Study on Scanning Squareness Measurement Method and Uncertainty Estimation

X. Chen and K. Takamasu

Department of Precision Engineering, The University of Tokyo, Hongo 7-3-1, Bunkyo-ku, Tokyo 113-8656, JAPAN

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Abstract. This paper presents a scanning squareness measurement method for large ultra-precision components. A rectangular block as the squareness reference is used. The squareness error of the rectangular block is eliminated according to the geometric principle that the sum of four internal angles of a rectangle is 360°. And the straightness error of each line of the rectangular block is eliminated by means of scanning method with two 1D probes. The above two error separation technologies are combined effectively and the data processing method is developed. Additionally, the standard uncertainties including tilt and squareness errors of the rectangular block, temperature drift and random errors of the measured values of the probes are analyzed theoretically. It is confirmed that a combined standard uncertainty of less than 1 arcsec can be obtained for typical values of the parameters.

Introduction

Squareness, which includes squareness between two fixed lines and motional squareness, etc., is an essential element of geometrical deviations. Generally, reference gauges such as right-angle gauges and levels are used for the measurement of squareness between two fixed lines. It is obvious that the accuracy of the reference gauge has to be higher enough than the aimed accuracy. However, there exist two problems for the measurement of large ultra-precision components. Firstly, it becomes very difficult and expensive to obtain a squareness reference with a higher accuracy than the ultra-precision component. Secondly, the straightness error of each line of the squareness reference can not be ignored anymore.

In this paper, a measurement system with a rectangular block as the squareness reference for measuring the squareness between fixed x and y axis mirrors is presented. The principle of how to eliminate the influences of the squareness errors and the straightness errors of the rectangular block is explained in detail. Additionally, the main sources of uncertainty including tilt errors and squareness errors of the rectangular block, temperature drift and random errors of the measured values, are estimated. Each of the standard uncertainties is derived mathematically and the combined standard uncertainty is calculated by substituting typical values of the parameters.

Scanning Squareness Measurement Method

As shown in Fig. 1 a rectangular block is used for measuring the squareness between fixed x and y axis mirrors. Here the squareness is defined as the crossing angle α between the least squares (LS) lines of x and y axis mirror. Also, the internal angles of the rectangular block are defined as $\beta_1 \sim \beta_4$ which are composed of the LS lines of the four sides of the rectangular block. With a coarse adjustment, two small angles (θ_1 and θ_2 shown in Fig. 1) can be obtained between two sides of β_1 and two sides of α. The geometrical relationship among α, β_1, θ_1 and θ_2 can be expressed as:

$$\alpha - \beta_1 = \theta_1 - \theta_2. \tag{1}$$

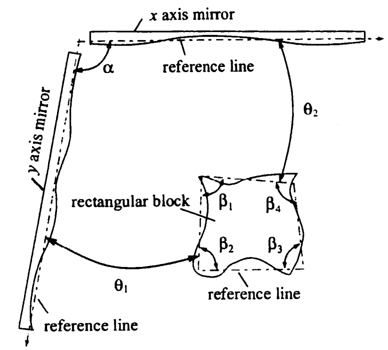

Fig. 1 Principle of squareness measurement between x-y mirror using rectangular block

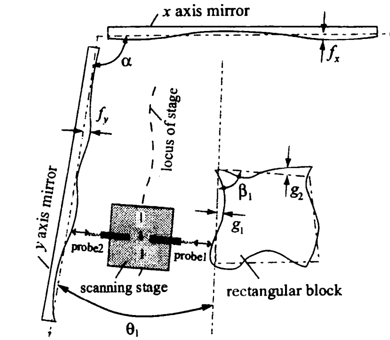

Fig. 2 Scanning method using two sensors for calculating θ_1

When the rectangular block is rotated 90° in clockwise direction, two small angles (θ_3 and θ_4 not shown in this paper) can be obtained between two sides of β_2 and two sides of α. Actually, by repeating the 90° rotation for 3 times, three pairs of small angles ($\theta_3 \sim \theta_8$) can be obtained in three positions and own similar relationships like Eq. (1) [1]. Because the sum of $\beta_1 \sim \beta_4$ is 360°, the squareness α can be expressed only by $\theta_1 \sim \theta_8$:

$$\alpha = 90° + \frac{1}{4}(\theta_1 + \theta_3 + \theta_5 + \theta_7 - \theta_2 - \theta_4 - \theta_6 - \theta_8). \tag{2}$$

To calculate $\theta_1 \sim \theta_8$ respectively, scanning technology is employed [2]. Fig. 2 shows the measurement system for calculating θ_1. Two 1D probes are oppositely set up on a scanning stage to scan a side of the rectangular block and y axis mirror respectively. If we take the sum of the measured values of two 1D probes, the motion errors of the scanning stage can be canceled and the following relationship can be obtained:

$$m_1(y_n) = f_y(y_n) + g_1(y_n) + \theta_1 \cdot y_n + c_1, \qquad n = [1, N], \tag{3}$$

where m_1 is the sum of the measured values of two 1D probes, f_y is the straightness error based on the LS line of y axis mirror, g_1 is the straightness error based on the LS line of one side of the rectangular block, y_n (= $(n-1) \times s$) is the lateral position, s is the sampling interval, N is the number of the scanning point, c_1 is the unknown constant. Eq. (3) can also be expressed as the following matrix equation:

$$\mathbf{M}_1 = \mathbf{F}_Y + \mathbf{G}_1 + \mathbf{A}^T\mathbf{X}_1, \qquad \text{where } \mathbf{A} = \begin{bmatrix} y_1, y_2, \ldots, y_N \\ 1, \ 1, \ \ldots, \ 1 \end{bmatrix} \text{ and } \mathbf{X}_1 = [\theta_1, c_1]^T. \tag{4}$$

Since f_y and g_1 are defined as the straightness errors based on their LS lines, the relationships ($\mathbf{AF}_Y = 0$ and $\mathbf{AG}_1 = 0$) are satisfied simultaneously. By means of left-multiplying Eq. (4) by $\mathbf{A}$, $\mathbf{X}_1$ can be solved as follows (least squares solution when $N \geq 3$):

$$\mathbf{X}_1 = \mathbf{BM}_1, \qquad \text{where } \mathbf{B} = (\mathbf{AA}^T)^{-1}\mathbf{A}. \tag{5}$$

It is clear that θ_1 is the first element of the parameter vector $\mathbf{X}_1$. By repeating scans along x and y axis for 8 times, $\theta_1 \sim \theta_8$ can be obtained totally.

Uncertainty Estimation

Firstly the tilt errors are considered when a 90° rotation of the rectangular block is performed. Fig. 3 shows an ideal rectangular block with an original scan line ABCD, which indicates that no tilt errors (yawing, rolling and pitching) occurred. It is obvious that yawing does not influence the measurement result. However, the rotation axis in the z direction should pass the gravity point of the block so that the same points on a side of the rectangular block can be scanned along the x and y directions respectively after the operation of a 90° rotation. Fig. 4 shows the influence when rolling and pitching occur at the same time. In this case, the actual scan line becomes E'F from AB. Note that the difference of straightness errors between E'F and AB can be ignored because of the very little rolling and pitching error. The angle difference u_γ occurred in face ABFE can be expressed as:

$$u_\gamma = \frac{EE'\times\gamma}{EE'\div\gamma} = \gamma^2, \tag{6}$$

where γ denotes maximum angle of rolling error or pitching error.

Next, we consider the squareness errors of the rectangular block itself accompanied with tilt errors. It is obvious that the squareness error of face ABCD does not influence the measurement result. Fig. 5 shows the influence when the squareness errors of face ABB'A' and face ADD'A', rolling and pitching occur at the same time. In this case, the actual scan line becomes EB from AB. As a result, the standard uncertainty u_r caused by the tilt and squareness errors of the rectangular block in eight scans can be expressed as:

$$u_r = \sqrt{\frac{8}{16}}\times u_\phi = \frac{1}{\sqrt{2}}\times\frac{(\phi+\gamma)\gamma}{1-\phi\gamma}, \tag{7}$$

where ϕ denotes maximum angle of the squareness errors of face ABB'A' or face ADD'A'. Fig. 6 shows the contour map of the standard uncertainty u_r versus the squareness error ϕ and the tilt error γ of the rectangular block.

Secondly, the temperature drift of the measured values of probes is considered. With a simple model, the standard uncertainty u_d in eight scans can be expressed as:

$$u_d = \frac{1}{\sqrt{2}}\times\frac{\Delta m}{L}, \tag{8}$$

where Δm is the drift during the period of one side scan, L is the length of one side scan.

Finally, the random errors of the measured values of probes are considered. It is assumed that the measured values vector $\mathbf{M}_1$ has random errors with normal distribution and no correlation. The error matrix of the parameters $\mathbf{S}_p$ can be expressed as [3]:

$$\mathbf{S}_p = \mathbf{B}\sigma_m^2\mathbf{I}_N\mathbf{B}^T = \sigma_m^2(\mathbf{A}\mathbf{A}^T)^{-1}, \tag{9}$$

where σ_m is the standard deviation of the random error of probes. As a result, the standard uncertainty u_m caused by the random error in eight scans can be expressed as:

$$u_m = \sqrt{\frac{8}{16}}\times \mathbf{S}_{p(1,1)} = \frac{\sigma_m}{s}\sqrt{\frac{6}{(N-1)N(N+1)}}. \tag{10}$$

Fig. 7 shows the contour map of the standard uncertainty u_m versus the number of scanning points N and the standard deviation of the random errors σ_m in the case of a sampling interval s = 1 mm.

According to the standard uncertainties list above, the combined standard uncertainty u_c can be calculated by Eq. (11) if we assume N = 50, s = 1 mm, σ_m = 50 nm, Δm = 100 nm, ϕ = 40 arcsec, γ = 150 arcsec.

$$u_c = \sqrt{u_r^2 + u_d^2 + u_m^2} = \sqrt{(0.098)^2 + (0.298)^2 + (0.072)^2} = 0.321 \text{ (arcsec)} \tag{11}$$

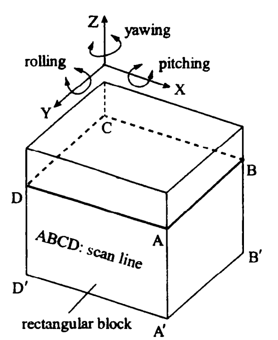

Fig. 3 Ideal rectangular block with original scan line: ABCD

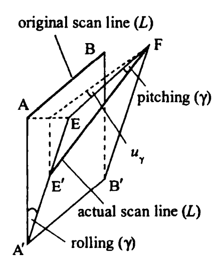

Fig. 4 Influence of tilt errors of rectangular block

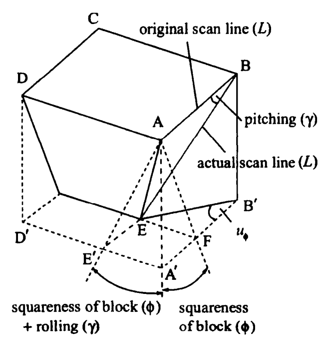

Fig. 5 Influence of squareness errors of rectangular block

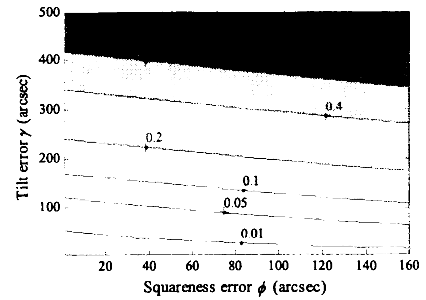

Fig. 6 Contour map of standard uncertainty (u_r: arcsec) versus squareness error of rectangular block and tilt error of rectangular block

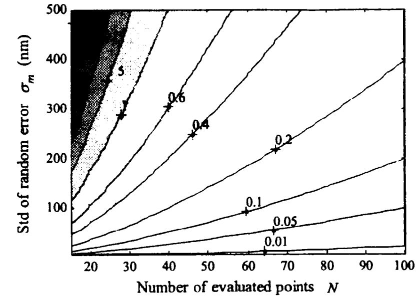

Fig. 7 Contour map of standard uncertainty (u_m: arcsec) versus number of evaluated points and Std of random error of probes in the case of a sampling interval s = 1 mm

Conclusions

A reference-free scanning squareness measurement method for large ultra-precision components has been proposed. We described the measurement system using the rectangular block and the data processing method by means of matrix equations. The standard uncertainties including tilt and squareness errors of the rectangular block, temperature drift and random errors of the measured values of the probes, have been estimated. It is shown that a combined standard uncertainty of less than 1 arcsec can be obtained for typical values of the parameters.

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