Uncertainty Estimation for Coordinate Metrology: Calibration, Form Deviation and Strategy of Measurement

K. Takamasu [1], S. Takahashi [1], M. Abbe [2] and R. Furutani [3]

[1] Department of Precision Engineering, School of Engineering, The University of Tokyo, Hongo 7-3-1, Bunkyo-ku, Tokyo, JAPAN

[2] Mitutoyo Corp., Mitutoyo Tsukuba Lab. Kamiyokoba 430-1, Tsukuba, Ibaraki, JAPAN

[3] Department of Precision Engineering, School of Engineering, Tokyo Denki University Kandanishiki-cho 2-2, Chiyoda-ku, Tokyo, JAPAN

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Abstract. Coordinate metrology utilizes complex measuring systems such as coordinate measuring machines, laser trackers, and triangulation systems. Therefore, the method of calibrating the coordinate measuring system using artifacts, e.g., the artifact calibration method, is a key technology. In this article, methods of estimating uncertainties using the coordinate measuring system after calibration are formulated. First, a calculation method which extracts the values of kinematic parameters using the least squares method is formulated. Secondly, the uncertainty of the specified measuring task is calculated using the error propagation method. A coordinate measuring system utilizing two line-cameras is analyzed as an example. Moreover, the influences of the form deviations of measured workpiece are calculated in the measurement of the features of a circle.

Introduction

The calibration methods of coordinate measuring machines are essential to accurate measurement and to the evaluation of measurement uncertainty [1]-[4]. Through the use of the feature-based metrology, we previously formulated a method to evaluate the uncertainty in coordinate metrology [5][6]. Furthermore, we proposed an error propagation method to estimate the uncertainty of kinematic parameters in the calibration of coordinate measuring machines [7]-[9].

These evaluations of parameter uncertainty are calculated in the machine coordinate system. However, the specified measurement tasks are done in a workpiece coordinate system after calibration. In this article, methods of estimating uncertainties using the calibrated coordinate measuring machine after calibration are formulated. First, a variance and covariance matrix on measuring points is calculated based on a variance and covariance matrix of the kinematic parameters of the calibrated measuring machine. Secondly, the uncertainties of a size measurement or a point measurement in a workpiece coordinate system are estimated using the error propagation method. Therefore, the methods of estimating the uncertainties on the specified measuring tasks are formulated in the feature-based metrology. Moreover, form deviation of the measured workpiece influences the uncertainty of measurement results. We formlated an uncertainty estimation method based on form deviations, and show an example involving circle feature measurement.

Uncertainty Evaluation of a Measuring Point After Calibration

Measurement of the uncertainties of a measuring point consists of three steps: 1) determining the kinematic parameters by calibration, 2) measuring the point after calibration, and 3) measurement under the conditions of the specified measuring task. The uncertainty of kinematic parameters can be calculated based on the uncertainty factors in the kinematic calibration of the coordinate measuring system using error propagation. The forward kinematics of the coordinate measuring system $\mathbf{f}$ is

defined by kinematic parameters $\mathbf{p}$ and readings of each encoder $\mathbf{q}$ as in Eq. (1). The uncertainty of kinematic parameters S_p is calculated based on the Jacobian matrix $\mathbf{A}$ and the error matrix S in the calibration procedure as in Eq. (2), where S_r is an uncertainty of the coordinate conversion parameters of artifacts and S_{pr} is a covariance between the kinematic parameters and the coordinate conversion parameters.

$$\mathbf{x} = \mathbf{f}(\mathbf{p},\mathbf{q}) = \begin{pmatrix} x \\ y \\ z \end{pmatrix},\quad \mathbf{p} = \begin{pmatrix} p_1 \\ p_2 \\ \vdots \end{pmatrix},\quad \mathbf{q} = \begin{pmatrix} q_1 \\ q_2 \\ \vdots \end{pmatrix} \tag{1}$$

$$\begin{pmatrix} \mathbf{S}_p & \mathbf{S}_{pr} \\ \mathbf{S}_{pr} & \mathbf{S}_r \end{pmatrix} = (\mathbf{A}^t\,\mathbf{S}^{-1}\,\mathbf{A})^{-1} \tag{2}$$

The uncertainties matrix of a measurement point $\mathbf{T}_1$ consists of variances of each coordinate sx^2, sy^2 and sz^2, and covariances between the XYZ coordinates sxy, sxz, and syz of a measuring point. Eq. (3) defines $\mathbf{T}_1$ as sum of three factors of uncertainties such as uncertainties from kinematic parameters $\mathbf{T}_p$, the uncertainty from encoders $\mathbf{T}_q$, and from probing $\mathbf{T}_m$. Each uncertainty can be calculated from Eqs. (1) and (2) using the error propagation method, where s_q is the uncertainty of encoders, s_m is the uncertainty of probing, and $\mathbf{E}$ is a unit matrix.

The uncertainty in Eq. (3) is evaluated regarding the measuring machine coordinate system. However, measurements are normally done regarding a workpiece coordinate system. Then, an uncertainty of measurement s_t is calculated as in Eq. (4), where, $\mathbf{A}_t$ is a Jacobian matrix by the specified measuring task and $\mathbf{T}_{1\text{-}n}$ is a variance and covariance matrix of the measuring points $\mathbf{x}_1$ - $\mathbf{x}_n$.

$$\mathbf{T}_1 = \begin{pmatrix} sx^2 & sxy & sxz \\ sxy & sy^2 & syz \\ sxz & syz & sz^2 \end{pmatrix} = \mathbf{T}_p + \mathbf{T}_q + \mathbf{T}_m = \mathbf{A}_p\,\mathbf{S}_p\,\mathbf{A}_p^t + s_q^2\mathbf{A}_q\,\mathbf{A}_q^t + s_m^2\,\mathbf{E} \tag{3}$$

$$s_t^2 = \mathbf{A}_t\,\mathbf{T}_{1-n}\,\mathbf{A}_t^t \tag{4}$$

Example of Calibration of Coordinate Measuring System Using Two Line-Cameras

We demonstrate an example of calculation using a two-dimensional coordinate measuring system employing by two line-cameras. Fig. 1 shows the coordinate system of the two-dimensional camera system; the two cameras are positioned at (0, 0) and (b, 0) on the XY coordinate plane. Parameters u_1 and u_2 are the offset angles of each camera from the Y axis and parameter b is the X coordinate of camera 2 of approximately 200 mm. The results of calibration of the two-dimensional line-camera system under the following conditions: no. of points: 25 in X and Y of 50-150 mm at 25 mm intervals. The standard deviations for u_1, u_2, and b are 0.0042 deg, 0.0042 deg, and 15.7 μm, respectively, under a random uncertainty of probing, while the obtained calibration and angle of each camera are 10 μm, 5 μm, and 0.001 deg, respectively.

From the results of calibration, we calculate the positioning uncertainty using Eq. (2). Fig. 2 shows the uncertainties of size measurement from a specified point (100, 100) as the symmetrical distribution, indicating that the selection method of the coordinate system and parameters does not influence the uncertainty evaluation results.

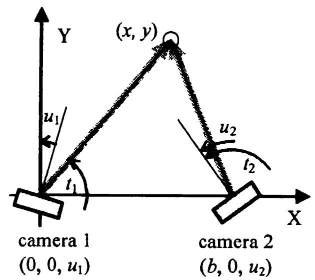


Fig. 1 A two-dimensional coordinate measuring system using two line-cameras.

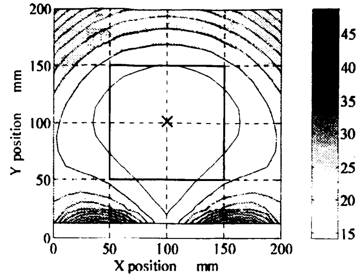


Fig. 2 Distribution of size measurement uncertainties from a specified point (100, 100) after calibration (unit is μm).

Uncertainty from Form Deviations of Measured Workpiece

Eqs. (5) and (6) show the method of calculating an error matrix of measured parameters **P**, where **A** is a Jacobian matrix, and S is an error matrix of measured points. From the error matrix **P**, we estimate the uncertainties of such measured parameters as the diameter and the coordinate of a center in the specified measured strategy.

$$\mathbf{C} = (\mathbf{A}'\mathbf{S}^{-1}\mathbf{A})^{-1}\mathbf{A}'\mathbf{S}^{-1} \quad (5)$$

$$\mathbf{P} = \mathbf{CSC}' = (\mathbf{A}'\mathbf{S}^{-1}\mathbf{A})^{-1} \quad (6)$$

From two types of error matrices $\mathbf{S}_{ran}$ and $\mathbf{S}_{cov}$, there are three types of uncertainties of measured parameters $\mathbf{P}_{ran}$, $\mathbf{P}_{cov}$, and $\mathbf{P}_{r+c}$ as defined in Eqs. (7), (8), and (9). $\mathbf{P}_{ran}$ is the uncertainty matrix of parameters when the form deviation is assumed as a random function. $\mathbf{P}_{cov}$ is the uncertainty matrix of parameters when the form deviation has a specified autocorrelation function and is calculated using the autocorrelation function. $\mathbf{P}_{r+c}$ is the uncertainty matrix of parameters when the form deviation has a specified autocorrelation function and is calculated using the normal least squares method without the autocorrelation function.

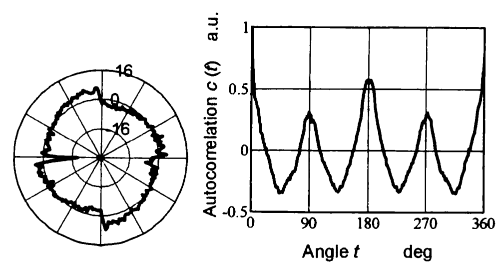
$$\mathbf{P}_{ran} = (\mathbf{A}'\mathbf{S}_{ran}^{-1}\mathbf{A})^{-1} = s_f^2(\mathbf{A}'\mathbf{A})^{-1} \quad (7)$$

$$\mathbf{P}_{cov} = (\mathbf{A}'\mathbf{S}_{cov}^{-1}\mathbf{A})^{-1} = s_f^2(\mathbf{A}'\mathbf{R}_{cov}^{-1}\mathbf{A})^{-1} \quad (8)$$

$$\mathbf{P}_{r+c} = ((\mathbf{A}'\mathbf{A})^{-1}\mathbf{A}')\,\mathbf{S}_{cov}\,((\mathbf{A}'\mathbf{A})^{-1}\mathbf{A}')' = s_f^2((\mathbf{A}'\mathbf{A})^{-1}\mathbf{A}')\,\mathbf{R}_{cov}\,((\mathbf{A}'\mathbf{A})^{-1}\mathbf{A}')' \quad (9)$$

Examples of Uncertainty Estimation for Circle Feature Measurement

Using a hole with a circularity of 28.8 μm shown in Fig. 3 (a) as an example, we calculate the uncertainty of measurement. Fig. 3 (b) illustrates the autocorrelation function of the hole. When the measuring points are set uniformly on the measured circle, $\mathbf{P}_{cov}$ and $\mathbf{P}_{r+c}$ are identical values. Fig. 4 illustrates the relationship between the number of data n and the uncertainty (standard deviation) of diameter s_d. In Fig. 4, the uncertainty of diameter in 4, 6, and 8 measured data is larger than those in an odd number of measured data. This is because the autocorrelation function of the measured circle (Fig. 3) has large 2- and 4- order frequency values.



(a) Circle feature (b) Autocorrelation

Fig. 3 Hole on aluminum board: diameter 20 mm, standard deviation $s_f^2 = 3.1$ μm, and circularity = 26.8 μm.

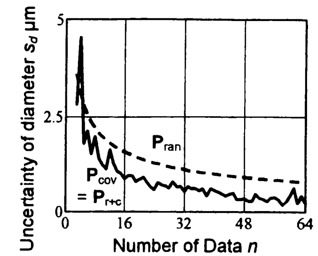


Fig. 4. Relationship between number of data n and uncertainty (standard deviation) of diameter s_d.

Summary

In this article, we formulate theoretically a method of evaluating uncertainty of measurement after calibration of the coordinate measuring system. Using this method, we can evaluate the uncertainty of specified measuring tasks such as size measurement in the workpiece coordinate system. Furthermore, we suggest that the distribution of the uncertainties of the size measurement of the coordinate measuring system demonstrates the performance of the coordinate measuring machine.

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