different center frequencies can be expressed as

$$\begin{aligned} L_1 &= L \times n_{\text{g}}(\lambda_1) = p \times \Lambda_{\text{vac}} \times n_{\text{g}}(\lambda_1) \\ L_2 &= L \times n_{\text{g}}(\lambda_2) = p \times \Lambda_{\text{vac}} \times n_{\text{g}}(\lambda_2). \end{aligned} \tag{4}$$

Here, p is the integer obtained when L is measured in units of Λ_{vac}.

Let us consider the formation of interference fringes in the interferometer. Because $n_{\text{g}}(\lambda_1)$ and $n_{\text{g}}(\lambda_2)$ are different, we can assume that $n_{\text{g}}(\lambda_1) < n_{\text{g}}(\lambda_2)$, and, consequently, we have $L_1 < L_2$. First, incoming pulses are split into two identical parts at the beam splitter. When the relative distance between the reference mirror and the object mirror is equal to $p \times \Lambda_{\text{vac}} \times n_{\text{g}}(\lambda_2)$, the i'th pulse reflected from the object mirror and the $(i + p)$'th pulse reflected from the reference mirror will overlap and interfere with each other. Next, the reference mirror is moved far away from the beam splitter. When the relative distance between the reference mirror and the object mirror is equal to $p \times \Lambda_{\text{vac}} \times n_{\text{g}}(\lambda_1)$, the j'th pulse reflected from the object mirror and the $(j + p)$'th pulse reflected from the reference mirror will overlap and interfere with each other.

By moving the reference mirror, we observe the following fringes

$$\begin{aligned} I_1(l) &\propto \exp(-\{2\sqrt{\ln 2}[l - p \times \Lambda_{\text{vac}} \times n_{\text{g}}(\lambda_1)]/L_{\text{coh\_1}}\}^2) \\ &\quad \otimes \delta[l - p \times \Lambda_{\text{vac}} \times n_{\text{g}}(\lambda_1)] \times \cos[2\pi \times l \times n_{\text{p}}(\lambda_1)/\lambda_1] \\ I_2(l) &\propto \exp(-\{2\sqrt{\ln 2}[l - p \times \Lambda_{\text{vac}} \times n_{\text{g}}(\lambda_2)]/L_{\text{coh\_2}}\}^2) \\ &\quad \otimes \delta[l - p \times \Lambda_{\text{vac}} \times n_{\text{g}}(\lambda_2)] \times \cos[2\pi \times l \times n_{\text{p}}(\lambda_2)/\lambda_2]. \end{aligned} \tag{5}$$

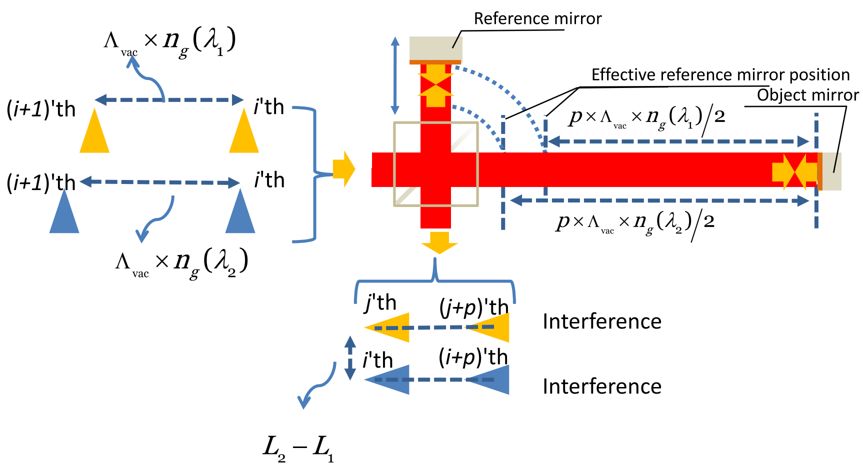

Fig. 1 Interference fringes between pulse trains of the fundamental wave and the second-harmonic wave. With optical-path delays of L_1 and L_2, the relatively delayed pulse trains will overlap, and the expected length difference can be observed from the delay between the interference fringes.

Here, l is the physical length difference between the reference mirror and the object mirror. We assume that the FOFC sources show Gaussian spectral distributions, and L_{coh_i} ($i = 1, 2$) is the temporal coherence length of one pulse.

We define

$$A = [n_g(\lambda_1) - 1]/[n_g(\lambda_2) - n_g(\lambda_1)]. \tag{6}$$

L' is an estimate of the length L, which can be calculated as

$$L' = L_1 - A \times (L_2 - L_1). \tag{7}$$

The difference $L_2 - L_1$ can be measured as the distance between the temporal coherence peaks of the two fringes. L_1 can be measured by using the interference fringes of multiple pulse trains.[15] For example, by observing the destructive interference between two pairs of pulse trains, we can determine the relative delay between them.

3 Experiments

Before describing the experiment, we provide a simple numeric calculation to show how the APRIL-based two-color method works. In order to match the parameters of the experiment, we perform our calculation for a pulse repetition frequency of 100.0 MHz, and we use 1560.0 and 780.0 nm as the two wavelengths. The APRIL in vacuum, calculated based on Eq. (1), was 2.99792458 m. Under standard environmental conditions (a temperature of 20°C, a pressure of 101.325 kPa, and 0% humidity), the A parameter, the distance between pulses for 1560.0 nm, and the distance between pulses for 780.0 nm, as calculated based on Eqs. (6) and (4), were, respectively, 48.05, 2.99711588 m, and 2.99709870 m. The distance between the two-color pulses was 1718 nm. Based on Eq. (7), L' can be estimated to be 2.99792436 m. The difference between L' and L is 223 nm. We assume that the average temperature has changed to 23°C. The distances between pulses for 1560.0 and 780.0 nm were, respectively, 2.99712409 and 2.99710709 m. The distance between the two-color pulses was 1700 nm. This distance was less sensitive to environmental conditions. Based on Eq. (7) (using an A parameter that was calculated for a temperature of 20°C), L' can be estimated to be 2.99792436 m. The difference between L' and L is 218 nm. More details can be found in Ref. 24.

An optical schematic of the experimental setup is shown in Fig. 2. The experiment is carried out with a system consisting of a polarization-mode-locked femtosecond fiber laser (Menlo Systems, Freistaat Bayern, München, Germany, FC1500), a second-harmonic generation (SHG) stage, and a detection stage. The central wavelength and the repetition frequency are, respectively, 1560.0 nm and 100.0 MHz; the repetition frequency was observed with a frequency counter with an accuracy on the order of 10^{-10}.

The pulse train of the FOFC is introduced into an optical amplifier (Keopsys, Lannion, Côtes-d'Armor, France, EDFA CBO27) via an optical fiber. The amplified FOFC light is introduced into the second-harmonic generation stage. The second-harmonic generation stage is a combination of a periodically poled lithium niobate (PPLN, China, HCP Photonics, HsinChu, Taiwan) crystal and two collimator lenses. One collimator lens is used to focus the light on the PPLN crystal, and the other is used to collect the FOFC-based SHG signal from the PPLN and couple it into an optical fiber. The PPLN is heated to 86°C.

The fundamental wave and the second-harmonic wave from the PPLN are introduced into the modified Michelson interferometer via an optical fiber. Two pairs of pulse trains (the fundamental wave and the second-harmonic wave) traverse the same optical path and overlap at the beam splitter. Another collimator lens forms images of each of the fringes on each of the two photodetectors (New Focus, Santa Clara, California, Model 2001, for the second-harmonic wave and New Focus, Model 2011, for the fundamental wave) via an optical fiber coupler and optical fibers, respectively.

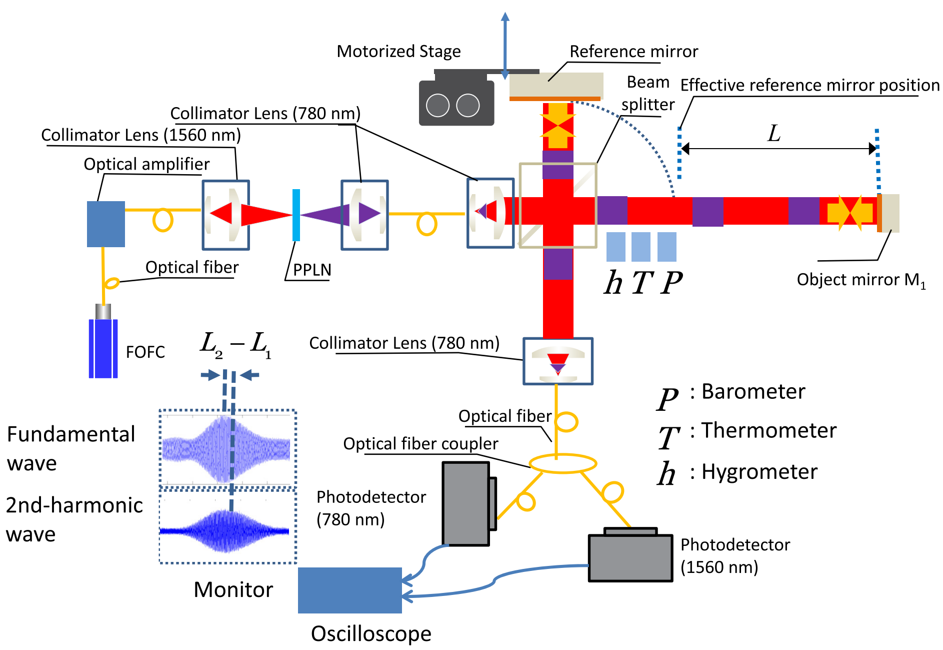

Fig. 2 Experimental setup used to demonstrate the feasibility of the adjacent pulse repetition interval length–based two-color method. FOFC: femtosecond optical frequency comb.

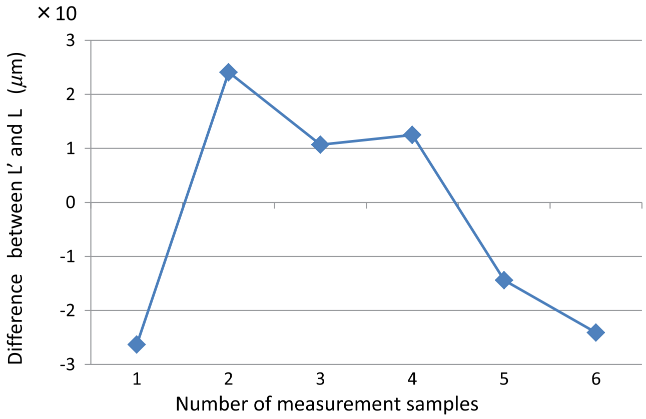

Fig. 3 Difference between the result measured by our optical system and the calculation from the value of the frequency counter.

We acquired interference fringes for the fundamental wave and the second-harmonic wave, respectively. We set the object mirror in a position such that $L_1 = \Lambda_{\text{vac}} \times n_g(\lambda_1) \approx$ 1.5 m. The two interference fringe signals exhibited a high contrast between the two pairs of pulse trains based on the relative optical displacements L_1 and L_2, respectively.

As shown in Fig. 3, six data points were obtained. Each data point is obtained by averaging over 10 measurements to suppress the effect of air fluctuations along the optical path. The measurements were done on six different days. The reported average value of the length $L\prime$ and uncertainty value was calculated on the basis of these six measurements. To obtain $n_g(\lambda_1)$ $[L_1 = \Lambda_{\text{vac}} \times n_g(\lambda_1)]$ and the A parameter, we used a thermometer (Testo, Lenzkirch, Freiburg, Germany, 735), a barometer (Sunoh, Tsukuba, Ibaraki, Japan, VR-18), and a hygrometer (VAISALA, Helsinki, Vantaa, Finland, HM70), respectively, to measure temperature, pressure, and humidity. $L_2 - L_1$ is measured from the peaks of the envelopes of the two interference fringes. The measured delay $L_2 - L_1$ is 5.83 ± 0.90 μm for the first data point. Based on Eq. (7), the measured average value of the length $L\prime$ was 2.997924 ± 0.000038 m. The difference between the result measured by our optical system and the calculation from the repetition-frequency value obtained from the frequency counter[17] was about ± 40 μm. Because the distance was calculated from the peak of the envelope of the interference fringe, (based on our analysis to date) 10 nm is the smallest displacement our system can measure. The measurement accuracy of the peak position limited the performance of the optical system. This error was magnified by the A parameter (by about a factor of 50). A detailed discussion of the complete process used to estimate the quantitative error will be reported in another paper.

In this experiment, we only measured a distance of ~3 m. Because $L_2 - L_1$ increases with the distance L, this value can be used to determine the number of APRILs in the length L (namely, the value of p). Due to the fixed pulse repetition frequency, we were only able to measure distances that were approximately a multiple of 3 m (such as 6 and 9 m). This limitation is not intrinsic to this method. The system can be improved by using a frequency comb with a variable pulse repetition frequency to realize a measurement of an arbitrary length, at the cost of a complicated configuration.

4 Conclusion

In summary, we proposed an unconventional type of two-color method, which we call the APRIL-based two-color method. The viability of the proposed method and its ability to measure an absolute length were demonstrated. The unique feature of the proposed method is that, while a length is measured by two pulses of different colors in the same manner as in the conventional two-color method, the only recorded data is the interference fringes of multiple pulse trains, and the length information is obtained by detecting envelope peaks. The proposed technique enables absolute interferometry that is free from 2π ambiguity and the need for a continuous line of sight to the target. The proposed method will open up the possibility of a universal, easily distributable standard for absolute length measurements, because in the near future we will be able to deliver an FOFC through optical fibers. To our knowledge, this is the first report on the general principle and experimental demonstration of the APRIL-based two-color method.

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