Parameter Calibration for non-Cartesian CMM

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Abstract:

It is described how to calibrate the kinematic parameters of articulating CMM and parallel CMM. In this paper, the artefact which consists of spheres is selected as a physical constraints for self-calibration. The number of spheres and the ability of calibration is discussed. The kinematic calibration of the articulating CMM is performed using the artefact of two spheres. The kinematic calibration of the parallel CMM is performed using the artefact of one sphere. Generally, it is not easy to determine the coordinates of the stylus of the parallel CMM. This paper describes how to determine the coordinates of that. Comparing the articulating CMM and the parallel CMM, the parameter of CMM with a translational input sensor can be calibrated using one sphere. It is just self-calibration.

1. Introduction

Different type of CMMs from Cartesian CMM have been developed. As the non-Cartesian CMMs have different structure and sensor inputs, when the parameter of non-Cartesian CMM is calibrated, the method and the artefact are also different from those of Cartesian CMM. Parameter calibration methods were classified into two methods in [1]. Two methods were following,

1. The method using redundant information from additional sensors.
2. The method using physical constraints.

The parameter calibration is performed by either method.

In [2], the system has the redundant sensors, the kinematic calibration is performed by first method.

In [3-5], the spheres are used as the artefact, then the kinematic calibration is performed by second method. In this paper, the second method, physical constraints, is adapted and the artefact type is considered. In some structure of CMM, the self-calibration of the kinematic parameters of CMM can be performed without the calibrated artefact.

First, the artefact and the model of calibration are considered with the articulating CMM.
It is shown that the ball-bar is the most adequate artefact.

Second, the model of calibration is considered with the 2D parallel CMM. It is shown that one sphere without any calibrated coordinates can be used as the artefact in this case. This method is approximately self-calibration method except using a sphere.

2. Principle

We considered the parameter calibration of Cartesian CMM, the articulating CMM and the parallel CMM. The articulating CMM and the parallel CMM have more than three degree of freedom of movement . As a result, those CMM could measure the same point in different orientation of stylus. When the artefact ,which constrains a position, is used, the orientation of stylus is redundant freedom. Therefore, when a point is measured, only one information is taken in the case of Cartesian CMM. A lot of information is taken in the case of the articulating and the parallel CMM in proportion to the number of orientations of stylus.

In this paper, the physical constrains consist of some spheres, i.e. points.
The number of spheres, measurements of each sphere and dispositions of the physical constraints are n_s , n_m and n_d respectively. The freedom of position is n_c . The number of the parameter is n_p . The number of parameter of transformation from coordinate system of physical constraints to world coordinate system is n_t .
The number of equations is

$$n_c n_s n_m n_d \quad (1)$$

The number of parameters is

$$n_p + n_d n_t \ , \quad (2)$$

when the coordinates of spheres are calibrated.
The parameter could be estimated in the case of

$$n_c n_s n_m n_d \geq n_p + n_d n_t \quad (3)$$

It will be discussed later whether it is necessary to calibrate the coordinates of spheres or not.

3. Articulating CMM

The articulating CMM is generally modelled as

$$x = f(p, q, t) \quad (4)$$

x, p, q and t are a generalized coordinate of stylus, parameters, sensor inputs and parameter of coordinate transformation. Generally, there are two type of joints, a rotational joint and a translational joint. The parameters p are divided to p_r and p_t , which are the rotational parameters and translational parameters. The sensor inputs q are also divided to q_r and q_t, which are rotational sensor inputs and translational sensor inputs.

$$x = f(p_r, p_t, q_r, q_t, t) \qquad (5)$$

4. Artefact for Articulating CMM

The artefact as ball plate has some spheres on plate and the center of spheres are measured accurately. As the artefact has a lot of spheres and its weight is heavy, it is difficult to handle it.

To reduce the number of spheres, the possibility of parameter calibration using the artefact described in Table 1 is checked.

Table 1: The artefacts consist of some spheres

Number of spheres	Number of parameter of coordinate system transformation	Freedom of CMM coordinate system	Mirror image
1	Position 3	Orientation 3	Exist
2	Position 3 + Orientation 2	Orienation 1	Exist
3	Position 3 + Orientation 3	None	Exist
More than 3	Position 3 + Orientation 3	None	Not exist

When the number of spheres is 2, the coordinate system of CMM can rotate around the line connecting two spheres. Both of the right-hand and the left-hand coordinate system can be possible. Where the number of spheres is 3, the coordinate system of CMM is fixed on the artefact except the vertical direction from the plane of three spheres. In this case, both of the right-hand and left-hand coordinate system can also be possible. When the number of spheres is more than 4, the coordinate system of CMM can be fixed on the artefact.

When the number of spheres is one, the center of sphere is origin in the artefact coordinate system. The artefact coordinate system does not have any direction defined on the artefact. Then, in Eq.(5), x is always zero. Therefore Eq.(5) multiplied by k becomes Eq.(6).

$$kx = kf(p_r, p_t, q_r, q_t, t) = 0 \qquad (6)$$

When the sensor inputs are only rotational sensor inputs, Eq.(6) becomes Eq.(7).

$$kx = kf(p_r, p_t, q_r, t) = f(p_r, kp_t, q_r, t) = 0 \qquad (7)$$

It is proved that the translational parameters can be multiplied by any number k. In this case, the parameter can be calibrated except the size. After the calibration, a standard length should be measured to determine the amplitude k.

When the sensor inputs are including a translational sensor input, Eq.(6) becomes Eq.(8).

$$kx = kf(p_r, p_t, q_r, q_t, t) \neq f(p_r, kp_t, q_r, kq_t, t) \qquad (8)$$

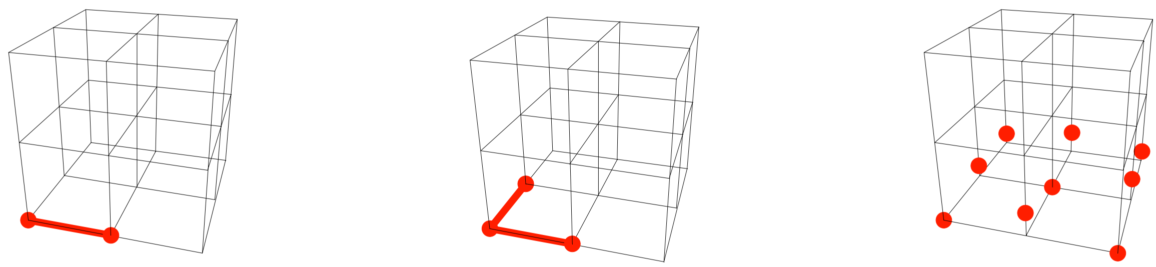
Because q_t is a translational sensor inputs and can't be multiplied by k. In this case, the parameter can be calibrated using only one sphere.

5. Parameter Calibration of Articulating CMM

Simple Artefact

Fig.1 Articulating CMM with six rotational joints and sensors

Fig. 1 shows the articulating CMM with six rotational joints. The artefact should have more spheres than one in order to calibrate the parameter of this type of CMM. Fig.2 shows the artefacts used in the simulation of parameter calibration.



(a) Artefact(b) (b) Artefact(c) (c) Artefact(d)

Fig.2 Artefact which consists of spheres

The articulating CMM has 21 kinematic parameters[5]. Here $n_c = 3$, $n_d = 1$ and $n_p = 21$, Eq.(3) becomes Eq.(9).

$$3n_s n_m \geq 21 + n_t$$
$$n_m \geq \frac{21 + n_t}{3n_s} \qquad (9)$$

The number of measurement which is necessary to calibrate the parameters is shown in Table 2.

Table 2: Relationship between the number of measurement and the number of spheres

Number of spheres	Number of parameters	Minimum number of measurements
2	26	5
3	27	3
9	27	1

In simulation of parameter calibration, the best calibration result is gotten using artefact(d). The worst calibration result is gotten using artefact(b), because artefact(b) covers a narrow part of measuring volume.

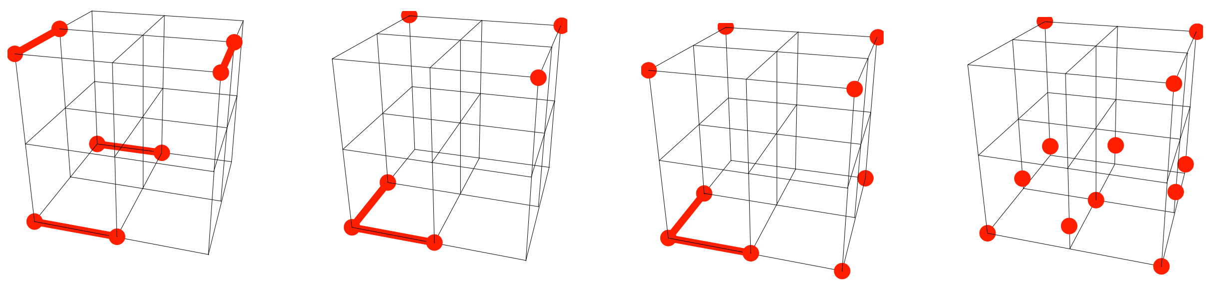
Combination of simple artefacts

The artefact(d) is useful for parameter calibration but heavy. The artefact(b) is not enough useful to calibrate the articulating CMM but light. In order to solve this problem, the artefact(b) is disposed at some locations and orientations. As a result, the most part of measuring volume of the articulating CMM is covered with the artefact(b).

Table 3 shows the dispositions of the artefacts shown in Fig.2. Fig.3 shows the example of the dispositions of the artefacts shown in Table 3.

Table 3: The disposition of the artefacts

	1-sphere	2-spheres	3-spheres	9-spheres	The number of measured spheres
A		1			2
B			1		3
C	3		1		6
D		4			8
E	6		1		9
F				1	9
G	3			1	12



(a) Artefact(C) (b) Artefact(D) (c) Artefact(E) (d) Artefact(G)

Fig.3 Example of combination of Simple Artefacts shown in Fig.2

Uncertainty of Articulating CMM

The uncertainty of articulating CMM is estimated from Eq.(4) according to [5,7]

$$u_x^2 = \left(\frac{\partial f}{\partial p}\right)^2 u_p^2 + \left(\frac{\partial f}{\partial q}\right)^2 u_q^2 + \left(\frac{\partial f}{\partial t}\right)^2 u_t^2 \qquad (10)$$

As the last term in Eq.(10) is requested to use the artefacts disposed at the different location and orientation, it can be neglected after parameter calibration, Eq.(11) can be gotten.

$$u_x^2 = \left(\frac{\partial f}{\partial p}\right)^2 u_p^2 + \left(\frac{\partial f}{\partial q}\right)^2 u_q^2 \qquad (11)$$

The uncertainty of the measurement calculated by Eq.(11) is shown in Table 4.

Table 4: The Uncertainty of Measurement

Artefact	Uncertainty of Measurement
A	0.4968
B	0.0715
C	0.0342
D	0.0299
E	0.0291
F	0.0342
G	0.0289

It is proved that two spheres artefact as Ball-Bar should be disposed at four different locations and orientations to accurately calibrate the parameters of the articulating CMM.

6. Parallel CMM

The parallel CMM is generally modelled as

$$g(x,p,q,t)=0 \qquad (12)$$

x, p, q and t are a generalized coordinate of stylus, parameters, sensor inputs and parameter of coordinate transformation. The parameters p and the sensor inputs are divided to p_r, p_t, q_r and q_t as the articulating CMM. The generalized coordinates of spheres are divided to position x_r and orientation x_t.

$$g(x_r,x_t,p_r,p_t,q_r,q_t,t)=0 \qquad (13)$$

7. Parameter Calibration of Parallel CMM

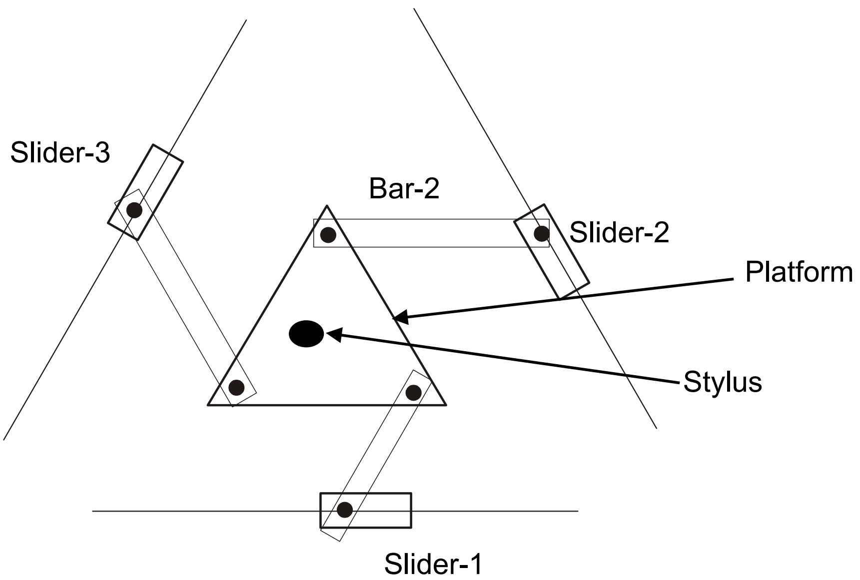


Fig.4 2D Parallel CMM

Fig.4 shows the 2D Parallel CMM with three translational sliders. It has three translational sensor inputs. The input sensors measure the location of one end of the bars, the stylus moves with the movement of the end of the bars. The platform has three degree of freedom of movement. Therefore, the platform can rotate around the probing point contacting with the same point. Then this type parallel CMM can get a lot of different sensor inputs measuring the same point.

Here, the parameter calibration using one sphere artefact can be considered. In this case, the sphere has no coordinate information as described above, Eq.(13) becomes Eq.(14) as Eq.(8).

$$kg(x_r,x_t,p_r,p_t,q_r,q_t,t)\neq g(x_r,kx_t,p_r,kp_t,q_r,kq_t,t) \qquad (14)$$

From Eq.(14), the parameter can be calibrated using one sphere artefact.The number of parameters of the parallel CMM shown in Fig.4 is 14. There are three parameters of shape of platform, three parameters of the origin and direction of sliders, the lengths of three bars, the stylus position and the parameters of coordinate transform in more details. Here $n_c = 3$ and $n_p = 14$,Eq.(3) becomes Eq.(15).

$$3n_m n_d \geq 14 + n_d(2 + n_m)$$
$$n_m \geq 2 + \frac{14}{n_d} \qquad (15)$$

Table 5 shows the relationship between the number of dispositions of a sphere and the number of measurement calculated from Eq.(15).

Table 5: The Relationship between the number of dispositions of a sphere and the number of measurement

Number of dispositions of a sphere	Minimum number of measurement
1	16
2	9
3	7
4	6
5,6	5
7-13	4
14 and more	3

In this case, it is proved that the coordinates of spheres are not necessary to be calibrated. Therefore, the parameter of the parallel CMM is calibrated by following procedure.

1. put and fix a sphere at any location.
2. measure a fixed sphere in any orientations.
3. 1-2 is iterated, if necessary.

When the kinematic parameters are estimated, there is a problem that the model of the parallel mechanism is described in implicit functions as Eq.(10). The coordinates of stylus of parallel CMM can not be solved analytically, then Eq.(10) generates some candidates of stylus coordinates. One of the candidates is physically correct coordinates and other is not. Generally it is difficult to distinguish physically correct coordinates from the others.

However, in this case, the knowledge that the same point is measured some times can be utilized ,i.e., Eq.(10) and sensor inputs in some measurements generate the sets of candidates of stylus coordinates, it is proved that the union of the sets of them is physically correct coordinates. The incorrect coordinates can be avoided by this way. As a result, the kinematic parameter can be estimated.

As the parameter calibration does not request the accurate calibrated coordinates of sphere, the accuracy of parameter calibration is dependent on the accuracy and the precision of sensors.

Uncertainty of Parallel CMM

The uncertainty of Parallel CMM is estimated from Eq.(12) according to [7],

$$u_x^2 = (AG_p)^2 u_p^2 + (AG_q)^2 u_q^2 + (AG_t) u_t^2 \qquad (16)$$

where,

$$G_x = \frac{\partial g}{\partial x}, G_p = \frac{\partial g}{\partial p}, G_q = \frac{\partial g}{\partial q}, G_t = \frac{\partial g}{\partial t}$$

$$A = \left(G_x^T G_x \right)^{-1} G_x^T$$

As the last term in Eq.(16) is requested to use the sphere disposed at the different location, it can be neglected as the case of Eq.(10). After parameter calibration, Eq.(17) can be gotten.

$$u_x^2 = (AG_p)^2 u_p^2 + (AG_q)^2 u_q^2 \qquad (17)$$

8. Summary

In this paper, the self-calibration of kinematic parameters of the articulating CMM and the parallel CMM are described. When the artefact consists of the spheres, the relationship between the number of sphere and the ability of the calibration is discussed. Especially, it is proved that the parameter calibration of CMM with a translational input sensor can be performed using one sphere. The parameter calibration of CMM without any translational input sensors can be performed using two spheres of which the distance shall be calibrated.

In 2D Parallel CMM, it is described how to distinguish physically correct coordinates (solution) from the others. It is shown that the parameter calibration can be performed using one sphere by this way. As the parameter calibration does not request the accurate calibrated coordinates of sphere, this calibration method is a self-calibration.

9. References

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