**ISMTII** 2005
7th International Symposium Series on Measurement Technology & Intelligent Instruments
6-8 September 2005, Huddersfield, UK
Page 63 - 66

# Estimation of uncertainty of measurements of 3D mechanisms after kinematic calibration

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**Abstract**. Calibration methods for 3D mechanisms are necessary to use the mechanisms as coordinate measuring machines. The calibration method of coordinate measuring machine using artifacts, the artifact calibration method, is proposed in taking account of traceability of the mechanism. There are kinematic parameters and form-deviation parameters in geometric parameters for describing the forward kinematic of the mechanism. In this article, the estimation methods of uncertainties using the calibrated coordinate measuring machine after the calibration are formulated. Firstly, the calculation method which takes out the values of kinematic parameters using least squares method is formulated. Secondly, the estimation value of uncertainty of the measuring machine is calculated using the error propagation method.

## 1. Introduction

Calibration methods of coordinate measuring machines are essential to measure accurately and to evaluate uncertainty of measurements [1][2]. In the feature based metrology, we formulated the method to evaluate the uncertainty in coordinate metrology [3][4]. Furthermore, we proposed the error propagation method to estimate the uncertainty of kinematic parameters in the calibration of the coordinate measuring machines [5][6].

These evaluations of uncertainty of the parameters are calculated in the machine coordinate system. However, the specified measurement tasks are done in a workpiece coordinate system after the calibration. In this article, the estimation methods of uncertainties using the calibrated coordinate measuring machine after the calibration are formulated. Firstly, a variance and covariance matrix on measuring points is calculated from a variance and covariance matrix of the kinematic parameters of the calibrated measuring machine. Secondly, uncertainties of a size measurement or a point measurement in a workpiece coordinate system are estimated using the error propagation method. Therefore, the estimation methods of uncertainties on the specified measuring tasks are formulated in the feature based metrology.

## 2. Uncertainty evaluation of a measuring point

2.1. Uncertainties of kinematic parameters

The uncertainty of kinematic parameters can be calculated from the uncertainty factors in the kinematic calibration of the coordinate measuring system using error propagation. The forward kinematics of the coordinate measuring system **f** is defined by kinematic parameters **p** and readings of each encoder **q** as in equation (1).

$$\mathbf{x} = \mathbf{f}(\mathbf{p}, \mathbf{q}) = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \quad \mathbf{p} = \begin{pmatrix} p_1 \\ p_2 \\ \vdots \end{pmatrix}, \quad \mathbf{q} = \begin{pmatrix} q_1 \\ q_2 \\ \vdots \end{pmatrix} \tag{1}$$

The uncertainty of kinematic parameters $\mathbf{S}_p$ is calculated from the Jacobian matrix **A** and the error matrix **S** in the calibration procedure as in equation (2). Where $\mathbf{S}_r$ is an uncertainty of coordinate conversion parameters of artifacts and $\mathbf{S}_{pr}$ is a covariance between the kinamatic parameters and the coordinate conversion parameters.

$$\begin{pmatrix} \mathbf{S}_p & \mathbf{S}_{pr} \\ \mathbf{S}_{pr} & \mathbf{S}_r \end{pmatrix} = (\mathbf{A}^t \, \mathbf{S}^{-1} \, \mathbf{A})^{-1} \tag{2}$$

2.2. Uncertainty of a measuring point after the calibration

The uncertainties matrix of a measuring point $\mathbf{T}_1$ consists of variances of each coordinate $sx^2$, $sy^2$ and $sz^2$, and covariances between XYZ coordinates *sxy*, *sxz* and *syz* of a measuring point. Equation (3) defines $\mathbf{T}_1$ as sum of three factors of uncertainties such as uncertainties from kinematic parameters $\mathbf{T}_p$, the uncertainty from encoders $\mathbf{T}_q$ and from probing $\mathbf{T}_m$. Each uncertainty can be calculated from equations (1) and (2) using error propagation method. Where $s_q$ is the uncertainty of encoders, $s_m$ is the uncertainty of probing and **E** is a unit matrix.

$$\mathbf{T}_1 = \begin{pmatrix} sx^2 & sxy & sxz \\ sxy & sy^2 & syz \\ sxz & syz & sz^2 \end{pmatrix} = \mathbf{T}_p + \mathbf{T}_q + \mathbf{T}_m = \mathbf{A}_p \, \mathbf{S}_p \, \mathbf{A}_p^t + s_q^2 \mathbf{A}_q \, \mathbf{A}_q^t + s_m^2 \, \mathbf{E} \tag{3}$$

2.3. Uncertainty of a specified measuring task

The uncertainty in equation (3) is evaluated on the measuring machine coordinate system. However, measurements are normally done on a workpiece coordinate system. Equation (4) defines a measuring task of size measurement in XY coordinate plane as an example of simple measuring task. The size *d* is calculated by two measuring points $\mathbf{x}_1$ and $\mathbf{x}_2$ in XY coordinate plane. From this equation, a Jacobian matrix $\mathbf{A}_d$ is defined using partial differential of coordinates of two points as in equation (5). Then, an uncertainty of size measurement $s_d$ is calculated as in equation (6). Where, $\mathbf{T}_{1\text{-}2}$ is a variance and covariance matrix of the two measuring points $\mathbf{x}_1$ and $\mathbf{x}_2$.

$$d = \mathbf{G}_d(\mathbf{x}_1, \mathbf{x}_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \tag{4}$$

$$\mathbf{A}_d = \begin{pmatrix} \dfrac{\partial \mathbf{G}_d}{\partial \mathbf{x}_1} & \dfrac{\partial \mathbf{G}_d}{\partial \mathbf{x}_2} \end{pmatrix} = \frac{\begin{pmatrix} -x_1 + x_2 & -y_1 + y_2 & -x_1 + x_2 & -y_1 + y_2 \end{pmatrix}}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}} \tag{5}$$

$$s_d^2 = \mathbf{A}_d \, \mathbf{T}_{1-2} \, \mathbf{A}_d^t \tag{6}$$

## 3. An example of calculation

We demonstrate an example of calculation using a two dimensional coordinate measuring system by two line cameras. Figure 1 shows the coordinate system of the two dimensional camera system; the tow cameras are positioned at (0, 0) and (*b*, 0) on XY coordinate plane. Parameters $u_1$ and $u_2$ are offset angles of each camera from Y axis and a parameter $b$ is X coordinate of camera 2 of approximately 200 mm.

A forward kinematics of the system can be expressed as in equation (7). Where $\mathbf{p}$ is a parameter vector of three kinematic parameters and $\mathbf{q}$ is an angle vector from images of each camera.

$$\begin{pmatrix} x \\ y \end{pmatrix} = \mathbf{x} = \mathbf{f}(\mathbf{p},\mathbf{q}) = \begin{pmatrix} \dfrac{b\tan(t_2-u_2)}{\tan(t_2-u_2)-\tan(t_1-u_1)} \\ \dfrac{b\tan(t_1-u_1)\tan(t_2-u_2)}{\tan(t_2-u_2)-\tan(t_1-u_1)} \end{pmatrix}, \quad \mathbf{p} = \begin{pmatrix} u_1 \\ u_2 \\ b \end{pmatrix}, \quad \mathbf{q} = \begin{pmatrix} t_1 \\ t_2 \end{pmatrix} \tag{7}$$

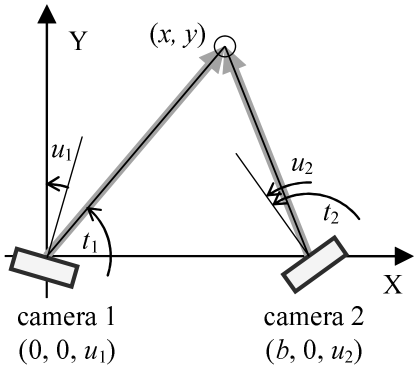


Figure 1. A two dimensional coordinate measuring system by two line cameras has three kinematic parameters such as offset angles of line cameras $u_1$ and $u_2$, and a length of baseline $b$.

### 3.1. Results of calibration

Table 1 shows the results of calibration of the two dimensional line camera system under the conditions; no. of points: 25 in X of 50-150 mm and Y of 50-150 mm at 25 mm intervals, the camera 1 is located at (0, 0) and the camera 2 is located at (200, 0). As errors in the calibration, a random uncertainty of probing is 10 μm, a measuring uncertainty of an external measuring system for the calibration is 5 μm and an angular uncertainty of each camera is 0.001 deg are obtained.

**Table 1.** Calibration results of the two dimensional line camera; no. of points: 25 in X of 50-150 mm and Y of 50-150 mm at 25 mm intervals. The standard deviations of the three kinematic parameters and the correlation coefficients between the parameters are calculated.

| | standard deviations | correlation coefficients for | |
|---|---|---|---|
| | | $u_2$ | $b$ |
| $u_1$ | 0.0042 deg | 0.0057 | -0.4764 |
| $u_2$ | 0.0042 deg | - | 0.4763 |
| $b$ | 15.7 μm | - | - |

### 3.2. Uncertainties after the calibration

From the results of calibration, we calculate the positioning uncertainty using equation (2). Figure 2 illustrates a contour map of root sum square value from uncertainties of X and Y coordinates of the two dimensional camera system. In this figure, it is assumed that an uncertainty of probing is 10 μm and an uncertainty of each camera is 0.001 deg. Figure 3 shows the size measurement uncertainties from a specified point (100, 100) using equation (6).

In figure 2, the distribution of uncertainty is effected by the selection method of the coordinate system and parameters in the calibration and the uncertainty of positioning is over estimated compare to the results in figure 3. In the other hand, figure 3 shows the symmetrical distribution, it means the selection method of the coordinate system and parameters does not influence on the evaluation results of uncertainties. Figure 4 illustrates the relationship between the measuring sizes and uncertainties in the measuring range of the coordinate measuring system. This relationship shows the performance of the coordinate system clearly.

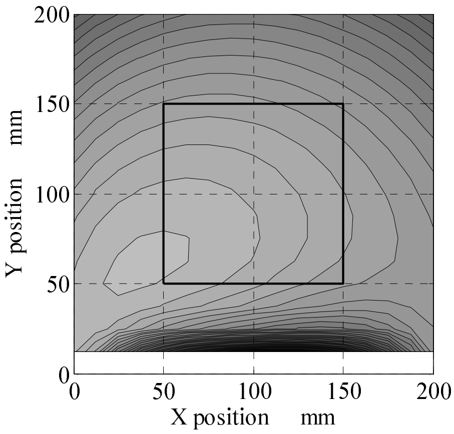


Figure 2. Distribution of position uncertainties in the measuring machine coordinate system after the calibration (unit is μm).

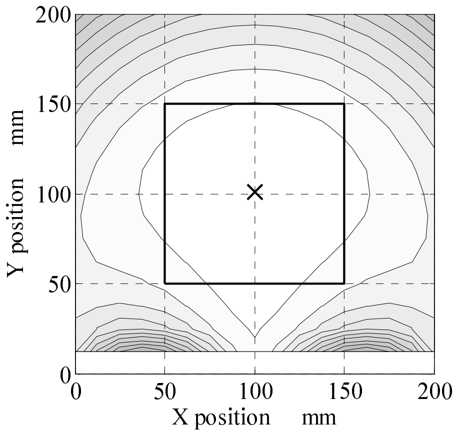


Figure 3. Distribution of size measurement uncertainties from a specified point (100, 100) after the calibration (unit is μm).

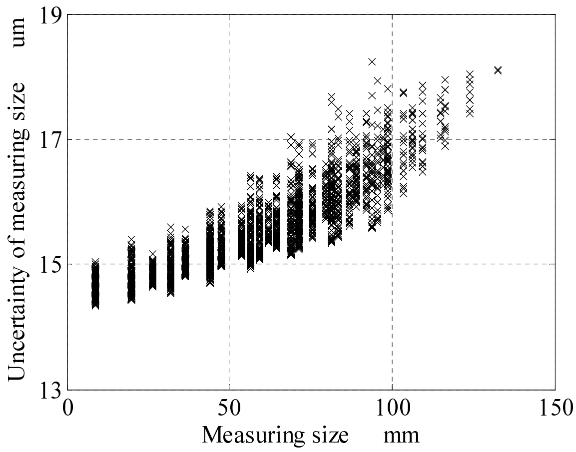


Figure 4. Relationship between the measuring size and the uncertainties of measuring size (unit is μm) in the measuring range.

## 4. Conclusions

In this article, we formulate theoretically the evaluation method of uncertainty of measurements after the calibration of the coordinate measuring system. Using this method, we can evaluate the uncertainty in the specified measuring tasks such as size measurement in the workpiece coordinate system. Furthermore, we suggest that the uncertainties distribution of size measurements of the coordinate measuring system shows the performance of the coordinate measuring machine.

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