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UNCERTAINTY EVALUATION OF CMM BY MODELING WITH SPATIAL CONSTRAINT

Abbe, M. and Nara M.
Mitutoyo Corp., Mitutoyo Tsukuba Lab.
Kamiyokoba 430-1, Tsukuba, Ibaraki, 305-0854, Japan

Takamasu, K.
The University of Tokyo
Hongo 7-3-1, Bunkyo, Tokyo, 113-8656, Japan

Abstract: *A noble simulation method for evaluating task specific uncertainty in coordinate metrology is proposed. Provided that specification or test result conforming to ISO 10360-2 is available, both variance of point coordinates and covariance expressing the mutual influence is handled to perform Monte Carlo simulation reflecting spatial constraint in error of CMM. Development and implementation of the over all procedures are pursued to apply them on real CMM. Comparison result between uncertainties obtained by the proposed method and that by experimental measurements shows good agreement. However the worst shows 1 μm over-estimated, and the functionality and the characteristic of ease to use are validated.*
Keywords: *CMM, uncertainty, modeling, simulation*

1. INTRODUCTION

Decision process for showing conformance or non-conformance to the specification is one of the major concerns in industry today related to geometrical quality control. Since uncertainty can be interpreted as an indication of sharpness of the decision process, the effective evaluation method of uncertainty has been increasingly interested (Kunzmann *et al*., 2005). The Monte Carlo simulation integrated with coordinate measuring machine (CMM) is recognized as a typical possibility on this subject especially for complex measurement task commonly specified on practical part drawings (Wilhelm *et al*., 2001).

Task specific uncertainty evaluation by Monte Carlo simulation was firstly realized as Virtual CMM by PTB (Trapet *et al*., 1999). The approach quantifies all the effective uncertainty contributors one by one as input parameters. Random generators are applied on the respective contributors to reproduce the population distribution through number of repeated trials. A simplified simulation method requiring limited number of input parameters, typically a specification value or a result of ASME B89 performance test is proposed (Phillips *et al*., 1997). The approach reproduces enough number of different CMM geometry on computer by Monte Carlo simulation. Another Monte Carlo simulation method which tries to quantify correlation effect on the basis of simultaneous estimation of the geometric errors and so on is presented as well (Balsamo *et al*., 1999). The above task specific uncertainty evaluation attracts users of CMM. However, the evaluation does not always spread in industry related to geometrical quality control. Limited number of application example has been practically utilized mostly at calibration laboratories or practical users of CMM. The current study is motivated by noticing ease to use for them to be one of the important issue. It is authors' intention to propose and implement a smart methodology based on modelling of spatial constraint naturally existing in errors of CMM.

2. CONSTRAINED MONTE CARLO SIMULATION

Typical flow of Monte Carlo Simulation built for task specific uncertainty evaluation of coordinate measure-ment is shown in Fig.1.

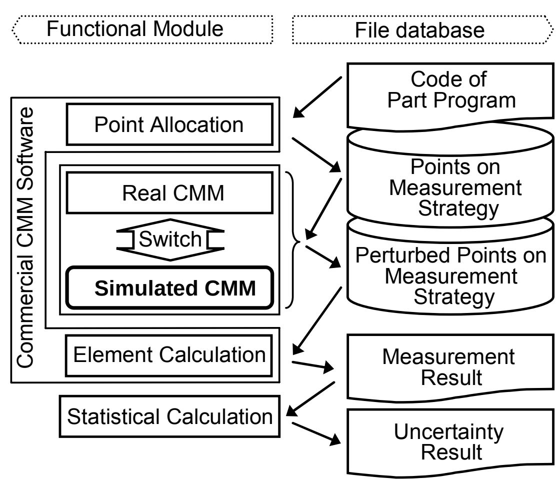

Fig. 1. Scheme of CMM uncertainty evaluation by Monte Carlo simulation

Functionality of a physical CMM is replaced by a software module modeling CMM error behaviour for the uncertainty evaluation purpose. The whole process starts from a part program for the CMM. It is translated from a measurement task specified typically on a drawing for the part to be inspected. The part program is then interpreted to a string of point coordinates to be measured. Actually sampled point coordinates are collected from the CMM. Geometrical elements are calculated as the drawing indicates. Processes down to evaluation of the elements are so repeated at this point with each time varied scenario of input influencing contributions as to accumulate enough number of the element evaluation results.

The whole process ends up by quantifying uncertainty in interest. The whole process can be operated like genuine CMM software. This is preferable for transparency and accountability of the uncertainty statement in industry.

2. 1. Proposed Modular Structure

A proposed modular structure for Constrained Monte Carlo Simulation (CMS) is shown in Fig. 2. Functionality of the CMS module starts from accepting a point coordinate interpreted by the CMM software. Variance-covariance information typically translated from ISO 10360-2 specification or the test result, described in later in the current study is utilized as input influencing contribution. The module ends up the function by returning a perturbed point coordinate which reflects possible geometrical deviation according to characteristics of the input influencing contributions, back to the CMM software.

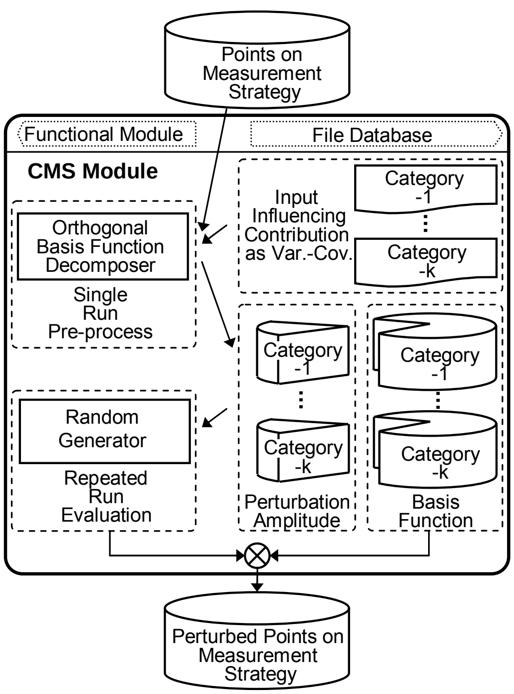

Fig. 2. Scheme of constrained Monte Carlo simulation

A single run pre-process is executed prior to repeated simulation for uncertainty evaluation. Having variance-covariance information, and also a string of point coordinates composing whole measurement strategy, task specific variance-covariance matrix with n by n dimension is assigned. Where, n corresponds to total number of point coordinates within the measurement strategy. The variance-covariance matrix is then decomposed into a series of orthogonal basis functions and the corresponding re-combination factors. The basis functions and the factors are transiently recorded for subsequent process.

The core operation of CMS consists of generation of n-times of random numbers each having variance equal to the re-combination factors. The re-composition process proceeds by taking a linear combination of the random numbers with the basis functions. Consequently a string of perturbed point coordinates within the measurement strategy is obtained at a single stroke. Repeating the core operation m-times to be able to obtain sufficient population of perturbed point coordinates, following element calculation and the statistical calculation is operated as conventional Monte Carlo simulation. As far as input influencing contribution is given by variance-covariance information, other contributions can easily be integrated into the simulation module.

3. MODELING OF SPATIAL CONSTRAINT

Technologically available measuring instruments today commonly adopt systematic error compensation to enhance the native performance. Major systematic error components are subject of compensation as far as they behave in reproducible manner within focused time and space. Experience on calibration service suggests that residual of systematic error compensation tends to derive rather moderately fluctuating wave form. This is because finite sampling in time and space as well as filtering operation for smoothing or noise suppression inevitably derives residual of calibration which characterizes behaviour of unknown but systematic.

3. 1.Unknown systematic contribution

The unknown systematic contribution may become a major player on uncertainty evaluation, although quite limited possibilities have been presented. Several studies introduce a basis function to be superimposed as unknown systematic contribution. Typically sinusoidal wave form is adopted by assigning suitable wavelength, amplitude and phase to the focusing phenomena. However, the sinusoidal wave is the best coherent basis. The coherence may unexpectedly interfere with measurement strategy. It intrinsically contains limitation of its usage for uncertainty evaluation by Monte Carlo simulation.

Another possibility is to adopt covariance or equivalent auto correlation (Takamasu *et al*., 2003; van Dorp and Haitjema, 2001) as resource for the basis function. Especially by combining the possibility with orthogonal decomposition, it is able to derive a series of mutually uncorrelated basis functions (Abbe & Takamasu, 2002). The nature enables a simple Monte Carlo simulation on the orthogonal basis with a simple random generator to reproduce trial series reflecting the correlation naturally observed in unknown systematic contribution.

3. 2. Ease to prepare input influencing contribution

Considering a typical procedure for uncertainty evaluation, all the influencing contributions are listed up and then quantified one by one. Easiness of the procedure for the operator strongly depends on number and complexity of preparation of input influencing contributions. It is desirable to establish a simplified procedure which is able to cope with both effectiveness of uncertainty evaluation and easiness of the preparation. A potential solution could be to focus on industry available performance test framework.

Utilization of specification or the test result of ISO 10360-2 test as principal information to prepare input influencing contribution for uncertainty evaluation (Abbe *et al*., 2000) is considered. The ISO standard is internationally accepted as an acceptance and reverification test procedure for CMM. The procedure consists of two major tests. One is E-test measuring one dimensional size with five different spacing over the whole measurement volume along seven different directions. Commercially available CMM conforming to this standard claims the specification in the form of the maximum permissible error (*MPE*). The test is so performed as to be able to show conformance to the specification by taking the test uncertainty in account. An example of a typical expression of the specification of the E-test can be e.g. $MPE_E = a + b \times l$. Applied the MPE_E on shorter size of l, it shows rather smaller deviation like a , while larger the size the expression allows deviation to enlarge correspondingly. This tendency falls basically in line with our daily experience, although deviation observed on a real CMM reveals random like fluctuation in combination with moderately varying wave form. The latter is mostly caused by auto reflection of the unknown systematic contribution. It is natural to grasp that the expression of the maximum permissible error varying positively with the size increment includes information of intrinsic spatial constraint of error of CMM.

A way of modeling is illustrated in Fig. 3 by recognizing specification or the test result of ISO 10360-2 to be an indirect expression of uncertainty in size measurement. This model with maximum permissible sense is inspired by a fact that the specification and interpretation of the test result shall be applied to any location and orientation in the CMM measurement volume.

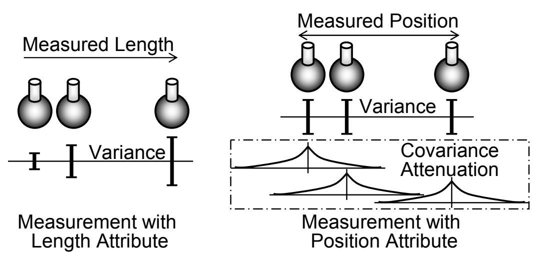

Fig. 3. Modelling of spatial constraint by covariance

4. IMPLEMENTATION

A concept of Constrained Monte Carlo Simulation (CMS) for evaluating task specific uncertainty in coordinate metrology was presented. A modelling procedure applicable to CMS was then proposed. This chapter advances practical implementation scheme to realize uncertainty evaluation functional module by combining them.

4. 1. Derivation of task specific covariance matrix

A derivation procedure of task specific covariance matrix from specification or test result of ISO 10360-2 is described. Considering a result of size measurement $l=x_2-x_1$ composed of two point coordinates x_1 and x_2, variance of the size measurement Var(l) results in equation 1 by introducing variance of respective point coordinates and the covariance $\mathrm{Cov}(x_1,x_2)$ as the mutual effect. Variance of the point coordinate is assumed to be homogeneous in the measurement volume as the worst scenario. The mutual effect is explained as dependant of the distance.

$$\begin{aligned}\mathrm{Var}(l) &= \mathrm{Var}(x_1)+\mathrm{Var}(x_2)-2\mathrm{Cov}(x_1,x_2)\\ &= 2\left[\mathrm{Var}(x)-\mathrm{Cov}(l)\right]\end{aligned} \quad (1)$$

Let us simplify discussion here by interpreting MPE_E be comparable to 95% probability limit of normal distribution.

$$\mathrm{Var}(x) = \left[\left(a+b\times l_{\max}\right)/2\right]^2 / 2 \quad (2)$$

$$\mathrm{Cov}(l) = \left[\mathrm{Var}(l_{\max})-\mathrm{Var}(l)\right]/2 \quad (3)$$

where, l_{max} expressing physically accessible longest length in the measurement volume is introduced. Assuming homogeneous variance in the volume and the mutual effect attenuated depending on the distance, variance-covariance of respective point coordinates composing a complex measurement strategy can be quantified in a straight forward manner.

4. 2. Decomposition into basis functions of error

Having variance-covariance information, a unique Monte Carlo simulation fully reflecting the given statistical characteristics is performed. Suppose we have a measurement strategy composed of discrete point coordinates, e.g. n-points, defined by CMM software, a task specific variance-covariance matrix $\boldsymbol{C}$ with n$\times$n dimension is assigned, and transferred to single run pre-process to decompose it orthogonally into the eigen vectors $\boldsymbol{V}$ and the corresponding eigen values $\boldsymbol{D}$.

$$\boldsymbol{C} = \boldsymbol{VDV}^{-1} \quad (4)$$

Where,

$$\boldsymbol{V} = \left[(\boldsymbol{v}_1)\ \ (\boldsymbol{v}_2)\ \ \cdots\ \ (\boldsymbol{v}_n)\right] \quad (5)$$

$$
\boldsymbol{D} = \begin{pmatrix} \lambda_1 & & & 0 \\ & \lambda_2 & & \\ & & \ddots & \\ 0 & & & \lambda_n \end{pmatrix} \quad (6)
$$

The important point to note is that the eigen vector means orthogonal basis function and the eigen value does re-combination factor for our considering these information to be used for CMS. An example of the basis function in case of one dimensional size measurement with 100 mm equi-distanced measurement strategy is shown in Fig. 4. A series of lower order exponential curve-like wave forms are observed.

4. 3. Trial Series Reflecting Variance-Covariance

Once task specific basis functions and the corresponding re-combination factors are obtained, a string of trial values ($\hat{x}$) for CMS can be generated easily.

$$
(\hat{x}) = \varepsilon_1 \boldsymbol{v}_1 + \varepsilon_2 \boldsymbol{v}_2 + \cdots + \varepsilon_n \boldsymbol{v}_n \quad (7)
$$

Where, ε_i is a random number satisfying:

$$
\mathrm{E}(\varepsilon_i) = 0 \text{, and } \mathrm{Var}(\varepsilon_i) = \lambda_i \quad (8)
$$

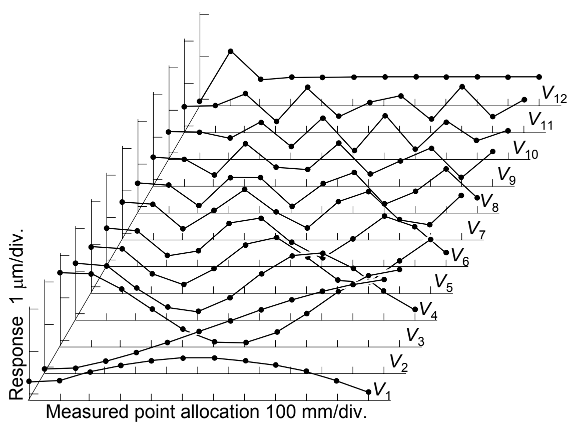

Fig. 4. Example of orthogonally decomposed basis functions in case of size measurement.

The simple equation 7 explains the fact that repeated run evaluation for the task specific uncertainty evaluation can be performed by applying only a simple random generator. That is to say, it takes linear combination with the basis function and the randomized re-combination factor. This re-composition operation derives a string of perturbed point coordinates included in the task specific variance-covariance matrix at a single stroke. The run evaluation is repeated m-times for m-times Monte Carlo simulation consisting of each n-point coordinates.

5. VERIFICATION ON REAL CMM

Constrained Monte Carlo Simulation module is integrated into CMM software adopted on a moving bridge CMM commercially available in industry as a test case. CMS module described in the former section for overall CMM geometry and that for probing characteristics are implemented. The CMM has specification of as follows is in unit of mm.

$$
\begin{aligned} MPE_{\mathrm{E}} &= 1.9 + 3/1000 \times l \ [\mu\mathrm{m}] \\ MPE_{\mathrm{P}} &= 1.9 \ [\mu\mathrm{m}] \end{aligned} \quad (9)
$$

The CMM has been located at a site for more than three years. Record of the environmental log shows the temperature deviation within 20 ± 1.5 deg. C. Temperature gradient in time and in space, which give major impact on the CMM performance, is within the CMM specification, however not superior. Geometrical characteristic of the CMM has been periodically inspected. It is believed that the long term and short term error behavior is empirically known as the CMM just to satisfy the specification. A calibrated cylinder artifact is chosen as an artifact to verify functionality of proposed CMS. A cylinder has a variety of features which can be well calibrated by conventional techniques (Trapet *et al*., 1999). The cylinder artifact having nominal dimension of 90 mm in diameter and 250 mm in size in the axial direction has been calibrated. The calibration uncertainty was reported as less than 0.7 µm for measurands. The calibration uncertainty is small comparing to practical performance of the CMM.

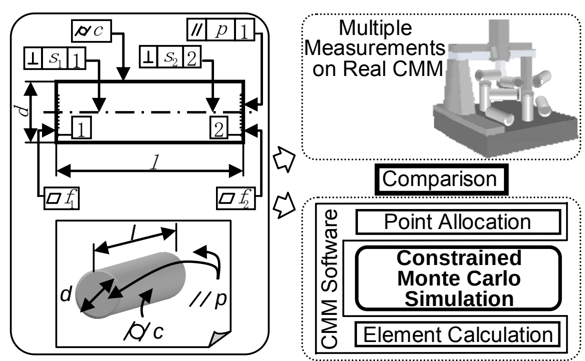

Fig. 5. Comparison between real measurement and CMS result

Comparison are made between results obtained through a series of physical measurement performed on the real CMM and that done on the CMS as schematically shown in Fig. 5. The physical measurement is repeated on the CMM for 256 times by varying location and orientation of the cylinder artifact, as well as the probing configuration of the CMM. The measurement result may provide an experimentally obtained task specific uncertainty statements by multiple measurements.

Simulation measurement by CMS is repeated 256 times as well. Both uncertainty evaluation processes and primary output population distribution of calculation results for respective features are of interest. Fig. 6 shows population distribution for some of evaluated features as examples, e.g. length, diameter, and squareness.

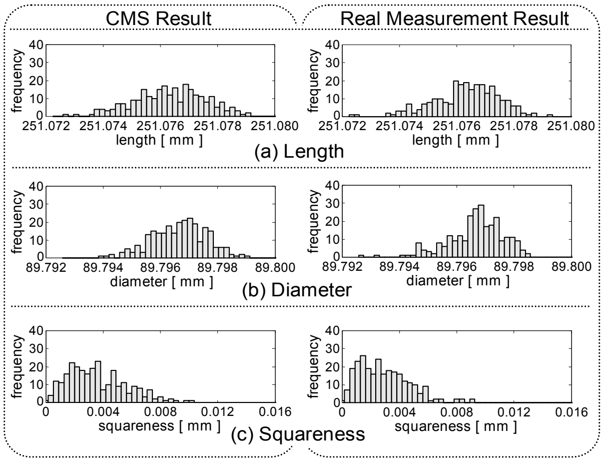

Fig. 6. Comparison of population distribution obtained by CMS and real measurement

6. CONCLUSION

A simulation method for evaluating task specific uncertainty applicable to industrially available CMM is proposed. The method performs Monte Carlo Simulation in orthogonally decomposed multi dimensional error space to be able to reproduce statistical behavior of unknown systematic error of CMM, provided the variance-covariance information is known prior to the simulation. Implementation of CMS for another category of input influencing contributions can be handled in very similar manner by having a procedure to quantify the variance-covariance information.

A potential limitation of the method may be dimension of the task specific variance-covariance matrix especially on application requiring a large number of discrete points. A possible solution is to section the task specific variance-covariance matrix into parts where the mutual influence is to be negligible.

We see wider possibility of the method to expand the application field in coordinate metrology by accumulating sufficient experience on quantification procedure for various input influencing contributions.

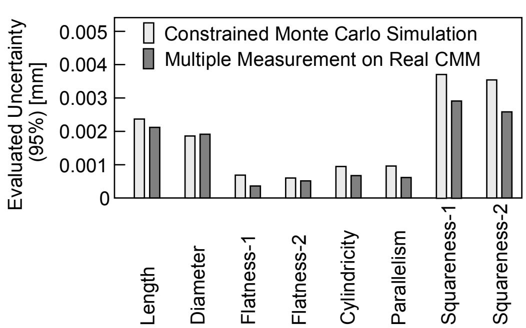

Fig. 7. Comparison of uncertainty evaluation result

The left three figures show population distribution from CMS result, and the right three figures show that from real measurement on the CMM. All the plots draw dimensional variation of features in the abscissa and the population in the ordinate. It is confirmed that the CMS is able to derive feasible population distribution generally conforming to that obtained by real measurement. Including the above three features, all the evaluation results of population distribution resulted by the CMS show agreement with that obtained on real measurement.

The comparison result of expanded uncertainties between CMS and the real measurement is summarized in Fig. 7. Both results conform to each other in case of uncertainty evaluation of a cylinder artifact although the worst shows 1 μm over estimated.

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