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Meas. Sci. Technol. **19** (2008) 084001 (5pp) doi:10.1088/0957-0233/19/8/084001

Uncertainty estimation for coordinate metrology with effects of calibration and form deviation in strategy of measurement

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Received 30 November 2007, in final form 2 March 2008
Published 4 July 2008
Online at stacks.iop.org/MST/19/084001

Abstract
Coordinate metrology utilizes complex measuring systems such as coordinate measuring machines, laser trackers and triangulation systems. Therefore, calibrating the coordinate measuring system using artifacts, e.g. the artifact calibration method, is a key technology. In this paper, methods of estimating uncertainties using the coordinate measuring system after calibration are formulated. First, a calculation method which extracts the values of kinematic parameters using the least-squares method is formulated. Second, the uncertainty of the specified measuring task is calculated using the uncertainty propagation method. A coordinate measuring system utilizing two line cameras is analyzed as an example. Moreover, the influences of the form deviations of the measured workpiece are calculated in the measurement of the features of a circle.

Keywords: uncertainty estimation, coordinate measuring machine, calibration, form deviation

1. Introduction

The calibration methods of coordinate measuring machines are essential for accurate measurement, for the establishment of traceability and for the evaluation of measurement uncertainty [1–6]. Through the use of feature-based metrology, we previously formulated a method to evaluate the uncertainty in coordinate metrology [7, 8]. Furthermore, we proposed an uncertainty propagation method to estimate the uncertainty of kinematic parameters in the calibration of coordinate measuring machines [9–11].

These evaluations of parameter uncertainty are calculated in the machine coordinate system. However, the specified measurement tasks are done in a workpiece coordinate system after calibration. In this paper, methods of estimating uncertainties using the calibrated coordinate measuring machine after calibration are formulated. First, a variance and covariance matrix on measuring points is calculated based on a variance and covariance matrix of the kinematic parameters of the calibrated measuring machine. Second, the uncertainties of a size measurement or a point measurement in a workpiece coordinate system are estimated using the uncertainty propagation method. Therefore, the methods of estimating the uncertainties on the specified measuring tasks are formulated in the feature-based metrology. Moreover, the form deviation of the measured workpiece influences the uncertainty of the measurement results. We formulated an uncertainty estimation method based on form deviations, and show an example involving a circle feature measurement. The proposed method can be a means of modeling according to GUM (*Guide to the Expression of Uncertainty in Measurement*) [4].

2. Uncertainty propagation in coordinate metrology

In coordinate metrology, associated features and associated derived features are calculated from measured data sets on real features by a CMM (coordinate measuring machine).

Then, the associated features are compared with the nominal features indicated on the drawings. In this data processing, the features are primal targets to calculate, evaluate and process. Consequently, this process is called feature-based metrology [7].

A key technique in feature-based metrology is to estimate the uncertainty of measurement [4] in the specific measuring strategy [1–3]. A method for estimating the uncertainties of measured parameters has already been proposed in which only random errors are under consideration [8]. The uncertainty of each measured point is defined by error analysis of the CMM and the probing system. We give an example of how to estimate the uncertainty of measuring points in section 3. From the uncertainty of measuring points, the uncertainty of the measured feature can be calculated statistically by using the following equations.

Equation (1) shows an observation equation, where $\mathbf{A}$ is the Jacobian matrix, $\mathbf{d}$ is a measurement vector and $\mathbf{x}$ is a parameter vector. The parameters of the feature are calculated by a least-squares solution in equation (2). We also estimated the uncertainty of parameters $\mathbf{P}$ of the feature by the uncertainty propagation in equation (3) and, the uncertainty matrix (error matrix) $\mathbf{S}$ is defined by the uncertainty of the measuring points. We analyze the uncertainty measurement of circular features as an example in section 4,

$$\mathbf{d} = \mathbf{Ax} \tag{1}$$

$$\mathbf{x} = (\mathbf{A}^t\mathbf{S}^{-1}\mathbf{A})\mathbf{A}^t\mathbf{S}^{-1}\mathbf{d} \tag{2}$$

$$\mathbf{P} = (\mathbf{A}^t\mathbf{S}^{-1}\mathbf{A})^{-1}. \tag{3}$$

In this paper, it is assumed that the bias of all factors in measurements is compensated. When there is no systematic error in the measuring process, the uncertainty matrix is the unit matrix multiplied by the random error. In this case, the uncertainty matrix does not affect the result of the calculation in equation (2). For known systematic errors, we can compensate for the measuring values. On the other hand, when unknown systematic errors influence the measuring results, the uncertainty matrix has factors of covariance due to the systematic errors. Using the uncertainty matrix, we can statistically estimate the influences from the unknown systematic errors.

3. Uncertainty evaluation of a measuring point after calibration

There are three steps to evaluate the uncertainties of a measuring point: (1) determining the kinematic parameters by calibration, (2) measuring the point after calibration and (3) measurement under the conditions of the specified measuring task. We formulated these steps theoretically, and analyzed a coordinate measuring system utilizing two line-cameras as an example.

3.1. Theoretical calculations for uncertainty of a measuring point

The uncertainty of kinematic parameters can be calculated based on the uncertainty factors in the kinematic calibration of the coordinate measuring system using uncertainty propagation. The forward kinematics of the coordinate measuring system $\mathbf{f}$ is defined by the kinematic parameters $\mathbf{p}$ and readings of each encoder $\mathbf{q}$ as in equation (4). The uncertainty of the kinematic parameters $\mathbf{S}_p$ is calculated based on the Jacobian matrix $\mathbf{A}$ and the uncertainty matrix $\mathbf{S}$ in the calibration procedure, as in equation (5), where $\mathbf{S}_r$ is the uncertainty of the coordinate conversion parameters of artifacts and $\mathbf{S}_{pr}$ is the covariance between the kinematic parameters and the coordinate conversion parameters:

$$\mathbf{x} = \mathbf{f}(\mathbf{p}, \mathbf{q}) = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \qquad \mathbf{p} = \begin{pmatrix} p_1 \\ p_2 \\ \vdots \end{pmatrix}, \quad \mathbf{q} = \begin{pmatrix} q_1 \\ q_2 \\ \vdots \end{pmatrix} \tag{4}$$

$$\begin{pmatrix} \mathbf{S}_p & \mathbf{S}_{pr} \\ \mathbf{S}_{pr} & \mathbf{S}_r \end{pmatrix} = (\mathbf{A}^t\mathbf{S}^{-1}\mathbf{A})^{-1}. \tag{5}$$

The uncertainty matrix of a measurement point $\mathbf{T}_1$ consists of variances of each coordinate sx^2, sy^2 and sz^2, and covariances between the *XYZ* coordinates sxy, sxz and syz of a measuring point. Equation (6) defines $\mathbf{T}_1$ as the sum of three factors of uncertainties, such as uncertainties from kinematic parameters $\mathbf{T}_p$, the uncertainty from encoders $\mathbf{T}_q$ and the uncertainty from probing $\mathbf{T}_m$. Each uncertainty can be calculated from equation (4) using the uncertainty propagation method; here, s_q is the uncertainty of the encoders, s_m is the uncertainty of probing and $\mathbf{E}$ is the unit matrix.

The uncertainty in equation (6) is evaluated with regard to the measuring machine coordinate system. However, measurements are normally done on a workpiece coordinate system. Then, the uncertainty of measurement s_t is calculated as in equation (7), where $\mathbf{A}_t$ is the Jacobian matrix by the specified measuring task and $\mathbf{T}_{1-n}$ is a variance and covariance matrix of the measuring points $\mathbf{x}_1 - \mathbf{x}_n$.

$$\mathbf{T}_1 = \begin{pmatrix} sx^2 & sxy & sxz \\ sxy & sy^2 & syz \\ sxz & syz & sz^2 \end{pmatrix} = \mathbf{T}_p + \mathbf{T}_q + \mathbf{T}_m$$
$$= \mathbf{A}_p\mathbf{S}_p\mathbf{A}_p^t + s_q^2\mathbf{A}_q\mathbf{A}_q^t + s_m^2\mathbf{E} \tag{6}$$

$$s_t^2 = \mathbf{A}_t\mathbf{T}_{1-n}\mathbf{A}_t^t. \tag{7}$$

3.2. Example of calibration of the coordinate measuring system using two line-cameras

We demonstrate an example of calculation using a two-dimensional coordinate measuring system employing two line-cameras. Figure 1 shows the coordinate system of the two-dimensional camera system; the two cameras are positioned at (0, 0) and (a, 0) on the *XY* coordinate plane. Parameters b_1 and b_2 are the offset angles of each camera from the *Y*-axis and parameter a is the *X* coordinate of camera 2 of approximately 200 mm.

The calibration conditions of the two-dimensional line-camera system are as follows: number of calibration points: 25 in *X* and *Y* of 50–150 mm at 25 mm intervals. The standard deviations (uncertainties) of the calibration result for b_1, b_2

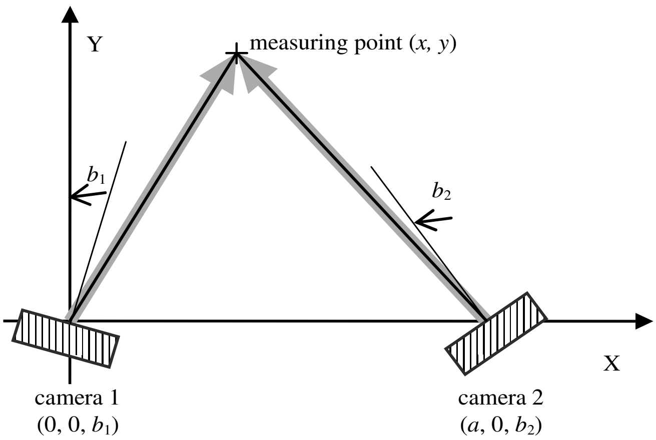


Figure 1. Two-dimensional coordinate measuring system using two line-cameras. The kinematic parameters of the system are b_1, b_2 and a.

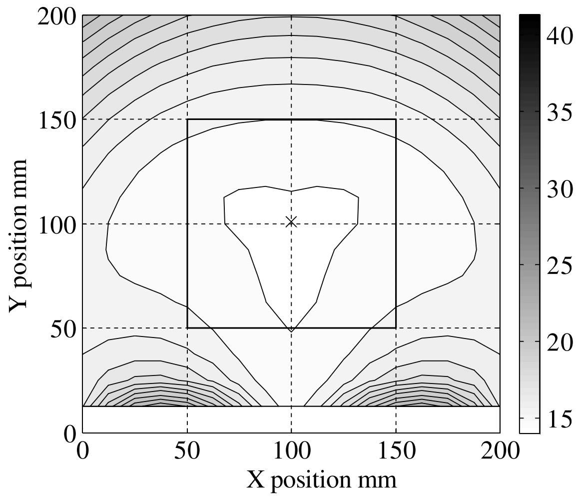


Figure 2. Distribution of size measurement uncertainties from a specified point (100, 100) after calibration (unit is μm).

and a are 0.0042°, 0.0042° and 15.7 μm, respectively, under a random uncertainty of probing, while the obtained calibration and the angle of each camera are 10 μm, 5 μm and 0.001°, respectively.

Equation (8) defines a measuring task of size measurement in the XY coordinate plane as an example of a simple measuring task. The size d is calculated by two measuring points $\mathbf{x}_1$ and $\mathbf{x}_2$ in the XY coordinate plane. From this equation, the Jacobian matrix $\mathbf{A}_d$ is defined using the partial differential of the coordinates of two points as in equation (9). Then, an uncertainty of size measurement s_d is calculated as in equation (10), where $\mathbf{T}_{1-2}$ is a variance and covariance matrix of the two measuring points $\mathbf{x}_1$ and $\mathbf{x}_2$. Figure 2 shows the uncertainty s_d of the size measurement from the specified point $\mathbf{x}_1$ ($x_1 = 100$, $y_1 = 100$) in equation (8). The distribution of the uncertainty is symmetric, which means that the selection method of the coordinate system and parameters does not influence the evaluation results of the uncertainties,

$$d = \mathbf{G}_d(\mathbf{x}_1, \mathbf{x}_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \tag{8}$$

$$\begin{aligned}\mathbf{A}_d &= \begin{pmatrix} \dfrac{\partial \mathbf{G}_d}{\partial \mathbf{x}_1} & \dfrac{\partial \mathbf{G}_d}{\partial \mathbf{x}_2} \end{pmatrix} \\ &= \frac{(-x_1 + x_2 \quad -y_1 + y_2 \quad -x_1 + x_2 \quad -y_1 + y_2)}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}\end{aligned} \tag{9}$$

$$s_d^2 = \mathbf{A}_d \mathbf{T}_{1-2} \mathbf{A}_d^t. \tag{10}$$

4. Uncertainty from form deviations of a measured workpiece

The form deviation of the measured workpiece influences the uncertainty of the measurement results. We formulated an uncertainty estimation method based on form deviations, and showed an example involving the circle feature measurement.

4.1. Theoretical calculations for uncertainty from form deviations

Equations (11) and (12) show the method of calculating an uncertainty matrix of the measured parameters $\mathbf{P}$, where $\mathbf{A}$ is the Jacobian matrix and $\mathbf{S}$ is an uncertainty matrix of the measured points. From the uncertainty matrix of parameters $\mathbf{P}$, we estimate the uncertainties of such measured parameters as the diameter and the coordinate of a center in the specified measured strategy:

$$\mathbf{C} = (\mathbf{A}^t \mathbf{S}^{-1} \mathbf{A})^{-1} \mathbf{A}^t \mathbf{S}^{-1} \tag{11}$$

$$\mathbf{P} = \mathbf{C}\mathbf{S}\mathbf{C}^t = (\mathbf{A}^t \mathbf{S}^{-1} \mathbf{A})^{-1}. \tag{12}$$

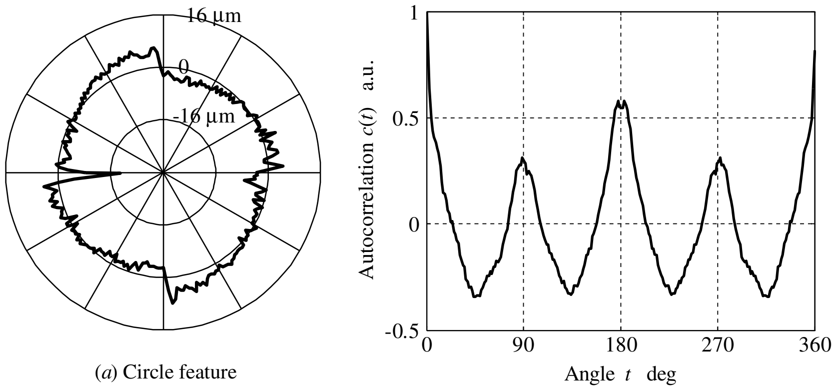
When the form deviation is a random function, the uncertainty matrix $\mathbf{S}_{\text{ran}}$ is defined by the unit matrix and the uncertainty s_f of the form deviation in equation (13). When the form deviation has a specified function, the uncertainty matrix $\mathbf{S}_{\text{cov}}$ is defined by the autocorrelation matrix $\mathbf{R}_{\text{cov}}$ and the uncertainty s_f of the form deviation in equation (14):

$$\mathbf{S}_{\text{ran}} = \begin{pmatrix} s_f^2 & & 0 \\ & \ddots & \\ 0 & & s_f^2 \end{pmatrix} = s_f^2 \begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix} = s_f^2 \mathbf{E} \tag{13}$$

$$\begin{aligned}\mathbf{S}_{\text{cov}} &= \begin{pmatrix} s_f^2 & s_{12} & \cdots & s_{1n} \\ s_{12} & \ddots & s_{ij} & \vdots \\ \vdots & s_{ij} & \ddots & s_{n-1n} \\ s_{1n} & \cdots & s_{n-1n} & s_f^2 \end{pmatrix} = s_f^2 \begin{pmatrix} 1 & r_{12} & \cdots & r_{1n} \\ r_{12} & \ddots & r_{ij} & \vdots \\ \vdots & r_{ij} & \ddots & r_{n-1n} \\ r_{1n} & \cdots & r_{n-1n} & 1 \end{pmatrix} \\ &= s_f^2 \mathbf{R}_{\text{cov}}.\end{aligned} \tag{14}$$

From two types of error matrices $\mathbf{S}_{\text{ran}}$ and $\mathbf{S}_{\text{cov}}$, there are three types of uncertainties of measured parameters $\mathbf{P}_{\text{ran}}$, $\mathbf{P}_{\text{cov}}$ and $\mathbf{P}_{\text{r+c}}$, as defined in equations (15)–(17), respectively, where $\mathbf{S}_{\text{ran}}$ is the uncertainty of the form deviation s_f multiplied by a unit matrix and $\mathbf{S}_{\text{cov}}$ is s_f multiplied by an autocorrelation matrix of the form deviation $\mathbf{R}_{\text{cov}}$ as equations (13) and (14).

$\mathbf{P}_{\text{ran}}$ is the uncertainty matrix of the parameters when the form deviation is assumed as a random function. $\mathbf{P}_{\text{cov}}$ is the uncertainty matrix of the parameters when the form deviation has a specified autocorrelation function and is calculated using the autocorrelation function $\mathbf{R}_{\text{cov}}$. $\mathbf{P}_{\text{r+c}}$ is the uncertainty matrix of the parameters when the form deviation has a specified autocorrelation function and is calculated using the normal



(*a*) Circle feature

(*b*) Autocorrelation

Figure 3. Hole in aluminum board: diameter 20 mm, form deviation (standard deviation) s_f 3.1 μm, number of measuring points of 256 and circularity 28.8 μm. (*a*) Circle feature, (*b*) autocorrelation.

least-squares method without the autocorrelation function $\mathbf{R}_{\rm cov}$. In the three types of uncertainties, $\mathbf{P}_{\rm cov}$ is the most accurate estimation when the form deviation of the measured workpiece has a specified autocorrelation function $\mathbf{R}_{\rm cov}$:

$$\mathbf{P}_{\rm ran} = \left(\mathbf{A}^t\mathbf{S}_{\rm ran}^{-1}\mathbf{A}\right)^{-1} = s_f^2(\mathbf{A}^t\mathbf{A})^{-1} \tag{15}$$

$$\mathbf{P}_{\rm cov} = \left(\mathbf{A}^t\mathbf{S}_{\rm cov}^{-1}\mathbf{A}\right)^{-1} = s_f^2\left(\mathbf{A}^t\mathbf{R}_{\rm cov}^{-1}\mathbf{A}\right)^{-1} \tag{16}$$

$$\begin{aligned}\mathbf{P}_{\rm r+c} &= ((\mathbf{A}^t\mathbf{A})^{-1}\mathbf{A}^t)\mathbf{S}_{\rm cov}((\mathbf{A}^t\mathbf{A})^{-1}\mathbf{A}^t)^t \\ &= s_f^2((\mathbf{A}^t\mathbf{A})^{-1}\mathbf{A}^t)\mathbf{R}_{\rm cov}((\mathbf{A}^t\mathbf{A})^{-1}\mathbf{A}^t)^t.\end{aligned} \tag{17}$$

4.2. Examples of uncertainty estimation for circle feature measurement

Using a hole with a circularity of 28.8 μm and form deviation (standard deviation) s_f of 3.1 μm as shown in figure 3(*a*) as an example, we calculated the uncertainty of measurement of the least-squares diameter. The number of measuring points is 256 uniformly on the measured circle and the uncertainty of measurement is included in the form deviation of the measured circle. Figure 3(*b*) illustrates the autocorrelation function of the hole. When the measuring points are set uniformly on the measured circle, $\mathbf{P}_{\rm cov}$ and $\mathbf{P}_{\rm r+c}$ are identical values. Figure 4 illustrates the relationship between the number of data n and the uncertainty of least-squares diameter s_d calculated by equations (15)–(17). In figure 4, the uncertainty of the diameter in the 4, 6 and 8 measured data is larger than those in an odd number of measured data. This is because the autocorrelation function of the measured circle (figure 3) has large 2- and 4-order frequency values.

Figure 5 illustrates the relationship between the number of data n and the uncertainty of least-squares diameter s_d, when the measuring points are set on quarter of the measured circle. In figure 5, $\mathbf{P}_{\rm r+c}$ is the overestimation and $\mathbf{P}_{\rm ran}$ is the underestimation from the accurate estimation of the uncertainty $\mathbf{P}_{\rm cov}$.

5. Summary

In this paper, we theoretically formulate a method of evaluating the uncertainty of measurement after calibration

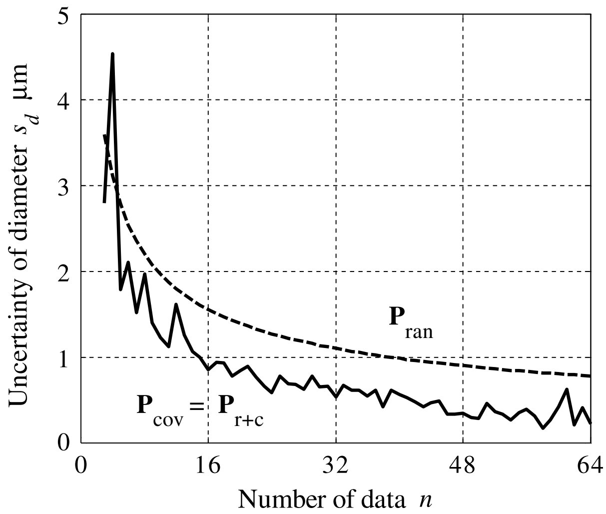


Figure 4. Relationship between number of data n and uncertainty of least-squares diameter s_d, when the measuring points are set uniformly on the measured circle.

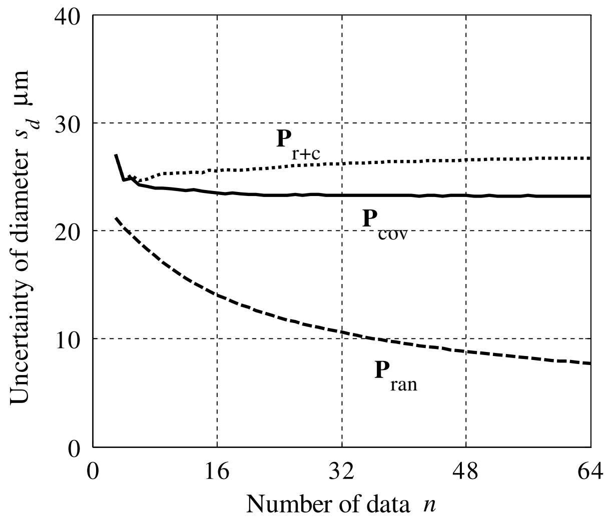


Figure 5. Relationship between the number of data n and uncertainty of the least-squares diameter s_d, when the measuring points are set on quarter of the measured circle.

of the coordinate measuring system. Using this method, we can evaluate the uncertainty of specified measuring tasks such as size measurement and circle feature measurement in the workpiece coordinate system. The uncertainty of the specified measuring task is calculated using the uncertainty propagation method. A coordinate measuring system utilizing two line-cameras is analyzed as an example. Moreover, the influences of the form deviations of the measured workpiece are calculated in the measurement of the features of a circle. These theoretical methods are applied to two-dimensional models in this paper. In the next step, we will expand these methods to three-dimensional complex models and execute experiments to verify the simulated results.

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