Calibration for the sensitivity of multi-beam angle sensor using cylindrical plano-convex lens

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A B S T R A C T
A highly sensitive and compact multi-beam angle sensor (MBAS), which utilizes the principle of operation of an autocollimator, was developed to detect the differential of the local slope components (angle difference) of a point on the mirror surface and using Fourier series, we can obtain the profile data from the angle difference. In order to investigate the application of the MBAS for high precision aspheric surface measurements, two types of calibration methods using plane mirror and cylindrical plano-convex lens has been proposed to measure the sensitivity of the MBAS. The calibration data analysis results using plane mirror agree well with the measurement results of the cylindrical plano-convex lens data. Comparison of the two methods confirms that the second method (using cylindrical plano-convex lens) is more adapted for measurement with ultra high level of uncertainty. Further, the second method is simple, corresponding to a direct calculate in the sensitive parameters aiming to minimize the cost.

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1. Introduction

Precision autocollimators are the most accurate, widely available means of measuring the tilt angle of a reflecting plane mirror. They have a wide range of applications such as optical metrology instruments, the semiconductor industry, and inspection equipment (Whitehouse, 1976; Chetwynd and Siddall, 1976; Vissiere et al., 2012; Whitehouse, 1974).

The autocollimator method is generally used to measure flatness due to its simple optical-path design (Ennos and Virdee, 1982; Xiao et al., 2012; Ennos and Virdee, 1983). At the Tohoku University, the optical was modified by fitting the photoelectric autocollimator with a resolution of 0.1′′ for angle measurements (Gao and Kiyono, 1997; Gao et al., 2002). The well-balanced harmonic response in the entire frequency range is a drawback of this method. Bručas proposed and developed two modified autocollimators by setting a charge-coupled device (CCD) to the optical units, which can be also used for the angle measurements (Bručas and Giniotis, 2009a, b, 2010). However, they are very time consuming and too many autocollimators make it difficult to adjust the direction of the sensor’s radius. At the Physikalisch-Technische Bundesanstalt institute, the autocollimators are calibrated by using a WMT 220 angle comparator, which demonstrated the direct traceability of high-resolution autocollimators to the SI unit of the plane angle (Just et al., 2009; Probst et al., 1998; Probst, 2008). It is expected that the measurement uncertainty of the angle will be further reduced by calibrating the angle autocollimator, so that in future the measurement can be attained with an even smaller uncertainty (Just et al., 2003; Yandayan et al., 2011; ISO/IEC Guide 98-3:2008).

In order to investigate the application of the multi-beam angle sensor (MBAS) for high precision roundness measurements, we previously reported the preliminary results of MBAS applications to measure the surface profiles of a cylindrical workpiece. Using Fourier series, we obtained the profile data from the angle difference (Chen et al., 2015a, 2014). Here, we present a new detailed, expanded two types of calibration methods for the sensitivity of MBAS using a plane mirror and cylindrical plano-convex lens. The calibration data analysis results using plane mirror agree well with the measurement results of the cylindrical plano-convex lens data. In the second method (using cylindrical plano-convex lens), the radius of curvature R and the differential spacing Δd can be used to calibrate the sensitivity of the MBAS. Despite the simplicity of the proposed method, the uncertainty budget for the sensitivity of the MBAS in the calibration experiment using plane mirror agrees well with the measurement results of the sensitivity in the cylindrical plano-convex lens experiment, which also verified the feasibility of the calibration for the MBAS sensitivity using a cylindrical plano-convex lens.
Comparison of the two methods confirms that the second method is more adapted for measurement with ultra high level of uncertainty. Further, the second method is simple, corresponding to a direct calculate in the sensitive parameters aiming to minimize the cost.

2. Principle of operation

2.1. Multi-beam angle sensor for flatness measurement of mirror

An autocollimator is an optical instrument for non-contact measurement of angles from a reflecting surface. The MBAS is based on a multi-autocollimator system using microlenses to measure deflections in an optical system. The MBAS works by projecting an image onto a beam splitter, and measuring the deflection of the returned image against a scale. The reflected angles at several points on the mirror surface can be measured using the sensor. Then, the sensor scans the workpiece while it is rotating.

Fig. 1 illustrates the optical system of the MBAS. The laser beam from a laser diode (LD) passes through a pinhole, and it is collimated by a collimator lens. The beam is then bent by a beam splitter and projected to the workpiece surface. The reflected beam from the workpiece surface passes totally through the beam splitter to the microlenses, and after being focused on the microlens, it is split into several beams. The resulting pattern is observed and recorded by a CMOS camera mounted along the vertical axis. The imaging can be observed on a TV monitor. In order to investigate the application of the multi-beam angle sensor (MBAS) for high precision roundness measurements, we previously reported the preliminary results of MBAS applications to measure the surface profiles of a cylindrical workpiece. Using Fourier series, we obtained the profile data from the angle difference (Chen et al., 2014).

Using the MBAS, we implemented the experimental system shown in Fig. 2. A workpiece was placed on the tilt stage, and the rotary platform is mounted between two XY-platforms. Therefore, the rotary table in this case acts as a small angle generator and the reference mean of angle measurement.

Fig. 3 illustrates how the two points A and B in the circumference of the circle with certain radius are carried out by rotating the workpiece step by step (Chen et al., 2015b). The workpiece flatness is calculated by applying the autocollimator principle of the angle difference at each of these two angles on the workpiece. The angle difference can be calculated from the intensity distribution of the spots on the CMOS (Chen et al., 2016). Then, the specimen profile at each location on the circle can be determined accurately. This procedure is repeated for circular scans of different radii, to yield the overall shape of the surface.

In the flatness measurement, the sensitivity $W$ and width of the lattice spacing $\Delta d$ (calculated from the angle difference using MBAS) have been used to estimate the radius of curvature $R$ of the cylindrical plano-convex lens (if $R$ is unknown). Here, the profile of cylindrical is also can be calculated from the curvature $R$. Therefore the sensitivity $W$ is an important parameter in the profile measurement.

Reversely, in the calibration experiment, the radius of curvature $R$ and width of the lattice spacing $\Delta d$ (calculated from the angle difference using MBAS) have been used to calibrate the sensitivity of the MBAS.

2.2. Measurement of the sensitivity $W$

Using the MBAS, we could measure the flatness of several tens micrometer with repeatability of several tens nanometer. Some flatness measurement results also imply that the MBAS can measure flatness with absolute accuracy under several tens nanometer if the sensitivity $W$ is less than 0.5% (Chen et al., 2015b, 2016). Therefore, an important prerequisite for the determination of the measurement uncertainty can be achieved with the accurate calibration of sensitivity of the MBAS.
2.2.1. The principle of the MBAS for measurement of variable-radius of curvature

In order to investigate the sensitivity of the MBAS in measurement of variable-radius of curvature, the principle for the flatness and aspheric surface measurement are discussed.

As a first example, we consider a common application in flatness measurement, the focusing of a perfectly collimated beam to a small spot, which is shown in Fig. 4(a). Here we have a laser beam, which is focused by a lens. From Fig. 4(a), we find that the slope is coupled to the position of the spot.

Another example is the nearly collimated beam focused by a lens in aspheric surface measurement, as shown in Fig. 4(b). Only the slope of the center of the lens is coupled to the center of the spot and the size of the spot increases. The problem is often stated in terms of focusing the output from a “parallel light source.”

A further analysis of the radius of curvature and sensitivity of the autocollimator will be discussed on this topic in Section 2.2.2.

2.2.2. Calculating the sensitivity W from the radius of curvature R

In order to obtain a precise estimate of the signal in the measurement of the aspheric surface, a multi-spot light beam has been developed for measurement of the local slope of the cylinder lens. Whereas in this situation, the MBAS is sequential with a highly flexible sampling pattern and measures aberrations of the incoming beam using only one sensor for an imaging system. Here, we have a laser beam as shown in Fig. 5, with radius of curvature $R$ and divergence $2\alpha$ that is focused by a lens of focal length $f$. Assuming that the lattice spacing of the lens is $d$, $k$ is the distance between the lens and aspheric surface, and the position of the beam coming from the center of lens on the aspheric surface is $c$, we can obtain the divergence $\alpha$ by the lens spacing between points A and B, $d_{AB}$ and radius of curvature $R$. The divergence $\alpha$ is then given by:

$$\alpha = \frac{d_{AB}}{2R}$$ (1)

The optical invariant then tells us that we must have Eq. (2), because the product of the radius and divergence angle must be constant. From Fig. 5, we obtain the lens spacing between points A and B, $d_{AB}$, by the following:

$$\frac{d_{AB}}{2} = c + 2\alpha k$$ (2)

From Eqs. (1) and (2), we obtain the position of the beam from center lens on the aspheric surface $c$, calculated by a simple equation:

$$c = \frac{Rd_{AB}}{2R + 4k}$$ (3)

From Fig. 5, we obtain the spot spacing between points A and B, $d_{RAB}$, by the following:

$$\frac{d_{RAB}}{2} = \frac{d_{AB}}{2} + \alpha(f_A + f_B)$$ (4)
From Eqs. (3) and (4), we obtain the differential between the lens spacing $d$ and the spot spacing $d_R$, $\Delta d_{AB}$ (the differential spacing), by the following:

$$\Delta d_{AB} = d_{AB} - d = \frac{(f_A + f_B) d_{AB}}{R + 2k}$$

(5)

Accordingly, a variation in the differential spacing of points $A$ and $B$, can be found from variation in the focal distance $f_{AB}$, lens spacing $d_{AB}$, radius of curvature of the workpiece $R$, and distance between lens and aspheric surface $k$.

In order to know, theoretically, the relationship between the radius of curvature $R$ and the sensitivity between points $A$ and $B$, we derived the following formula:

$$R = \frac{(f_A + f_B) d_{AB}}{\Delta d_{AB}} - 2k = \frac{W_{AB}}{\Delta d_{AB}} - 2k$$

(6)

where $W_{AB}$ is the sensitivity between points $A$ and $B$, which can be described as follows:

$$W_{AB} = (f_A + f_B) d_{AB}$$

(7)

Here, the sensitivity in the calibration experiment using plane mirror can be calculated from Eqs. (7).

We note that the sensitivity $W_{AB}$ can also be denoted as:

$$W_{AB} = (R + 2k) \Delta d_{AB}$$

(8)

Similarly, the sensitivity in the calibration experiment using cylindrical plano-convex lens can be calculated from Eqs. (8).

In order to know, theoretically, the relationship between the radius of curvature $R$ and the sensitivity $W_{AB}$ between points $A$ and $B$, we derived formula:

$$R = \frac{W_{AB}}{\Delta d_{AB}} - 2k$$

(9)

In the flatness measurement, using Formula (9), the sensitivity $W$ and width of the lattice spacing $\Delta d$ (calculated from the angle difference using MBAS) have been used to estimate the radius of curvature $R$ of the cylindrical plano-convex lens (if $R$ is unknown). Here, the profile of cylindrical is also can be calculated from the curvature $R$. Therefore the sensitivity $W$ is an important parameter in the profile measurement.

Reversely, in the calibration experiments, using Formula (7), we can calculate the sensitivity $W$ by measuring focal lens $f$ of two spots and the lens spacing $d$ between two spots; using Formula (8), we can calculate the sensitivity $W$ by measuring the width of the lattice spacing $\Delta d$ between two spots (calculated from the angle difference using MBAS).

2.3. Calibration of the MBAS

For the determination of the sensitivity of MBAS, two independent calibration methods can be applied using the Formulas (7) and (8) in Section 2.2.2.

One fundamental calibration method is based on a comparison of all displacements in a measurement range with certain steps of a plane mirror. The selection of measurement points and precise adjustment of the angles specified is performed with the aid of a tilt stage and laser hologage. Using Formula (7), we can calculate the sensitivity $W$ by measuring focal lens $f$ of two spots and the lens spacing $d$ between two spots.

Another novel calibration method is independent of any assistive tools, which is based on the angle difference measurement using a cylindrical plano-convex lens (if the radius of curvature $R$ is known). Using Formula (8), we can calculate the sensitivity $W$ by measuring the width of the lattice spacing $\Delta d$ between two spots (calculated from the angle difference using MBAS).

2.3.1. Using plane mirror and tilt stage

Calibration of the autocollimator is always a complicated task since small angle steps must be generated with a very high precision. In principle, the measurement step width can be selected by the resolution of the autocollimator; however, limited by the repeatability of the MBAS readout, the selection of micro area measurement steps is necessary to detect short period deviation of the autocollimator. As the scope of the calibration must be kept within reasonable limits and additional requirements for long-term stability must be met in the case of long measuring times, calibrations in micro area measurement steps are carried out only over selected small sections of the measurement range. The most accurate available method is to perform calibrations in different measurement ranges with appropriate measurement steps.

Fig. 6 illustrates the construction of calibration performed by comparison with the sensitivity of the autocollimator in different points. The measured displacement of the spot values of the MBAS with different $t$ indicated for the sensitivity difference of the autocollimator. The selection of measurement points and precise adjustment of the angles specified is performed with the aid of a laser hologage. For data acquisition, 30 single values are read out in each adjustment for $10 \mu$m displacement of the tilt stage and mean values of the displacement of spot are calculated. To eliminate the linear drift influences during calibration, three measurement series are carried out to obtain the standard deviation.

Here, the distance between the fulcrum and observation point $l$ is 132 mm, the displacement of the tilt stage $h$ is 10 $\mu$m (measurement step). Thus, the angle of inclination $t$ (in $\mu$rad) can be described by Eq. (10).

$$t = \frac{1000h}{l}$$

(10)

The sensitivity of the autocollimator $s$ can be expressed as:

$$s = \frac{t}{p}$$

(11)

where $p$ represents the ratio between the angle of inclination $t$ and sensitivity of the autocollimator $s$. From Eqs. (10) and (11), we can derive the sensitivity of the autocollimator $s$ (in $\mu$rad/pixel), which is expressed as follows:

$$s = \frac{1000h}{pt}$$

(12)

The focal length of the microlens array $f$ (in $\mu$m) can be denoted as:

$$f = \frac{d_i}{2s}$$

(13)

From Eqs. (12) and (13), we can derive the focal length of the microlens array $f$ (in mm), which is expressed as follows:

$$f = \frac{pld_i}{2h}$$

(14)

To evaluate the sensitivity $W$ based on the calibration method using real datasets, the measurement components were also analyzed.

Explanations regarding the components are:

$(p_A, p_B)$ Here $p$ represents the ratio between the angle of inclination $t$ and sensitivity of the autocollimator $s$. $p_A$ and $p_B$ is the ratio in point A and B, respectively. According to the calibration experiment using plane mirror, the ratio $p$ can be calculated by the mean value of typically 3–5 repeat measurements.

$(\frac{d_i}{2s})$ Here the distance between the fulcrum and observation point $l$ is 132 mm, the sensitive area of the CMOS $d_i$ is 2.2 $\mu$m and the displacement of the tilt stage $h$ is 10 $\mu$m. We can calculate the coefficient of the focal distance is 14.520 (in mm/pixel).
Fig. 6. Calibration system (principle): $h$ is the displacement (the measurement value of the laser hologate) of the tilt stage, $l$ is the distance between the fulcrum and observation points, and $t$ is the angle of inclination.

Fig. 7. Algorithm flowchart of the measurement: from the differential spacing $\Delta d$ to the sensitivity $W_{AB}$ using radius of curvature $R$.

$$(f_A, f_B)$$ Here $f_A$ and $f_B$ is the focal length in point A and B, respectively. From Eq. (14), we can calculate the focal length of microlens array at each point. 

(15) Calculation of the mean value of typically 3–5 repeat measurements. The standard average value of the differential spacing between points A and B is found by simple arithmetic:

$$d_{AB} = (d_A - d_B) / s$$  \(s\text{ (15)}$$

From Eq. (15), we also can calculate pitch of the microlens array $d_{AB}$.

Finally, the sensitivity $W_{AB}$ between points A and B can be calculated from Eq. (7). Similarly, we can obtain the sensitivity $W$ for other points.

### 2.3.2. Using cylindrical plano-convex lens

Fig. 7 shows the algorithm flowchart of the measurement. The angle difference value can be measured by MBAS, then to evaluate the sensitivity $W$ based on the measurement of the cylindrical plano-convex lens using real datasets, the measurement components are analyzed as follows.

Explanations regarding the components are:

(16) We note that the relationship between the differential spacing $\Delta d_{AB}$ and the angle difference $\Delta_c$ can be denoted as:

$$\Delta d_{AB} = \frac{\Delta c_{AB}}{1000 \times d_i}$$  \(s\text{ (16)}$$

Here the sensitive area of the CMOS $d_i$ is 2.2 $\mu$m. 

(17) Calculation of the mean value of typically 5–10 repeat measurements. The standard average value of the distance between the lens and aspheric surface $k$ is 55.5 (in mm). Here, as shown in Table 3, the radius of curvature $R$ of the cylindrical plano-convex lens is 519 (in mm).

(WAB) Furthermore, we can obtain the sensitivity $W_{AB}$ from Eq. (8).

Consequently, the sensitivity $W_{AB}$ can be denoted as the differential spacing $\Delta d_{AB}$ using the radius of curvature $R$. Similarly, we can obtain the sensitivity for other points.

The characteristics of the algorithm chart can be estimated by its transfer function, which defines the relationship between the differential spacing $\Delta d_{AB}$ and the sensitivity $W_{AB}$.

### 2.4. Estimation of uncertainty

In statistics, propagation of error is the effect of variable uncertainty in the measurement of plane mirror. We can estimate the uncertainty in the calibration method using an error propagation matrix that is deformed by neglecting correlations or assuming independent variables, yielding a common formula to calculate error propagation, the variance equations (Ku, 1966):

$$\sigma_s^2 = \left(\frac{\partial s}{\partial h}\right)^2 \sigma_h^2 + \left(\frac{\partial s}{\partial p}\right)^2 \sigma_p^2 + \left(\frac{\partial s}{\partial l}\right)^2 \sigma_l^2$$  \(s\text{ (17)}$$

where $\sigma_s$ represents the standard deviation of the function $s$, $\sigma_h$ represents the standard deviation of $h$, $\sigma_p$ represents the standard deviation of $p$, and $\sigma_l$ represents the standard deviation of $l$. From Eqs. (12) and (17), the standard deviation of the function $s$ is obtained as Eq. (18).

$$\sigma_s = \sqrt{\left(\frac{1000}{pl}\right)^2 \sigma_h^2 + \left(\frac{1000h}{p^2l^2}\right)^2 \sigma_p^2 + \left(\frac{1000h}{p^2l^2}\right)^2 \sigma_l^2}$$  \(s\text{ (18)}$$

$$\sigma_s = \sqrt{s^2 \left(\frac{\sigma_h}{h}\right)^2 + s^2 \left(\frac{\sigma_p}{p}\right)^2 + s^2 \left(\frac{\sigma_l}{l}\right)^2}$$  \(s\text{ (19)}$$

From Eq. (19), the standard deviation of the function $s$ can also be expressed in terms of the standard deviations of the other functions, given by:

$$\sigma_s = \sqrt{\left(\frac{\sigma_h}{h}\right)^2 + \left(\frac{\sigma_p}{p}\right)^2 + \left(\frac{\sigma_l}{l}\right)^2}$$  \(s\text{ (20)}$$

We note that the relationship between standard deviation of the function $\sigma_s$ and sensitivity of the autocollimator $s$ can be denoted as:

$$\sigma_s / s = \sqrt{\left(\frac{\sigma_h}{h}\right)^2 + \left(\frac{\sigma_p}{p}\right)^2 + \left(\frac{\sigma_l}{l}\right)^2}$$  \(s\text{ (21)}$$

It is important to note that this formula is based on the linear characteristics of the gradient of $s$ and therefore it is a good estimation for the standard deviation of $s$ as long as $\sigma_h$, $\sigma_p$, $\sigma_l$ are small compared to the partial derivatives (Clifford, 1973).
Fig. 8. Measurement arrangement for calibration of the MBAS: main setup of the pre-experiment consisted of the MBAS, a laser hologage, a tilt stage, a rotary stage, and two XY-platforms.

Table 1
Specifics of devices in MBAS (Fig. 8).

<table>
<thead>
<tr>
<th>Device</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Laser diode</td>
<td>Output power: 35 mW (CW)</td>
</tr>
<tr>
<td></td>
<td>Wavelength: 658 nm</td>
</tr>
<tr>
<td>Pinhole</td>
<td>Diameter: 400 μm</td>
</tr>
<tr>
<td>Aperture</td>
<td>Diameter: 4 mm</td>
</tr>
<tr>
<td>Microlens array</td>
<td>Focal length (f): 46.7 mm</td>
</tr>
<tr>
<td>CMOS</td>
<td>Size: 5.6 mm × 4.2 mm</td>
</tr>
<tr>
<td></td>
<td>Valid pixels: 2560 pixels × 1920 pixels</td>
</tr>
<tr>
<td></td>
<td>Sensitivity area (d_s): 2.2 μm × 2.2 μm</td>
</tr>
<tr>
<td>Laser hologage</td>
<td>Resolution: 0.01 μm</td>
</tr>
<tr>
<td></td>
<td>Repeatability: 0.02 μm</td>
</tr>
<tr>
<td>Tilt stage</td>
<td>Size: 160 mm × 220 mm</td>
</tr>
<tr>
<td></td>
<td>Tilt range: (α axis) ±0.7°; (β axis) ±0.6°</td>
</tr>
</tbody>
</table>

Similarly, from Eqs. (7) and (8), the standard deviation of the function W_{AB} can also be expressed in terms of the standard deviations of the other functions, given by:

\[
\sigma_{W_{AB}} = \sqrt{\left(\frac{d_{AB}}{d_{f}}\right)^2 \sigma_{d_{f}}^2 + \left(\frac{d_{AB}}{f}\right)^2 \sigma_{f}^2 + \left(\frac{f}{f} + d_{f}\right)^2 \sigma_{d_{f}}^2} \tag{22}
\]

\[
\sigma_{W_{AB}} = \sqrt{\left(\Delta d_{AB}\right)^2 \sigma_{d_{AB}}^2 + 4\left(\Delta d_{AB}\right)^2 \sigma_{f}^2 + (R + 2k)^2 \sigma_{d_{f}}^2} \tag{23}
\]

3. Experiment and result

3.1. Configuration of the experiment

The pre-experimental arrangement is shown in Fig. 8. In the pre-experiment, the MBAS is based on a multi-autocollimator system using a microlens array. The main setup of the pre-experiment consisted of the MBAS, a laser hologage, a tilt stage, a rotary stage, and two XY-platforms. The workpiece is centered with respect to the axis of rotation; the autocollimator (MBAS) is aligned such that the horizontal measuring axis and the optical axis of the MBAS lie in the workpiece plane. A tilt stage is mounted between the rotary stage and workpiece, which can be used to adjust the workpiece plane so that it reflects the beam of the MBAS at defined angles.

The resulting pattern (in Fig. 9) is observed and recorded by a CMOS camera mounted along the vertical axis. Table 1 shows the specifications of the devices in Fig. 8. By using the intensity distribution, the angle difference data can be calculated (Chen et al., 2015a).

3.2. Experiment of the plane mirror

In the following, the displacement of the spot (averaged over three measurement series) results for the autocollimator will be discussed. As a measure of reproducibility, Fig. 10 shows the standard average value of 30 single values of the autocollimator for a calibration over a measurement range of ±150 μm in measurement steps of 10 μm. The ratio of the output and displacement of the tilt stage at different points (A, B, C, and D in Fig. 10) is compared, where the ratio is the displacement of the spot on average.

According to the principle in Section 2.3.1 and the results shown in Fig. 10, the sensitivity W_{AB} between points A and B is 47.084. Similarly, we can obtain the sensitivity W for other points, as shown in Table 2.

The theoretical value of the sensitivity of the autocollimator S_0 (in μrad/pixel) can be denoted as:

\[
S_0 = \frac{d_i}{2f} \tag{24}
\]

Here the sensitive area of the CMOS d_i is 2.2 μm and the focal length of microlens array f is 46.7 mm. From Eq. (24), we can calculate the theoretical value of the sensitivity of the autocollimator S_0 = 23.55 μrad/pixel.

Assuming that the σ_r (the standard deviation of h) is 0.01, from Eq. (21), we can calculate the ratio between standard deviation of the function σ and the sensitivity of the autocollimator s = 0.5%.
Table 2
Summary of the results for the measurement of plane mirror.

<table>
<thead>
<tr>
<th>Type</th>
<th>Estimate/pixel</th>
<th>Coefficient</th>
<th>Estimate/(mm/pixel)</th>
<th>Component</th>
<th>Estimate/mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_A$</td>
<td>3.176</td>
<td>$d_{b}$</td>
<td>14.520</td>
<td>$f_A$</td>
<td>46.120</td>
</tr>
<tr>
<td>$p_B$</td>
<td>3.168</td>
<td>$d_{b}$</td>
<td>14.520</td>
<td>$f_B$</td>
<td>45.994</td>
</tr>
<tr>
<td>$d_{AB}$ - $d_{B}$</td>
<td>232.340</td>
<td>$d_{AB}$</td>
<td>0.0022</td>
<td>$d_{AB}$</td>
<td>0.511</td>
</tr>
</tbody>
</table>

Sensitivity W/mm$^2$

| AB  | 47.084 |
| BC  | 46.826 |
| CD  | 46.890 |

Table 3
Specifications of the cylindrical plano-convex lens.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size</td>
<td>30 mm $\times$ 30 mm</td>
</tr>
<tr>
<td>Focal length</td>
<td>1000 mm</td>
</tr>
<tr>
<td>Edge thickness</td>
<td>4.8 mm</td>
</tr>
<tr>
<td>Center thickness</td>
<td>5 mm</td>
</tr>
<tr>
<td>Radius of curvature</td>
<td>519 mm</td>
</tr>
</tbody>
</table>

Fig. 11. Measured angle data at points A, B, C, and D.

Fig. 12. Measured angle difference data for a cylindrical plano-convex lens measured by the MBAS.

similarly, when the $\sigma_p$ is 0.01 pixel, the ratio is 1.5%; when the $\sigma_l$ is 1 mm, the ratio is 0.75%.

3.3. Experiment of the cylindrical plano-convex lens

In the experiment, cylindrical plano-convex lenses were used as specimens. Table 3 shows the specifications of the workpiece; the cylindrical plano-convex lens has a convex curvature in the vertical direction and has no curvature in the horizontal direction.

Fig. 11 shows the angle data $c_3$, $c_4$, $c_5$, and $c_6$ measured by the MBAS system for a cylindrical plano-convex lens. The horizontal axis is the rotation angle and the vertical axis is the angle data value. The angles of the specimen at points A, B, C, and D are 414.2, 411.0, 413.3, and 419.5 (in pixel), respectively.

According to the principle in Section 2.3.2 and the measured angle difference data in Fig. 12, the sensitivity $W_{AB}$ between points A and B is 47.082. Similarly, we can obtain the sensitivity $W$ for other points, as shown in Table 4.

Despite the simplicity of the proposed method, the calibration data analysis results (Table 2) agree well with the measurement results of the cylindrical plano-convex lens data (Table 4), which verified the feasibility of the calibration for the sensitivity of the MBAS using cylindrical plano-convex lens.

3.4. Uncertainty budget for the sensitivity of MBAS from two experiments

To evaluate the developed methodology based on the calibration method using real datasets, the measurement uncertainties were also analyzed. Table 5 shows a list of all relevant uncertainty components for the calibration of the MBAS of the type, and states the estimates, the sensitivity coefficient function, and the resulting uncertainty contribution as standard uncertainty of the sensitivity (from Eq. (22)).

Explanations regarding the uncertainty components are:
- $(\sigma_{f_A}, \sigma_{f_B}$ and $\sigma_{d_{AB}}$) Calculation of the standard deviation of the mean value of typically 3–5 repeat measurements.
- $(f_A + f_B$ and $d_{AB})$ The sensitivity coefficient of the MABS according to Section 3.2 (Fig. 10).
- $(\sigma_{W_{AB}})$ According to Eq. (22), the total uncertainty contributions for the calibration of the sensitivity $\sigma_{W_{AB}}$ is ±0.0270 mm$^2$, we find the biggest function standard deviation error is $\sigma_{d_{AB}}$.

Similarly, we can obtain uncertainty of the sensitivity for other points, as shown in Table 5.

Table 6 shows a list of all relevant uncertainty components for the cylindrical plano-convex lens measurement of the type, and states the estimates, the sensitivity coefficient function, and the resulting uncertainty contribution as standard uncertainty of the sensitivity (from Eq. (23)).

Explanations regarding the uncertainty components are:
- $(\sigma_{f_k}, \sigma_{f_k}$ and $\sigma_{d_{AB}}$) Calculation of the standard deviation of the mean value of typically 3–5 repeat measurements. To evaluate the standard deviation for the radius of curvature $R$ of the cylindrical plano-convex lens, an experiment was developed using conventional high-precision machines (MITUTOYO FALCIO707) to measure the same cylindrical plano-convex lens. It has a radius of curvature of 519.084 mm (the uncertainty of the measured radius $R$ is 0.816 mm). Here, the indication accuracy of the MITUTOYO FALCIO707 is (1.9+4L/1000) μm.
- $(\Delta d_{AB})$ The sensitivity coefficient of the MABS according to Section 3.3 (Fig. 12).
- $(R + 2k)$ Calculation of the mean value of typically 5–10 repeat measurements. The standard average value of the distance between the lens and aspheric surface $k$ is 55.5 (in mm). Here, as shown in
Table 5
Uncertainty budget of the MBAS for plane mirror measurement (from Eq. (22)).

<table>
<thead>
<tr>
<th>Type</th>
<th>Uncertainty component</th>
<th>Estimate/mm</th>
<th>Sensitivity coefficient</th>
<th>Estimate/mm</th>
<th>Uncertainty contribution/mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{fA}$</td>
<td>Standard deviation of the focal distance in point A</td>
<td>0.0000476</td>
<td>$d_{A}\delta$</td>
<td>0.511</td>
<td>0.0000243</td>
</tr>
<tr>
<td>$\sigma_{fB}$</td>
<td>Standard deviation of the focal distance in point B</td>
<td>0.0000460</td>
<td>$d_{B}\delta$</td>
<td>0.511</td>
<td>0.0000235</td>
</tr>
<tr>
<td>$\sigma_{sAB}$</td>
<td>Standard deviation of the lens spacing</td>
<td>0.0000293</td>
<td>$f_{A} + f_{B}$</td>
<td>92.114</td>
<td>0.0270</td>
</tr>
</tbody>
</table>

Table 6
Uncertainty budget of the MBAS for cylindrical plano-convex lens measurement (from Eq. (23) and the radius of curvature of the workpiece is 519 mm).

<table>
<thead>
<tr>
<th>Type</th>
<th>Uncertainty component</th>
<th>Estimate/mm</th>
<th>Sensitivity coefficient</th>
<th>Estimate/mm</th>
<th>Uncertainty contribution/mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{s}$</td>
<td>Standard deviation of the radius of curvature</td>
<td>0.816</td>
<td>$\Delta d_{A}$</td>
<td>0.0747</td>
<td>0.0609</td>
</tr>
<tr>
<td>$\sigma_{l}$</td>
<td>Standard deviation of the distance between lens and workpiece surface</td>
<td>0.1</td>
<td>$\Delta d_{B}$</td>
<td>0.0747</td>
<td>0.00747</td>
</tr>
<tr>
<td>$\sigma_{\Delta d_{A}B}$</td>
<td>Standard deviation of the differential spacing</td>
<td>0.00000169</td>
<td>$R+2k$</td>
<td>630</td>
<td>0.00106</td>
</tr>
</tbody>
</table>

Table 7
Uncertainty budget of the MBAS for cylindrical plano-convex lens measurement (from Eq. (23) and the radius of curvature of the workpiece is 363.3 mm).

<table>
<thead>
<tr>
<th>Type</th>
<th>Uncertainty component</th>
<th>Estimate/mm</th>
<th>Sensitivity coefficient</th>
<th>Estimate/mm</th>
<th>Uncertainty contribution/mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{s}$</td>
<td>Standard deviation of the radius of curvature</td>
<td>0.816</td>
<td>$\Delta d_{A}$</td>
<td>0.0747</td>
<td>0.0609</td>
</tr>
<tr>
<td>$\sigma_{l}$</td>
<td>Standard deviation of the distance between lens and workpiece surface</td>
<td>0.1</td>
<td>$\Delta d_{B}$</td>
<td>0.0747</td>
<td>0.00747</td>
</tr>
<tr>
<td>$\sigma_{\Delta d_{A}B}$</td>
<td>Standard deviation of the differential spacing</td>
<td>0.00000169</td>
<td>$R+2k$</td>
<td>630</td>
<td>0.00106</td>
</tr>
</tbody>
</table>

Table 3, the radius of curvature $R$ of the cylindrical plano-convex lens is 519 (in mm).

($\sigma_{W_{AB}}$) According to Eq. (23), the same order of magnitude can be assigned to the bounds on the uncertainty of the sensitivity in ±0.0628 mm², where in the cylindrical plano-convex lens measurement, the biggest function standard deviation error is $\sigma_{R}$.

The uncertainty of the sensitivity is also measured for different curvature radii. Table 7 shows the uncertainty budget of the MBAS for the cylindrical plano-convex lens measurement when the radius of curvature of the workpiece is tuned to 363.3 mm. When the radius of curvature decreased, the uncertainty of the sensitivity decreased by several nanometers. Therefore, the calibration for the sensitivity of the MBAS can use cylindrical plano-convex lenses with different curvature radii.

Comparison of the two methods confirms that the second method (using cylindrical plano-convex lens) is more adapted for measurement with ultra high level of uncertainty. Further, the second method is simple, corresponding to a direct calculate in the sensitive parameters aiming to minimize the cost.

4. Conclusion

The results of this paper are summarized as follows:

1. A high accuracy micro aspheric measuring machine (micro-AMM) for accurate measurement of the small aspheric profile has been proposed in this paper. The schematic of the micro-AMM includes three main parts: a multi-beam angle sensor (MBAS), a rotary unit, and a bearing system. The MBAS was developed to detect the differential of the local slope components (angle difference) of a point on the mirror surface and using Fourier series, we can obtain the profile data from the angle difference.

2. To achieve the absolute accuracy of the flatness measurement under several tens nanometer, the sensitivity of MBAS should less than 0.5%. Therefore, for the determination of the sensitivity of MBAS, two independent calibration methods can be applied using plane mirror and cylindrical plano-convex lens has been proposed to measure the sensitivity of the MBAS. The measurement data analysis results using plane mirror agree well with the measurement results of the cylindrical plano-convex lens data, which verified the feasibility of the calibration for the sensitivity of MBAS using cylindrical plano-convex lens.

3. To evaluate the developed methodology based on the calibration method using real datasets, the measurement uncertainties was also analyzed. We evaluated sensitivity for each lens-array; afterwards we investigated the influence of the function standard deviation error on the calibration measurement using plane mirror and the cylindrical plano-convex. The biggest function standard deviation errors are $\sigma_{d}$ and $\sigma_{R}$ in the calibration experiment and the cylindrical plano-convex lens measurement, respectively. Therefore, the precision of the radius of curvature (cylindrical plano-convex lens) may be the main reason for the calibration for the sensitivity of the MBAS.

Comparison of the two methods confirms that the second method (using cylindrical plano-convex lens) is more adapted for measurement with ultra high level of uncertainty. Further, the second method is simple, corresponding to a direct calculate in the sensitive parameters aiming to minimize the cost.
References


