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Uncertainty Estimation Using Monte-Carlo Method Constrained by Correlations of the Data

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Abstract. Simple and easy uncertainty estimation method is proposed. Provided that specification or simple experimental result is available, possible variance and covariance in error are estimated and Monte-Carlo simulation reflecting constraint caused by the covariance can be performed. Comparison between uncertainties obtained by the proposed method and that by actual measurements on real CMM shows good agreement within 1 μm over-estimation.

Introduction

In recent years, the concept of traceability of measurement is spreading and the importance of uncertainty comes to be recognized[1]. Reality, however, is that uncertainty can be analytically calculated only in simple model case. For example, thinking about measurement with coordinate measurement machine (CMM), it has not only the complex structure but also the user may arbitrarily configure measurement task and the procedure for evaluating measured result. It requires enormous effort for users to evaluate uncertainty of the measurement, and the difficulty prevents the application on complicate measurement.

In order to solve this problem, uncertainty estimation by Monte-Carlo simulation is becoming mainstream. This method is firstly realized as Virtual CMM (VCMM) by PTB[2]. However, VCMM also requires times and effort for the simulation, because it needs a lot of experimental data.

In this report, very easy and simple uncertainty estimation method "Constrained Monte-Carlo Simulation (CMS) method" is proposed. This method is intended to estimate measurement uncertainty quickly with reasonable reliability and reduce user's burden for it.

Constrained Monte-Carlo Simulation

In this method, Monte-Carlo simulation reflecting constraint caused by possible correlations in error is performed. The method consists of three steps. At first a covariance matrix of measurement errors is estimated. Secondly the matrix is decomposed into eigen vectors and eigen values. In the last step the eigen vectors are linearly coupled with random coupling coefficients, where variance of coefficients correspond to eigen values respectively, and we can obtain a trial measurement's error.

Derivation of the covariance matrix, the first step, is a key point of this method[3]. We may derive it from Machine's specifications. For example, a specification of CMM is represented as a maximum permissible error in form of

$$\mathrm{MPE}_E = a + b \cdot l \ [\mathrm{m}], \tag{1}$$

where a and b are constant terms and l is measuring length. We consider the MPE_E has a information of variance and covariance of the measurement. This means the machine potentially shows error up to $a+b\cdot l_{\max}$, where $l_{\max}$ is maximum measurement range, but shorter the length than $l_{\max}$ the error is reduced due to covariance effect.

For ease of explanation, we consider one dimensional length measurement. Two coordinate values x_1 and x_2 result the size measurement $l=|x_2-x_1|$. $Var(l)$, the variance of the size measurement, may be written as Eq. (2). Here, let the variance of x_1 and x_2 be independent of allocation of the coordinate values and the covariance be a function of l. Furthermore, assuming that when the length between them is longer than $l_{\max}$ the covariance becomes 0, we have Eq. (3).

$$Var(l)=Var(x_2)+Var(x_1)-2Cov(x_1,x_2)$$
$$=2Var_x-2Cov(l)\ , \tag{2}$$
$$Var_x=\frac{1}{2}Var(l_{\max})\ , \tag{3}$$

Using Eqs. (2) and (3), $Cov(l)$ is represented as Eq. (4). Here, let us simplify discussion by interpreting MPE_E be comparable to 95% probability limit of normal distribution. Then, $Var(l)$ may also be written as Eq. (5) from Eq. (1).

$$Cov(l)=\frac{1}{2}\{Var(l_{\max})-Var(l)\}\ , \tag{4}$$
$$Var(l)=\{(a+b\cdot l)/2\}^2\ , \tag{5}$$

From Eq. (4) covariance can be calculated as a function of length l. Suppose our measurement strategy have a series of n point coordinates, covariance matrix $\mathbf{C}$ with $n\times n$ dimension is filled. Having the variance and covariance information, a unique Monte-Carlo simulation fully reflecting the given statistical characteristics is performed.

In the second step, the covariance matrix $\mathbf{C}$ is decomposed into the eigen vectors $\mathbf{V}$ and the corresponding eigen values $\mathbf{D}$.

In the last step, trial value $\hat{\mathbf{x}}$ is obtained by recomposing with random number ε_i as Eq. (7), where ε_i is a random number satisfying $E(\varepsilon_i)=0$ and $Var(\varepsilon_i)=\lambda_i$. It utilizes technique of principal component analysis. This step is repeated up to desired number of times and same number of trial values are produced.

$$\mathbf{C}=\mathbf{VDV}^T\ , \tag{6}$$
where $\mathbf{V}=[[\mathbf{v}_1]\ [\mathbf{v}_2]\ \cdots\ [\mathbf{v}_n]]$, $\mathbf{D}=\mathrm{diag}[\lambda_1\ \cdots\ \lambda_n]$,
$$\hat{\mathbf{x}}=\varepsilon_1\mathbf{v}_1+\varepsilon_2\mathbf{v}_2+\cdots+\varepsilon_n\mathbf{v}_n\ , \tag{7}$$

Integrating Effect Caused by Probe

In the previous chapter, we propose a procedure for deriving covariance matrix from MPE_E. However, constant term of the equation includes probing directional error which would be represented as a function of azimuth angle ϕ and elevation angle θ of probing direction. We have to reflect the effect separately.

We empirically generate probe correlation model. Error of commercially available touch trigger probes have periodic component in azimuth direction and bias component in elevation direction[4]. These two components are firstly separated by performing measurement of reference sphere over the upper hemi sphere and averaging the measured result in azimuth direction (as Fig. 1). Then, the periodic component is fitted by two-dimensional function $f(\phi, \theta)$, it would be cyclic in azimuth direction. Operating this procedure by changing probe setting angle in azimuth by 7.5 deg., totally 48 measurements and the function $f_i(\phi,\theta)$, where i = 1 : 48 is probe setting number, are obtained. Considering probing directions appeared in the measurement tasks, we can build an error matrix with $n \times 48$ dimension as shown in Eq. (8), and a covariance matrix representing probing directional error can be calculated as Eq. (9). Here, variance caused by probing directional error Var_{prb} can be estimated likewise. $Var(l)$ in the previous chapter should be reduced as Eq. (10). Consequently Monte-Carlo simulation reflecting the probing directional error on CMS scheme can be realized.

$$X = \begin{bmatrix} f_1(\phi_1,\theta_1) & \cdots & f_1(\phi_n,\theta_n) \\ & \vdots & \\ f_{48}(\phi_1,\theta_1) & \cdots & f_{48}(\phi_n,\theta_n) \end{bmatrix}, \tag{8}$$

$$C_{prb} = X^T X , \tag{9}$$

$$Var(l) = \left\{ \left(a + b \cdot l \right) / 2 \right\}^2 - Var_{prb} , \tag{10}$$

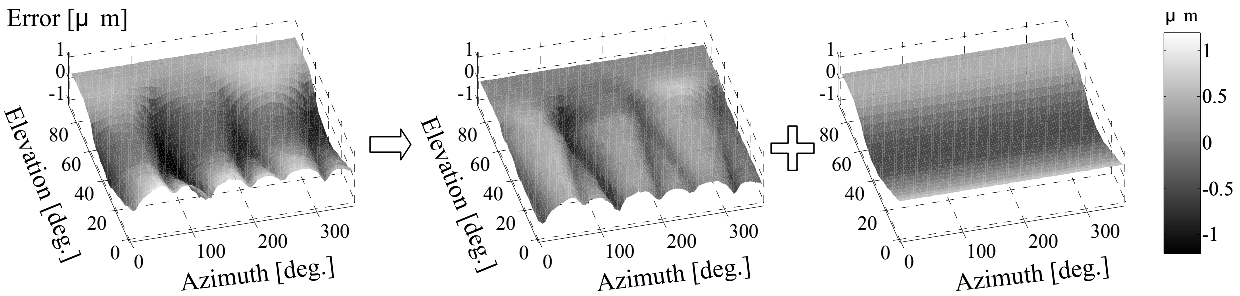


Fig.1 Separation of bias component from spatially periodic component

Simulation with Input Parameter from Real CMM

To confirm the ability of CMS, we perform simulation measurement by CMS with input parameter from a real CMM with the specification and the variance of probing directional error as Eqs. (11) and (12), where l is in unit of mm. Correlation of probing directional error is experimentally estimated as proposed in the last chapter. The simulation and the corresponding real measurement are performed for two measurement tasks namely measurement of a step gauge and that of a cylinder artifact, and both results are compared.

$$MPE_E = 1.9 + 3/1000 \times l \ \ [\mu\text{m}], \tag{11}$$

$$Var_{prb} = 0.06 \ [\mu\text{m}^2], \tag{12}$$

The result of ten times step gauge measurements is shown in Fig. 2. The left figure shows the simulation result and the other one does the result from the real CMM. Physical measurements are performed on ten different CMMs but same model. We confirm both results resemble each other well by comparing them.

The cylinder artifact (ϕ 90 mm, L 250 mm in nominal size) has been calibrated by an NMI. The calibration uncertainty was reported as less than 0.7 μm for measurands and it is minor comparing to practical performance of the CMM. The physical measurement is repeated 256 times on the real CMM by varying location and orientation of the artifact. The measurement result may provide an experimentally obtained task specific uncertainty statements by multiple measurements. Simulation measurement by CMS is repeated 256 times as well. The comparison result of expanded uncertainties between CMS and real measurement is summarized in Fig. 3. However the worst shows 1 μm over estimated, this result shows the effectiveness of this method.

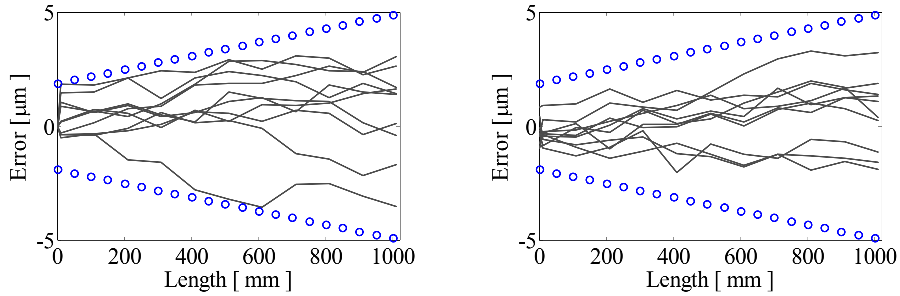
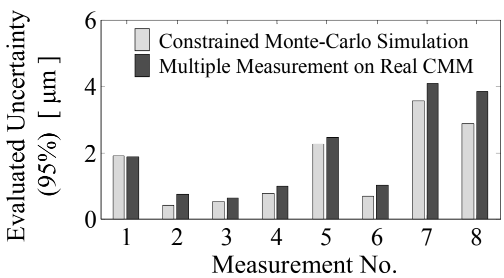


Fig.2 Comparison of simulation with real measurement (step gauge measurement).



Measurement No.
1. Length
2. Diameter
3. Flatness-1
4. Flatness-2
5. Cylindricity
6. Parallelism
7. Squareness-1
8. Squareness-2

Fig.3 Comparison of uncertainty evaluation result (cylinder artifact).

Conclusion

Very simple and easy uncertainty estimation method for industrially available CMM is proposed. This method performs Monte-Carlo simulation in orthogonally based error space and generate multiple trial measurement results which can be used for calculating task specific uncertainty. Comparison between uncertainties obtained by the proposed method and that by actual measurements on real CMM shows good agreement.

This method can be widely applied to various measuring instruments in general, provided that variance and covariance information can be estimated. The method benefits the users to evaluate uncertainty easily even in case of complex measurement. The method is expected to promote awareness of importance and usefulness of traceability and uncertainty in measurement.

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