Advanced optical metrology of geometrical quantity based on pulse trains' destructive interference

○Dong WEI, Satoru TAKAHASHI, Kiyoshi TAKAMASU, and Hirokazu MATSUMOTO
Department of Precision Engineering, The University of Tokyo
Corresponding Author / E-mail: weidong@nanolab.t.u-tokyo.ac.jp TEL: +81-3-5841-6472, FAX: +81-3-5841-6472

KEYWORDS : Ultrafast phenomena, Super-resolution, Coherence, Interferometry, Metrology

The interferometric measurement using the femtosecond optical frequency comb (FOFC) is in progress at present. We had analyzed the temporal coherence function of the FOFC since it is the fundamental description of interference phenomenon. As a result, it had been understood that the coherence peak exists during the time which is equal to the repetitions interval in the traveling direction of the FOFC. In order to make the best use of the temporal coherence characteristic, we propose a novel interferometric technique using an FOFC to observe the destructive interference between two pairs of pulse trains with different relative delays. Theoretical and simulation analysis of the FOFC shows that it offers a significantly different challenge for length measurement. This technique can be applied not only to surface profilometry and tomography, but also to optical super-high resolution metrology.

Manuscript received: July 15, 2009 / Accepted: August 15, 2009

NOMENCLATURE

FOFC = femtosecond optical frequency comb
TCF = temporal coherence function

1. Introduction

Measurements of lengths are strongly demanded for not only science purposes but also industry requirements. Due to its increased frequency stability and very broad frequency band, the femtosecond optical frequency comb (FOFC) is a good choice for high precision long-distance measurements.

In 1998, Chekhovsky et al. suggested a possible scheme for a pulse distance-measuring interferometer using an FOFC as a low coherence light source. [1] In 2000, Minoshima and Matsumoto measured the phase shift of the FOFC beat component between longitudinal modes for long distance measurements. [2] In 2001, Yasui et al. proposed a method that high temporal coherence of an unbalanced optical-path Michelson interferometer produced an adjacent pair of pulse trains from an FOFC for length measurement to test stabilization of the FOFC light source. [3] Following that pioneering work, various experiments were proposed using the high temporal coherence peak between a pair of pulse trains for length measurements. In 2002, Yamaoka et al. reported a new interferometry technique by using high temporal coherence of different pair of pulse trains from an FOFC to precisely calibrate the group refractive index of air for improving length measurements. [4] In 2004, Ye proposed to use high temporal coherence between a different pair of pulse trains from an FOFC for a long arbitrary length measurement to be less than an optical fringe by changing the pulse repetition frequency of the FOFC. [5] In 2006, Ki-Nam Joo and Seung-Woo Kim reported a method that enables to measure the length between two consecutive pulse trains by observing the interference fringes in frequency domain. [6] In 2008, Cui et al. experimentally demonstrated a possible new scheme to apply the proposal of Ye, scanning the reference mirror of the unbalanced optical-path Michelson interferometer between the pulse repetition periods of the FOFC to obtain the interference fringes between different pairs of pulse train instead of changing the pulse repetition frequency of the FOFC. [7] In 2009, A numerical model of pulse propagation in air was reported and was applied to length measurements by using interference fringes between chirped pulses in short displacements by Balling et al. [8] and long displacements by Cui et al. [9].

Generally speaking, using only one FOFC light source for a length measurement to be less than an optical fringe that means one must to observe the interference fringes between pairs of pulse train. For a long arbitrary length measurement to be less than an optical fringe that means one must to observe the interference fringes between pairs of pulse train in the range of the pulse repetition periods of the FOFC as proposed by Ye to change the pulse repetition frequency of the FOFC and as proposed by Cui et al. to scan the reference mirror between the pulse repetition periods of the FOFC. However, it is a trade-off relation between the variability and the stability of frequency of the FOFC. And there is no simple device which can do scanning the range of the pulse repetition periods of the FOFC such as 30 cm [7] to 6 m [2] with sub-nanometer resolution.

Using two FOFC light source potentially provide another solution. But to obtain the intermode beat [10] or the interference fringes [11] between different pulse train from different FOFC light sources that means the following things of the two FOFC light sources. The very good stability of phase locks and the differential frequency between the pulse repetition frequencies are required. The measurement system will become expensive and complicated.

However, to obtain the interference fringes between pairs of pulse train, the use of high temporal coherence in these experiments [1-9] is restricted to a pair of pulse trains. We have investigated the temporal coherence function of a pulse train from an FOFC. [12, 13] The results show that same high temporal coherence peaks exist during the period equal to the repetition intervals in the traveling direction of the FOFC. We have presented a novel interferometric technique using an FOFC to simultaneously observe "separated" high temporal coherence peak between two pairs of pulse trains with different relative delays [14-16]. As a special demonstration, measurement of the average temperature change over a distance of 3 m was also demonstrated. [14]

Based on these previous works, in this study, we demonstrated a modified Michelson interferometer to simultaneously observe "overlapped" high temporal coherence between different pairs of pulse trains from an FOFC for a long length measurement to less than an optical fringe. This is, to the best of our knowledge, the first demonstration of simultaneous observation of overlapped high temporal coherence peaks between two pairs of pulse trains with different relative delays. From a different perspective, our results show that this technique can be used as direct link between the FOFC and length measurement. Fortunately, this new approach to high-accuracy length metrology, by combining an ordinary Michelson interferometer and an unbalanced optical-path reversed-phase interferometer, has its own advantages. First, as shown below, this technique maintains the simplicity of the equipment. Second, the displacement metrology can be achieved without fringe analysis which typically restricted the measurement speed in an ordinary Michelson interferometer scheme. For simplicity of explanation, we have neglected the dispersion and absorption of the optical devices over the FOFC's illumination bandwidth.

The outline of this report is as following: Section 2 gives a review of principles for the FOFC light source, the TCF of the FOFC and interference fringes' formation between different pairs of pulse train, Section 3 investigates simulation, and lastly Section 4 presents a summary.

2. Principles

2.1 An FOFC

In the time domain, the "carrier" pulse moves with the center carrier frequency f_c of the FOFC. When the electric field packet repeats at the pulse repetition period T_R, the "carrier" phase slips by $\Delta\varphi_{ce}$ to the carrier-envelope phase because of the difference between the group and phase velocities. In the frequency domain, a mode-locked FOFC generates equidistant frequency comb lines with the pulse repetition frequency $f_{rep} \propto 1/T_R$, and due to phase slip $\Delta\varphi_{ce}$, the whole equidistant-frequency comb is shifted by f_{CEO}.

2.2 TCF of an FOFC

For convenience of explanation, we herein briefly review and summarize the most important characteristic of the temporal coherence function of an FOFC, which can be founded in derails in Ref.[12]

The power spectrum of an FOFC light source can be expressed as

$$P(f) \propto A(f - f_c) \times \text{comb}(f_{rep}),$$
$$\text{comb}(f_{rep}) \equiv \sum_{m=-\infty}^{+\infty} \delta(f - mf_{rep}), \qquad (1)$$

where $A(f - f_c)$ is the envelope function of the FOFC power spectrum. Based on the Wiener–Khintchine theorem, the interferometric signal of the autocorrelation function is given by the inverse Fourier transform of the spectrum of the source, and we have

$$\gamma(\tau) \propto \mathrm{F}^{-1}\left[A(f - f_c)\right] \otimes \text{comb}(T_R),$$
$$\text{comb}(T_R) \equiv \sum_{m=-\infty}^{+\infty} \delta(\tau - mT_R). \qquad (2)$$

From Eq. (2), the temporal coherence function periodically displays a high temporal coherence peak where the pulse trains signal of the FOFC displays a high-intensity peak with the pulse repetition period T_R.

2.3 Interference fringes' formation between two pairs of pulse train

The optical comb mode-lock technique results in interference fringes reappearing between different pairs of pulse trains. When one pulse train E_{train1} and the relatively delayed pulse train E_{train2} finally overlap in space, one can expect that interference fringes can be observed. After performing the time integration we obtain the interference fringes as

$$I(t) \propto |\gamma(\tau)| \cos[\mathrm{mod}(h \times \Delta\varphi_{ce}, 2\pi)]. \qquad (3)$$

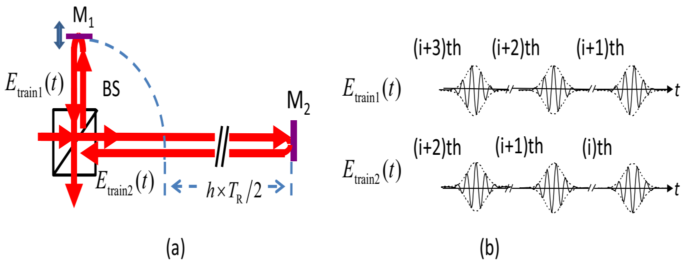


Fig. 1. Relative delay between pulse trains formed by an unbalanced optical-path Michelson interferometer (a) Simplified optical layout for interferometry. (b) Relative positions between two pulse trains (integer h=1)

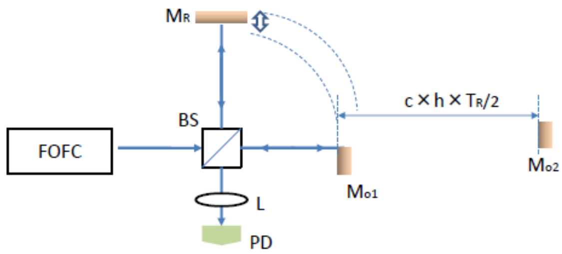


Fig. 2. Simplified optical layout for the proposed interferometry.

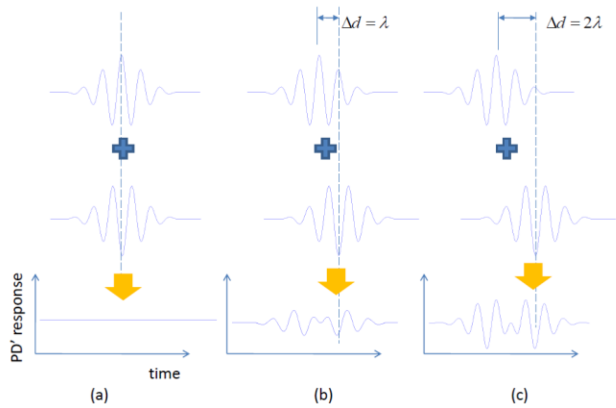
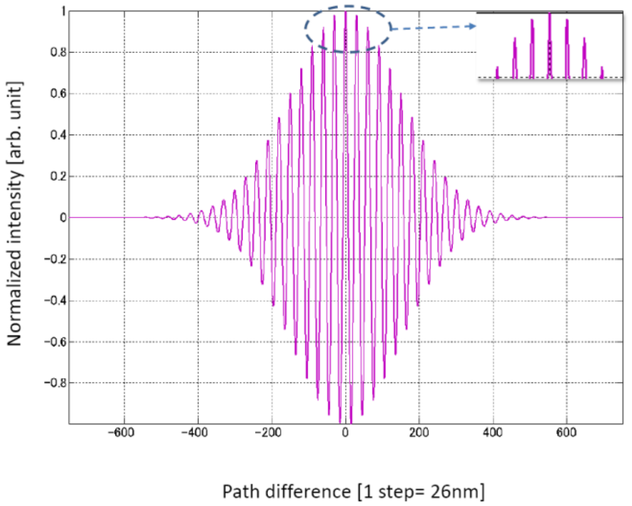


Fig. 3. Interference fringes' formation with the different displacements.

(a)



(b)

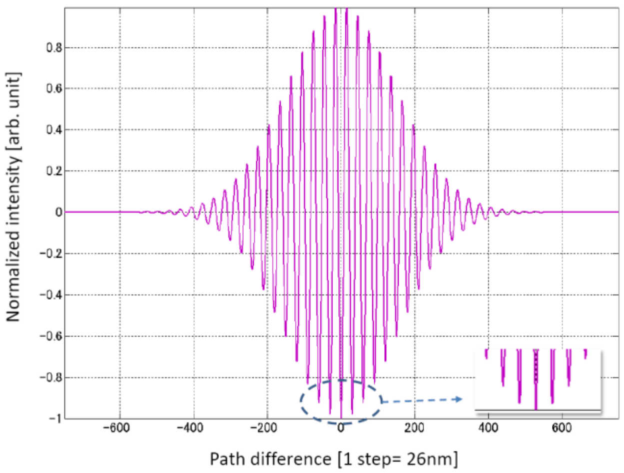


Fig. 4. Interference fringes. (a) Between M_R and M_{o1}. (b) Between M_R and M_{o2}.

2.4 Interference fringes' formation between three pairs of pulse train

Let us consider the interference fringes formed by a modified Michelson interferometer, as shown in Fig. 2.

The pulse train from the FOFC is introduced into the modified Michelson interferometer. The modified Michelson interferometer is a combination of an ordinary Michelson interferometer and an unbalanced optical-path reversed-phase Michelson interferometer. The ordinary Michelson interferometer is composed of a beam splitter BS, a reference mirror M_R, and an object mirror M_{o1}. The unbalanced optical-path reversed-phase Michelson interferometer is composed of the same BS and M_R, and a different object mirror M_{o2} to vary the relative delay between the pulse trains, which are reflected by the surface of the mirror M_{o1}. The mirror M_{o2} is located far from the surface of the M_{o1} at a distance of $c \times h \times T_R/2$ m (where $h \times \Delta\varphi_{ce} = \pi$.) in space.

During the measurement, by moving the common reference mirror M_R of the two interferometers by means of a computer-controlled and calibrated ultrasonic stepping motor SM, we observed the following interference fringes.

After traveling different path lengths, these two pulse trains overlap at the beam splitter. Lens L_1 images the interference fringe pattern onto a photodetector PD.

In Fig. 3, the pairs of interference fringe pattern, which are respectively formed between the reference mirror M_R and the surface of mirror M_{o1}, and between the reference mirror M_R and the surface of the mirror M_{o2}, have the same value of amplitude and move in the same direction, but the slippage of the interference fringe phase to the carrier-envelope between them is reversed (namely, π). The result is that the two interference fringes completely cancel each other out when there is complete overlap (see Fig.3(b)). At the state of complete overlap, no interference fringes can be observed at the BS. In the case, that the two interference fringes partly overlapped each other, the amplitude of the obtained interference fringes will be reduced. (see Fig.3(a,c))

In essential, by observing the relative peak value of the acquired interference fringes, one can obtain the length information between two separated points.

3. Simulation

The simulation is carried out with a Gaussian profile polarization mode locked FOFC. The pulse duration, the repetition rate of the FOFC are 180 femtoseconds and100 MHz, respectively. The output wavelength of the pulse is centered at λ=1550 nm with a spectral width of $\Delta\lambda$ = 80 nm. The corresponding coherence length is Lc = 26.5 µm.

Figure 4 illustrates the acquired interference fringes peaks formed for the surface of M_R and M_{o1} and the surface of M_R and M_{o2}, respectively. As shown in Fig.4, we can confirm the interference fringe phase slip π to the carrier-envelope between acquired interference fringes.

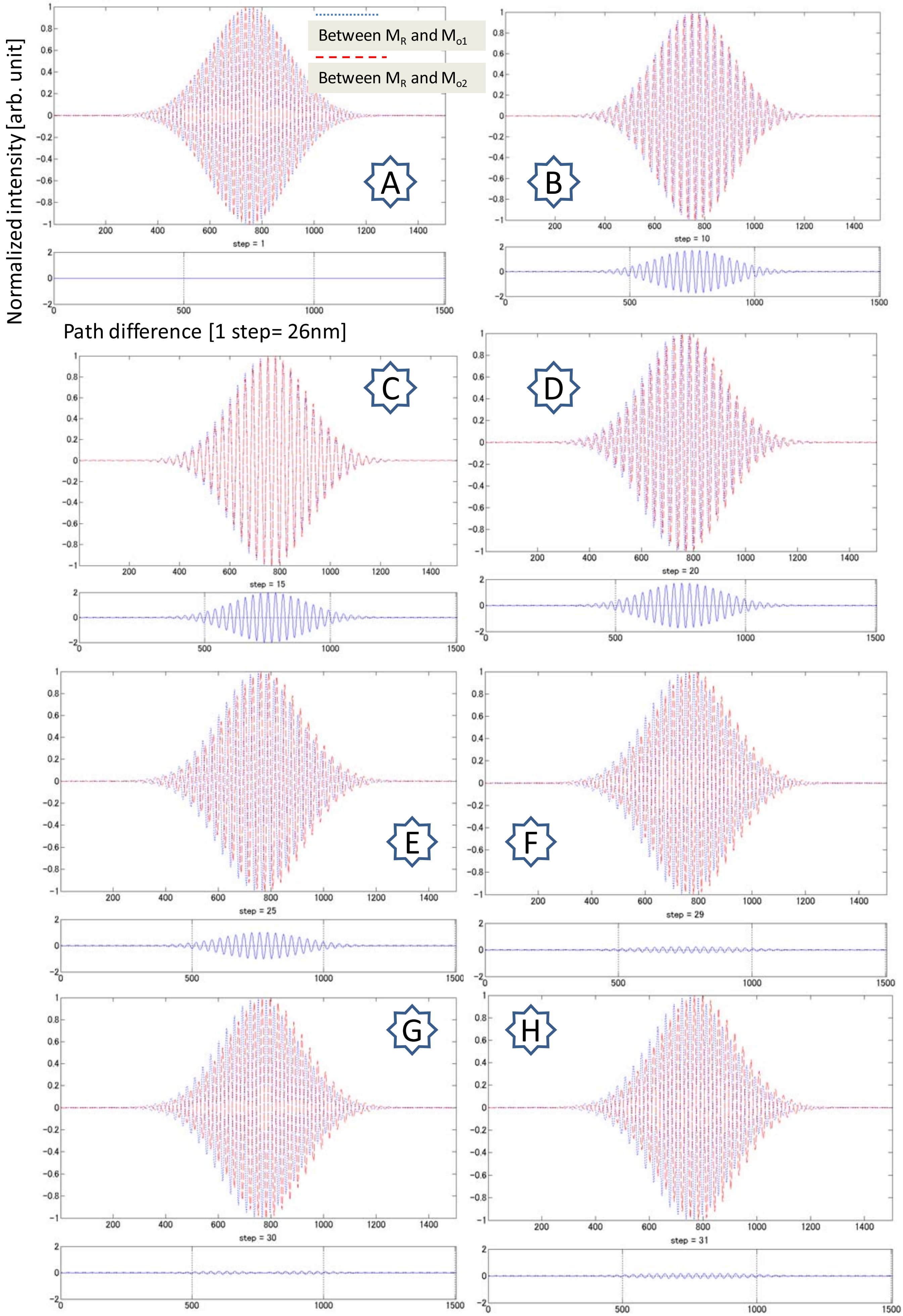
As noted in Fig.5, the peak formed for the surfaces of M_R and M_{o2} comes close to the peak formed for the surface of M_R and M_{o1}, by reducing the displacement between the two mirrors M_{o1} and M_{o2}, the maximum peak value of the acquired interference fringes and the the sum of the surface integral between the acquired interference fringes and the x axis are periodically reduced. When the two interference fringes peaks completely overlapped, by reducing the relative delay displacement between the two mirrors M_{o1} and M_{o2} equals to zero, the two interference fringes cancelled each other out. Then the acquired interference fringes value is zero.

4. Summary

We have presented a novel interferometric technique using an FOFC to observe the destructive interference between pairs of pulse trains with different relative delays for a length measurement. It is important to note that because it was possible to simultaneously observe "overlapped" high temporal coherence between different pairs of pulse trains with the reversed-phase, the relative positions of two separated point could be measured without fringe analysis. The results of this investigation show that, with an appropriate optical system, the FOFC can be used to observe the destructive interference for a length measurement. Finally, it is anticipated that the present technique will be a powerful metrological tool not only for surface profilometry and tomography but also for super-resolution metrology.

ACKNOWLEDGEMENTS

Part of this research work was supported by the Global Center of Excellence (COE) Program on "Global Center of Excellence for Mechanical Systems Innovation" granted to The University of Tokyo, from the Japanese Government. We are also grateful to NEOARK Corporation for providing the femtosecond fiber laser. Dong Wei gratefully acknowledges the scholarship given from Takayama International Education Foundation, Heiwa Nakajima Foundation, Ministry of Education, Culture, Sports, Science, and Technology of Japan, respectively.


Between M_R and M_{o1}
Between M_R and M_{o2}
Normalized intensity [arb. unit]
Path difference [1 step= 26nm]
A
step = 1
B
step = 10
C
step = 15
D
step = 20
E
step = 25
F
step = 29
G
step = 30
H
step = 31

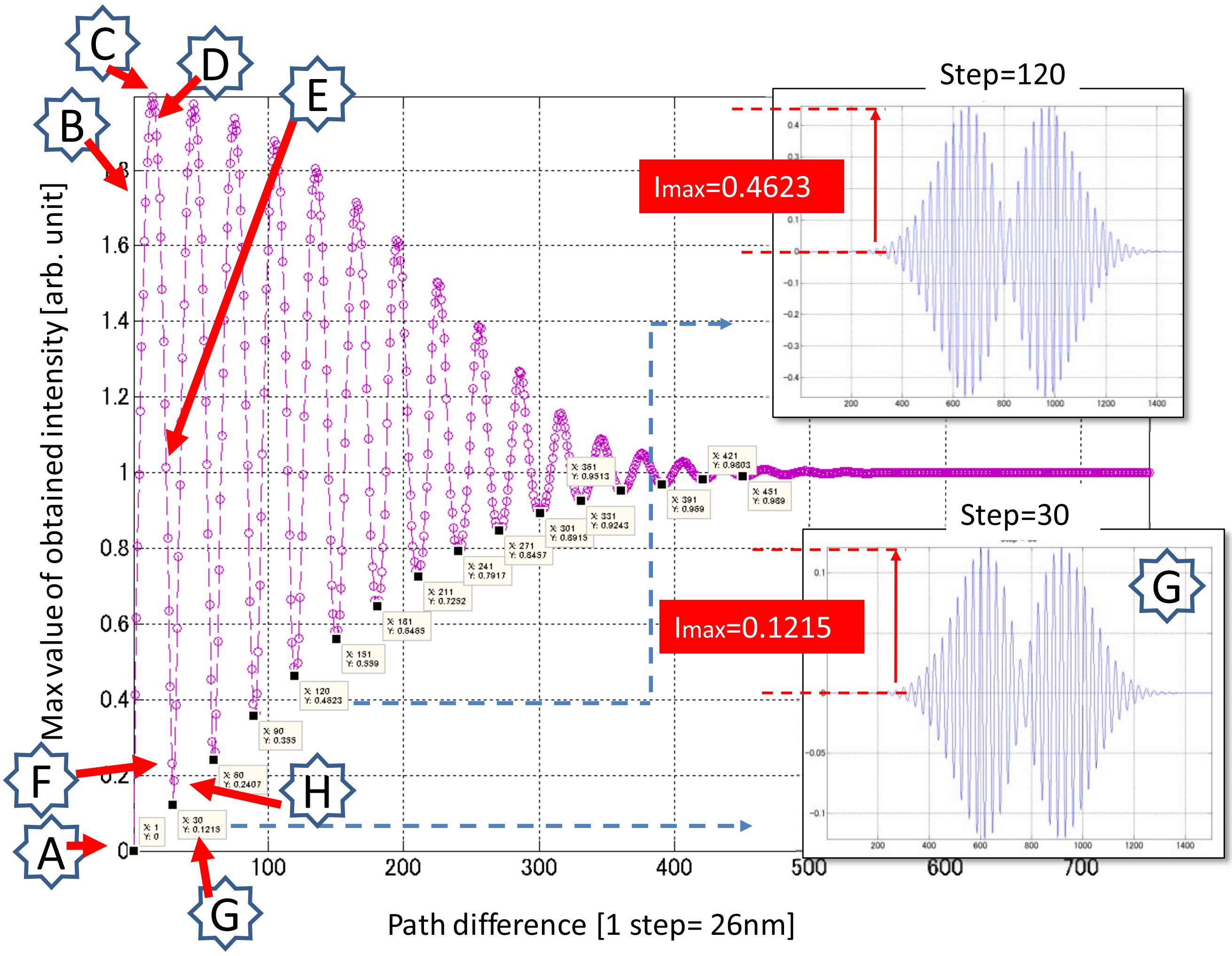


Fig. 5. Destructive interference between two pairs of pulse trains with the different displacements.

REFERENCES

1. A. M. Chekhovsky, A. N. Golubev, and M. V. Gorbunkov, "Optical Pulse Distance-Multiplying Interferometry," Appl. Opt. **37**, 3480-3483 (1998).

2. K. Minoshima and H. Matsumoto, "High-Accuracy Measurement of 240-m Distance in an Optical Tunnel by Use of a Compact Femtosecond Laser," Appl. Opt. **39**, pp, 5512-5517 2000.

3. T. Yasui, K. Minoshima, and H. Matsumoto, "Stabilization of femtosecond mode-locked Ti:sapphire laser for high-accuracy pulse interferometry," IEEE Journal of Quantum Electronics **37**, 12-19 (2001).

4. Y. Yamaoka, K. Minoshima, and H. Matsumoto, "Direct Measurement of the Group Refractive Index of Air with Interferometry between Adjacent Femtosecond Pulses," Appl. Opt. **41**, 4318-4324 (2002).

5. J. Ye, "Absolute measurement of a long, arbitrary distance to less than anoptical fringe," Opt. Lett. **29**, 1153-1155 (2004).

6. K.-N. Joo and S.-W. Kim, "Absolute distance measurement by dispersive interferometry using a femtosecond pulse laser," Opt. Express **14**, 5954-5960 (2006).

7. M. Cui, R. N. Schouten, N. Bhattacharya, and S. A. Berg, "Experimental demonstration of distance measurement with a femtosecond frequency comb laser," J. Eur. Opt. Soc. Rapid Publ. **3**, 08003 (2008).

8. P. Balling, P. Křen, P. Mašika, and S. A. van den Berg, "Femtosecond frequency comb based distancemeasurement in air," Opt. Express **17**, 9300-9313 (2009).

9. M. Cui, M. G. Zeitouny, N. Bhattacharya, S. A. van den Berg, H. P. Urbach, and J. J. M. Braat, "High-accuracy long-distance measurements in air with a frequency comb laser," Opt. Lett. **34**, 1982-1984 (2009).

10. Y. Shuko, Y. Takeshi, A. Tsutomu, and H. Yuki, "A Distance Meter Using a Sub-Terahertz Intermode Beat in an Optical Frequency Comb," in *Conference on Lasers and Electro-Optics/Quantum Electronics and Laser Science Conference and Photonic Applications Systems Technologies*, OSA Technical Digest (CD) (Optical Societ y of America, 2008), CMEE5.

11. I. Coddington, W. C. Swann, L. Nenadovic, and N. R. Newbury, "Rapid and precise absolute distance measurements at long range," Nat Photon **3**, 351-356 (2009).

12. D. Wei, S. Takahashi, K. Takamasu, and H. Matsumoto, "Analysis of the temporal coherence function of a femtosecond optical frequency comb," Opt. Express **17**, 7011-7018 (2009).

13. D. Wei, S. Takahashi, K. Takamasu, and H. Matsumoto, "Study on the temporal coherence function of a femtosecond optical frequency comb," in *Optical Measurement Systems for Industrial Inspection VI*, (SPIE, 2009), 73891R-73898.

14. D. Wei, S. Takahashi, K. Takamasu, and H. Matsumoto, "Simultaneous Observation of High Temporal Coherence between Two Pairs of Pulse Trains Using a Femtosecond-Optical-Frequency-Comb-Based Interferometer," Jpn. J. Appl. Phys. **48**, 070211 (2009).

15. D. Wei, S. Takahashi, K. Takamasu, and H. Matsumoto," Femtosecond optical frequency comb-base tandem interferome-ter," J. Eur. Opt. Soc. Rapid Publ. (in press)

16. D. Wei, S. Takahashi, K. Takamasu, and H. Matsumoto," Experimental observation of pulse trains' destructive interference with a femtosecond optical frequency comb-based interferometer," Opt. Lett. (in press)