ESTIMATION OF UNCERTAINTY IN FEATURE-BASED METROLOGY

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Abstract: In coordinate metrology, a feature (Gaussian associated feature) is calculated from a measured data set of CMM (Coordinate Measuring Machine) using least squares method. This data processing flow is called as "feature-based metrology". In the feature-based metrology, it is a key technique to estimate the uncertainty of measurement in the specific measuring strategy. The estimation method for uncertainties of measured parameters has been already proposed when the only random errors are put in the consideration. In this paper, the effects of systematic errors are theoretically analyzed to estimate the uncertainties in feature-based metrology. The center position error and the diameter error of the ball probe are occurred from the random errors of probing in calibration process. These errors propagate as unknown systematic errors to the uncertainties of measured parameters such as the center position and the diameter of a measured circle. The series of simulations for this method in statistical way directly implies that the concept and the basic data processing method in this paper are useful to the feature based metrology.

Keyword: CMM, Coordinate Measuring Machine, Uncertainty, Feature-Based Metrology

1 FEATURE-BASED METROLOGY

In coordinate metrology, an associated feature (Gaussian associated feature) is calculated from a measured data set on a real feature by CMM (Coordinate Measuring Machine). Then, the associated features are compared with the nominal features indicated on a drawing (see figure 1). In this data processing, the features are primal targets to calculate, to evaluate and to process [1][2]. Consequently, this process is called as "Feature-Based Metrology".

In the feature-based metrology, it is a key technique to estimate the uncertainty of measurement in the specific measuring strategy. The estimation method for uncertainties of measured parameters has been already proposed when the only random errors are put in the consideration [3]-[5]. The uncertainty of each measured point is defined by error analysis of the CMM and the probing system. From the uncertainty of measured point, the uncertainty of measured feature can be calculated statistically using following equations [6]-[9].

Equation (1) shows an observation equation, a regular equation and a least squares solution, where **A** is Jacobian matrix, **d** is measurements vector, **x** is a parameter vector and **S** is an error matrix.

$$\begin{aligned}&\mathbf{d} = \mathbf{A}\mathbf{x}\\&\mathbf{A}^t\mathbf{S}^{-1}\mathbf{A}\mathbf{x} = \mathbf{A}^t\mathbf{S}^{-1}\mathbf{d}\\&\mathbf{x} = (\mathbf{A}^t\mathbf{S}^{-1}\mathbf{A})\mathbf{A}^t\mathbf{S}^{-1}\mathbf{d}\end{aligned} \tag{1}$$

Using the propagation law of errors to least squares method, the error matrix of parameter **P** is calculated as equations (2). The matrix **P** indicates the variances and covariances of the parameters.

$$\mathbf{P} = (\mathbf{A}^t\mathbf{S}^{-1}\mathbf{A})^{-1} \tag{2}$$

Figure 2 shows an example of error analysis form twelve measured points on a flat plane. Middle plane is least squares plane, the upper and the lower planes are the upper and the lower limits of confidential zone of feature respectively. We note that the upper and the lower limits of confidential zone is equal to the range of the uncertainty of measured feature.

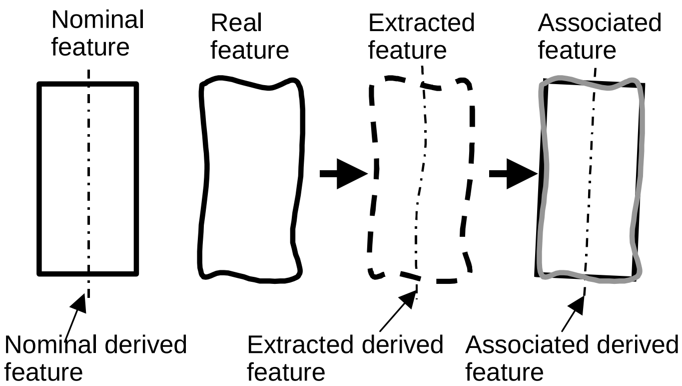


Figure 1. Data processing flow in feature-based metrology.

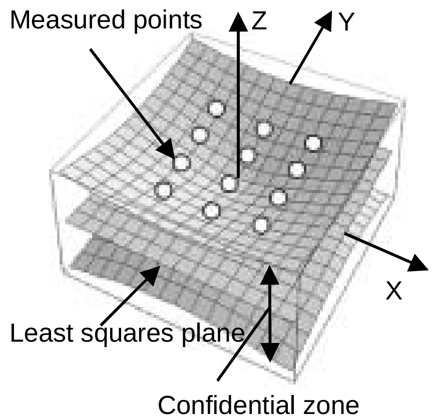


Figure 2. Confidential zone of measure plane.

2 RANDOM ERRORS

In this paper, the effects of systematic errors are theoretically analyzed to estimate the uncertainties in feature-based metrology. The center position error and the diameter error of the ball probe are taken up for the examples of the effects of systematic errors. These errors are occurred from the random errors of probing in calibration process and propagate as unknown systematic errors to the uncertainties of measured parameters such as the center position and the diameter of a measured circle.

Figure 3 shows the model for the theoretical analysis. Firstly, diameter and center position of a probe ball are calibrated by measuring a reference circle. The diameter of the reference circle is calibrated with the uncertainty s_c. After the calibration, a measured circle is measured by the ball probe with random measured error s_p.

When the only random errors are put in the consideration and n measured points are probed uniformly on the measured circle, the uncertainties of measured diameter and center position of the measured circle can be calculated as equations (3), (4), (5) and (6) using equations (1) and (2). Where the position of probed point is displayed by t_i and r_i in figure 4.

$$\mathbf{P} = \begin{pmatrix} s_x^2 & s_{xy} & s_{xd} \\ s_{xy} & s_y^2 & s_{yd} \\ s_{xd} & s_{yd} & s_d^2 \end{pmatrix} = (\mathbf{A}^t \mathbf{S}^{-1} \mathbf{A})^{-1} \tag{3}$$

$$\mathbf{S} = \begin{pmatrix} s_p^2 & & 0 \\ & \ddots & \\ 0 & & s_p^2 \end{pmatrix} = s_p^2 \begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix} \tag{4}$$

$$\mathbf{A} = \begin{pmatrix} -\cos t_1 & -\sin t_1 & -\frac{1}{2} \\ \vdots & \vdots & \vdots \\ -\cos t_n & -\sin t_n & -\frac{1}{2} \end{pmatrix} \tag{5}$$

$$\mathbf{P} = \begin{pmatrix} \frac{2}{n} s_p^2 & 0 & 0 \\ 0 & \frac{2}{n} s_p^2 & 0 \\ 0 & 0 & \frac{4}{n} s_p^2 \end{pmatrix} \tag{6}$$

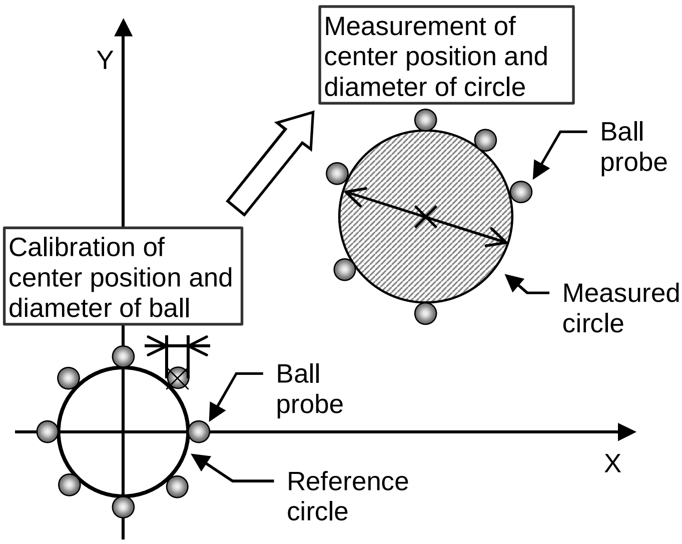


Figure 3. Model for calibration of ball probe and measurement of circle

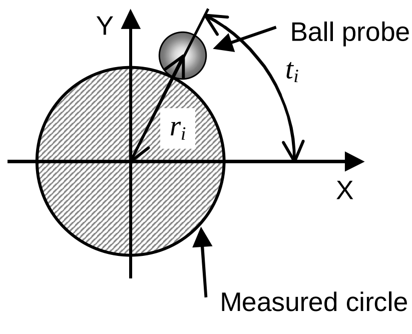


Figure4. Measured positions by angle t_i

3 UNKNOWN SYSTEMATIC ERRORS

From the calibration process of the ball probe, the unknown systematic errors of diameter and center position of the ball probe are occurred. The effect of these unknown systematic errors is not same as the effect of the random errors.

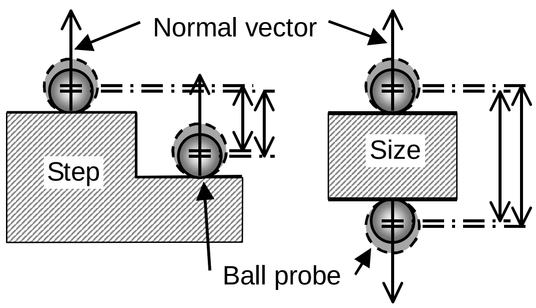
3.1 Diameter of ball probe

Figure 5 displays the influences of uncertainty of the diameter of the ball probe for the step measurement and size measurement. The uncertainty of diameter effects the only size measurement. Figure 6 and equation (7) show the measuring errors dr_1 and dr_2 from diameter errors on the measured circle. The variance and the covariance from diameter errors are shown in equation (8), where c_d is diameter error. In this case the error matrix of the parameters is calculated as equation (9).

$$\begin{aligned} dr_1 &= p_1 + \frac{d}{2} \\ dr_2 &= p_2 + \frac{d}{2} \end{aligned} \tag{7}$$

$$\begin{aligned} s_1^2 = s_2^2 &= s_p^2 + \frac{c_d^2}{4} \\ s_{12}^2 &= \frac{c_d^2}{4} \end{aligned} \tag{8}$$

$$\mathbf{P} = \begin{pmatrix} \frac{2}{n}s_p^2 & 0 & 0 \\ 0 & \frac{2}{n}s_p^2 & 0 \\ 0 & 0 & \frac{4}{n}s_p^2 + c_d^2 \end{pmatrix} \tag{9}$$



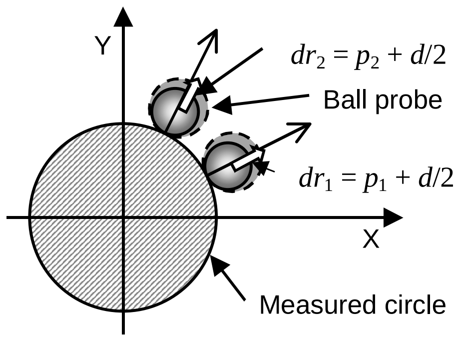


Figure 5. Effect of diameter errors of ball probe on step dimension and size dimension **Figure 6.** Correlation of two measured points by effects from diameter error

3.2 Center position of ball probe

Figure 7 displays the influences of uncertainty of center position of the ball probe. Figure 8 and equation (10) show the measuring errors dr_1 and dr_2 from center position errors on the measured circle. The variance and the covariance from center position errors are shown in equation (11), where c_x is center position error.

$$\begin{aligned} dr_1 &= dx\cos t_1 + dy\sin t_1 \\ dr_2 &= dx\cos t_2 + dy\sin t_2 \end{aligned} \tag{10}$$

$$\begin{aligned} s_1^2 &= s_2^2 = c_x^2\cos^2 t_1 + c_y^2\sin^2 t_1 = c_x^2 \\ s_{12}^2 &= c_x^2\cos t_1\cos t_2 + c_y^2\sin t_1\sin t_2 \\ &= c_x^2\cos(t_1 - t_2) \end{aligned} \tag{11}$$

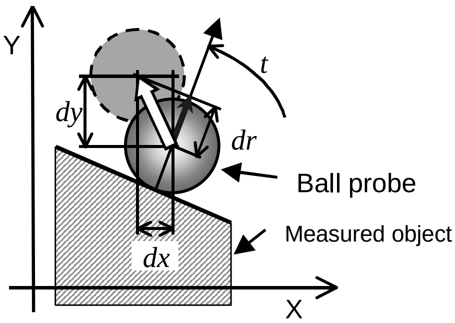


Figure 7. Effect of center position errors of ball probe

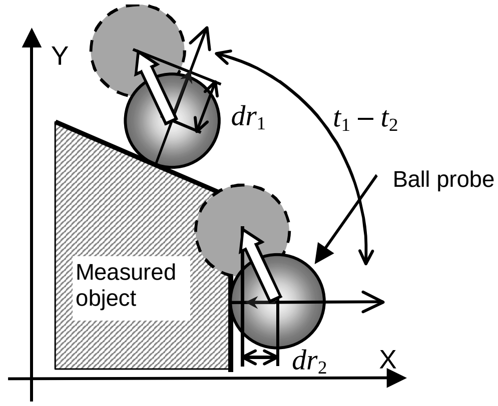


Figure 6. Correlation of two measured points by center position errors

4 CONCLUSION

In this paper, we theoretically analyzed the effects of the unknown systematic errors in feature-based metrology. The center position error and the diameter error of the ball probe are occurred from the random errors of probing in calibration process. These errors propagate as unknown systematic errors to the uncertainties of measured parameters such as the center position and the diameter of a measured circle. The method to calculate the error matrix was derived when the center position and the diameter of the circle are measured.

Using this method, the uncertainties of the measured parameters can also be calculated in the complex measuring strategy. The series of simulations for this method in statistical way directly implies that the concept and the basic data processing method in this paper are useful to the feature based metrology.

REFERENCES

[1] ISO 10360-1 : Geometrical Product Specifications (GPS) – acceptance and reverification test for coordinate measuring machine (CMM) – Part 1: Terms and definitions.

[2] K. Takamasu, R. Furutani, S. Ozono, Basic concept of feature-based metrology, Measurement, 26 (1999) 151.

[3] Guide to the expression of uncertainty in measurement (GUM). BIPM, IEC, IFCC, ISO, IUPAC, IUPAP, OIML, 1st edition, 1995.

[4] K. Takamasu, R. Furutani, S. Ozono, Reliability of parameters for associated features in coordinate metrology, Journal of Japan Society for Precision Engineering, 62, 7（1996）964.

[5] Traceability of Coordinate Measurements According to the Method of the Virtual Measuring Machine, Part 2 of the Final Project MAT1-CT94-0076, 1999.

[6] S. D. Phillips, B. Borchardt, W. T. Estler and J. Buttress: The Estimation of Measurement Uncertainty of Small Circular Features Measured by Coordinate Measuring Machine, Prec. Eng., 22, 2 (1998) 87.

[7] W. Choi, T. T. Kurfess and J. Cagan: Sampling Uncertainty in Coordinate Measurement Data Analysis, Prec. Eng., 22, 3 (1998) 153.

[8] A. Wechenmann, M. Knauer and H. Kunzmann: Influence of Measurement Strategy on the Uncertainty of CMM-Measurements, CIRP Ann., 47, 1 (1998) 451.

[9] M. Knauer and A. Wechenmann: A Software System to Estimate the Uncertainty Contribution of the Workpiece Shape in Coordinate Metrology, Proc. IMEKO XV, (1999) 15.

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