APPLICATION OF FEMTOSECOND OPTICAL FREQUENCY COMB'S TEMPORAL COHERENCE CHARACTERISTIC TO A DISTANT ESTIMATION

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Abstract

The improved frequency-stability and the very broad frequency-band of femtosecond mode-locked optical frequency comb (FOFC) have led to new applications in several precision metrology application areas such as high-precision optical frequency metrology, spectroscopy, and distance measurements. We have analyzed the temporal coherence function of the FOFC. As a result, it has been understood that the temporal coherence peaks exist during the time which is equal to the repetition interval in the traveling direction of the FOFC. In order to make the best use of the temporal coherence characteristic of an FOFC, we propose an FOFC-based multiplex Michelson interferometer that can use as a distant estimation. The present technique is expected to be useful for high-precision measurement of distances for not only science purposes but also industry requirements.

Introduction

Due to their improved frequency-stability and very broad frequency-band, there is considerable interest in the development of novel optical measurement techniques based on the characteristics of femtosecond optical frequency combs (FOFC). In the initial stage, the pioneering works [1-8] were done by the groups at the National Institute of Advanced Industrial Science and Technology (AIST). For example, in 2000, Minoshima and Matsumoto reported a high resolution long distance measurement method by using the phase shift of the stable intermode beats [3].

In recent years, the FOFC-based length measurement methods have become of great interest because of its fundamental role not only in science purposes but also industry requirements. For scientific space mission projects such as DARWIN and LISA, the FOFC is a good choice for high-precision absolute long-distance measurements. For example, see reference [9]. For industry requirements, because the FOFC is expected as a new standard tool of the unit system of “Length” and “frequency”, how to perform displacement metrology which directly linked to a frequency standard is a new challenge. And in 2009, the FOFC was specified for a new specified standard instrument of length in Japan. For example, see reference [10].

Most of the works have been done on FOFC-based applied metrology, but there are few reports on the temporal coherence function (TCF) of an FOFC, which is important for interference phenomenon. We had analyzed the temporal coherence function of the FOFC [11-12]. As a result, it has been understood that the coherence peak exists during the time which is equal to the repetitions interval in the traveling direction of the FOFC. Based on this new understanding, new applications can be proposed fairly readily [13-16]. For example, measurement of the average temperature change over a distance of 3 m was demonstrated [13]. Moreover, we proposed FOFC-based tandem interference, the possibility of the length measurement and the length delivery by the proposed system were also reported [14].

The present work focuses mainly on development of FOFC-based metrology methods which make the best use of the temporal coherence characteristic of an FOFC. Here, we describe an FOFC-based multiplex Michelson interference that can use as length estimation by using undersampled interference fringes.

Fortunately, this new approach to distance estimation, by combining an ordinary Michelson interferometer and an unbalanced optical-path interferometer, has its own advantages. First, as shown below, this technique maintains the simplicity of the equipment. Second, the displacement metrology can be achieved without fringe analysis which typically restricted the measurement speed in an ordinary Michelson interferometer scheme.

For simplicity of explanation, we have neglected the dispersion and absorption of the optical devices over the FOFC’s illumination bandwidth.

The outline of this report is as followings: Section 2 gives a review of an FOFC light source, the TCF of the
FOFC and interference fringes’ formation between different pairs of pulse train, Section 3 investigates experiments, and lastly Section 4 presents a summary.

Principles

For convenience of explanation, first, a summary description of an FOFC in the time domain and in the frequency domain is described. Second, a mathematical description of the TCF of the FOFC is given. Last, to observe the TCF of the FOFC, the interference fringe pattern formed by a modified Michelson interferometer is considered.

FOFC

For convenience of explanation, we herein briefly review and summarize the most important characteristic of the temporal coherence function of an FOFC, which can be founded in detailes in Ref.[11]

The power spectrum of an FOFC light source can be expressed as

\[ P(f) \propto A(f - f_\text{c}) \times \sum_{m=-\infty}^{\infty} \delta(f - mf_\text{c}) \, , \]

(1)

where \( A(f - f_\text{c}) \) is the envelope function of the FOFC power spectrum. Based on the Wiener–Khintchine theorem, the interferometric signal of the autocorrelation function is given by the inverse Fourier transform of the spectrum of the source, and we have

\[ \gamma(t) \propto F^{-1} \left[ A(f - f_\text{c}) \right] \otimes \sum_{m=-\infty}^{\infty} \delta(t - mT_\text{F}) \, . \]

(2)

From Eq. (2), the temporal coherence function periodically displays a high temporal coherence peak where the pulse trains signal of the FOFC displays a high-intensity peak with the pulse repetition period \( T_\text{F} \).

Interference fringes’ formation

Next, let us consider the interference fringe pattern formed by a modified Michelson interferometer shown in Fig. 2. Firstly the incoming pulse train is split into two identical parts \( E_\text{inc}^{(1)}(t) \) and \( E_\text{inc}^{(2)}(t) \) at the beam splitter BS, relatively \( E_\text{inc}^{(1)}(t) \) delays to \( E_\text{inc}^{(2)}(t) \) and they finally are recombined them at the BS. (see Fig. 3).

Fig. 1 . Optical comb. (a) Time-domain and (b) frequency domain

Fig. 2. Optical layout for a modified Michelson interferometer.
When the pulse trains overlap in space, one would expect that interference fringes can be observed, as shown in Fig. 1(b). After performing the time integration we obtain

$$I(\tau) \propto |\gamma(\tau)| \cos[\text{mod}(h \times \Delta \varphi_n, 2\pi)]. \quad (3)$$

**Experiment**

For convenience of explanation, first, the generation of the interference fringes formed between different pulse trains is confirmed with the modified Michelson interferometer as introduced by the principle. Second, a description of a multiple Michelson interferometer is given. Last, applying the FOFC-based multiplex Michelson interference, a technique that can use as length estimation by undersampled interference fringes is described.

**Interference fringes formed with different pulse trains**

The basic scheme of the first experiments is considered which is as same as the setup as shown in Fig. 2. The experiment is carried out with a system consisting of an FOFC (FC1500, Menlo Systems, \(f_{\text{rep}} = 100\,\text{MHz}\)), the modified Michelson interferometer, and system controls. The pulse trains from the FOFC is expanded and collimated by a collimator \(C_1\) and introduced into the modified Michelson interferometer. The modified Michelson interferometer are composed of a beam splitter \(\text{BS}\), a reference mirror \(M_1\), and an object mirror \(M_2\). During the measurement, by moving the reference mirror \(M_1\) of the interferometer by means of a computer-controlled ultrasonic stepping motor \(\text{SM}\), we observed the interference fringes. To obtain the interference fringes formed between different pulse trains, the object mirror is placed by the relative optical displacement \(Z_2\) (about 1.5 m) and \(Z_3\) (about 3 m) from the position \(Z_1\), respectively. After travelling different path lengths, these two pairs of pulse trains overlap at the BS. Lens \(L_1\) images the interference fringes onto a photo detector \(\text{PD}\).
Figures 4-6 illustrate the acquired interference fringes with the different relative optical displacement.

**Interference fringes formed by a multiple Michelson interferometer**

The basic scheme of the second experiments is shown in Fig. 7. The modified Michelson interferometer is a combination of an ordinary Michelson interferometer and two unbalanced optical-path Michelson interferometer. The ordinary Michelson interferometer is composed of a beam splitter BS, a reference mirror M1, and an object mirror HM1. The unbalanced Michelson interferometer is composed of the same BS and M1, and a different object mirror set BS2 and M2 to vary the relative delay between the three pulse trains, which are reflected by half-reflecting mirror HM1, BS2 and mirror M2, respectively. During the measurement, by moving the common reference mirror M1 of the three interferometers by means of a computer-controlled ultrasonic stepping motor SM, we could observe the interference fringes. After travelling different path lengths, these three pairs of pulse trains overlap at the BS.

![Fig. 7. Optical layout for a multiple Michelson interferometer. Abbreviations are defined in the text.](image)

In Fig. 8 we show the acquired interference fringes recorded when after travelling different path lengths, these three pairs of pulse trains sequentially overlapped at the beam splitter. The interference fringe signals also exhibit high contrasts between the three pairs of pulse trains by the relative displacements about 0.0 m, 1.5 m and about 3.0 m.

**Undersampled interference fringes formed by an ordinary Michelson interferometer**

Figure 9 illustrates the acquired interference fringes with the relatively large optical displacement (> 1 mm). Due to the restricted resolution of the oscilloscope, the undersampled interference fringes are recorded. Because the sampling theorem did not satisfy as noted in Fig.9, the Fourier transform method was not able to be applied for fringe processing. However, to obtain the relative average depth difference we need to get length information from the fringes pattern as shown in Fig.9.

**Undersampled interference fringes formed by a multiple Michelson interferometer and distance estimation**

Figure 10 illustrates the acquired interference fringes formed by a multiple Michelson interferometer. (For simplicity of explanation, the half-reflecting mirror HM2 is removed from the second experimental system.)

Figure 10 shows the result when a 500-ȝm step displacement by the shift of M2 was repeatedly induced over a distance range of 1000 μm. As noted in Fig. 4, the second interference fringes peak moves away from the first interference fringes peak when the object mirror is moved. We fitted the centers of the peaks, and measured the relative scan steps between the two peaks. The obtained scan step value of the change in relative displacement is 2.0±0.1 μm.
And as show in Table 1, we obtained the good linearity between the change in the step displacement shift of M2 and the relative scan steps. The relative average depth difference of two surfaces was measured as 2.9 ± 0.1 mm. According to the catalog, the theoretical value of the relative displacement is 3.0 ± 0.1 mm. The measured result of distance is in good agreement with the set distance value within a standard deviation.

Table 1 Relationship between the step displacement shift of M2 and relative scan steps

<table>
<thead>
<tr>
<th>Shift of M2 [μm]</th>
<th>50</th>
<th>100</th>
<th>100</th>
<th>400</th>
<th>500</th>
<th>500</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shift of Peak [steps]</td>
<td>48</td>
<td>108</td>
<td>95</td>
<td>395</td>
<td>504</td>
<td>500</td>
</tr>
</tbody>
</table>

Summary

In summary, we have presented a novel interferometry technique using an FOFC to measure the depth difference by undersampled interference fringes with a multiple Michelson interferometer. The unique feature of an FOFC-based multiple Michelson interferometer is that high temporal coherence peaks can be observed between different pairs of pulse trains from the FOFC light source. And the length information can be recorded and calibrated by interference fringes between different pairs of pulse trains which are located in separated places. The proof-of-the-principle experiments were presented and validated the proposed technique. The present technique is expected to be useful for not only surface profilometry but also tomography.

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References


