Japanese Journal of Applied Physics
Vol. 47, No. 9, 2008, pp. 7386–7389

Remote Measurement of Refractive Index of Air Using Tandem Interferometer over Long Optical Fiber

Hirokazu MATSUMOTO*, Kaoru SASAKI, and Akiko HIRAI

National Metrology Institute of Japan, National Institute of Advanced Industrial Science and Technology (AIST),
1-1-1 Umezono, Tsukuba, Ibaraki 305-8563, Japan

(Received March 11, 2008; accepted May 8, 2008; published online September 12, 2008)

The refractive index of air is accurately measured using a vacuum cell and a tandem low-coherence interferometer with a communication optical fiber of 30 km length with standard uncertainties of 0.20 ppm for the phase refractive index and 0.29 ppm for the group refractive index. Experimental results are compared with the value calculated from the equation for the refractive index of air using data obtained from measurements of the surrounding air, with a standard uncertainty of 0.22 ppm, and the difference between the average values is within 0.1 ppm. This method is applicable to the *in situ* high-precision measurement of optical length. [DOI: 10.1143/JJAP.47.7386]

KEYWORDS: refractive index, optical fiber, low-coherence interferometer, remote measurement, tandem interferometer, optical length

1. Introduction

The number of requirements for the measurement of the refractive index of air has been increasing in the area of optical length measurement by high-precision techniques, such as interferometry and intensity modulation. Many papers on interferometric measurements of the phase and group refractive indices of air have been published, in which a vacuum cell is used in an interferometer (which we here refer to as the "vacuum cell method").[1–4] Empirical equations for the refractive index of air were derived from the experimental data obtained by Edlén, Ciddor, and other researchers. For convenience, the refractive index of air is often determined from air temperature, air pressure, air humidity and CO_2 gas content using the equations and data obtained from sensors with high accuracy (which we call the "calculation method").[5–7]

Recently, remote or *in situ* interferometric length measurements have become necessary for fast, low-cost and high-accuracy calibration. Therefore, we have developed a remote measurement technique using a tandem low-coherence interferometer and an optical-fiber network.[8,9] In this case, the refractive index of air during the calibration of length is important for the correction of the remotely measured optical length.

In this study, we propose a new technique for measuring the group and phase refractive indices of air using a 100-mm-long vacuum cell. Two low-coherence interferometers are connected by a single-mode optical fiber of 30 km length in a wavelength range of 1.5 µm. The measurements of the phase and group refractive indices were achieved with standard uncertainties of 0.20 and 0.29 ppm, respectively, which are reasonable compared with the values obtained by the calculation method.

2. Principle of Tandem Interferometer

The optical setup of the new tandem interferometer is shown in Fig. 1. It consists of two Michelson-type interferometers, which are set in a well-air-conditioned room. One interferometer has a translation mechanism for generating low-coherence interference fringes, and the other interferometer has a vacuum cell, which is composed of two parallel glass plates (thickness: 10 mm, diameter: 40 mm) and a glass tube of 100 mm length and 20 mm diameter. The interferometers are connected by a 30-km-long single-mode optical fiber in the air-conditioned room and a 2.6-km-long single-mode optical fiber outside the building, and the optical paths of the tandem interferometer are composed of the optical paths of the two low-coherence interferometers. The low-coherence interference fringes are generated when the overall optical path difference of the tandem interferometer is zero.

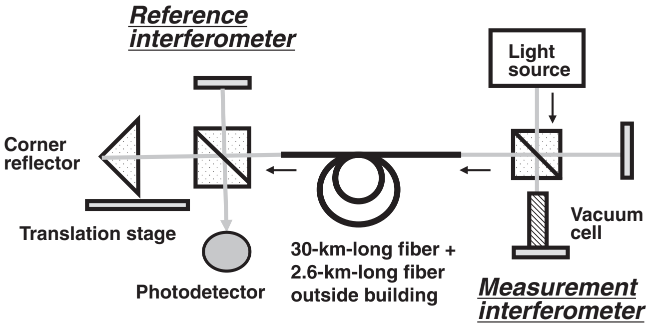

Fig. 1. Outline of experimental setup.

A light beam is introduced through the single-mode optical fiber from a near-infrared light source generated by amplified spontaneous emission (ASE) and is collimated to a diameter of about 5 mm, as shown in Fig. 2. In another experiment, a superluminescent diode (SLD) is also used. The collimated beam of 5 mmϕ is incident to the beam splitter of the measurement interferometer of the tandem interferometer, one surface of which has a half-reflecting metal coating on a plane parallel plate, whereas the other surface is reflecting and has a metal coating. The light beams from the beam splitter initially travel inside the vacuum cell [Fig. 2(a)]. Next, the vacuum cell is moved using an automatic translation stage, so that the two beams are inside and outside the cell [Fig. 2(b)]. The measurement is performed with the vacuum cell in two states to correct the offset path difference of the beam splitter. The output

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*Present address: Department of Precision Engineering, The University of Tokyo, 7-3-1 Hongo, Bunkyo-ku, Tokyo 113-8656, Japan.
E-mail address: hi.matsumoto@nanolab.t.u-tokyo.ac.jp

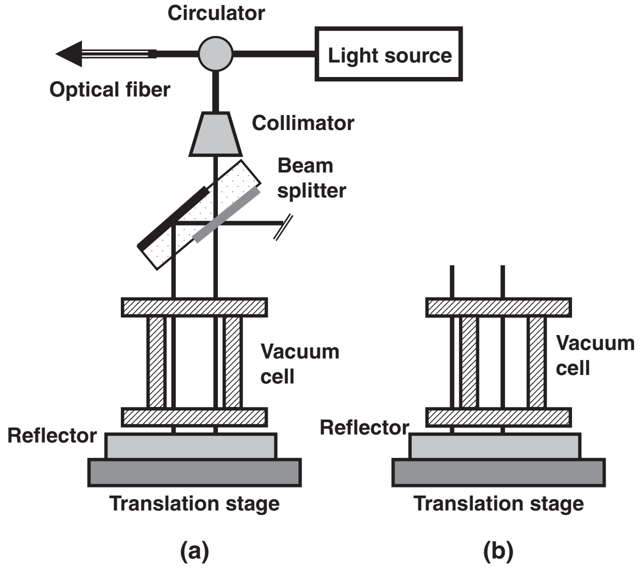

Fig. 2. Measurement interferometer of the tandem interferometer: (a) first measurements for obtaining offset of optical path difference and (b) second measurements for obtaining air refractive index.

beam from the measurement interferometer travels to the reference interferometer of the tandem interferometer through an optical-fiber circulator and the 30-km-long optical fiber. The optical fiber provides a common path for the beams; thus, the interference fringes are not affected by the fiber.

After passing through the optical fiber, the optical beam is collimated to a diameter of about 7 mm, as shown in Fig. 3. One of the reflectors of the reference interferometer is mounted on a piezoelectric transducer to scan an optical path longer than 40 µm, and then on a linear translation stage to change the optical path by the distance offset (about 13.3 mm) by the beam splitter of the measurement interferometer. Finally, the output beam is detected by an InGaAs photodiode (sensitive area: 0.3 mmϕ) after passing through a single-mode optical fiber.

First, the low-coherence interference fringes are generated when the optical path difference in the reference interferometer is zero, and then the interference fringes are generated when the optical path difference in the reference interferometer is equal to the offset of the beam splitter in the measurement interferometer, which is $2d_1$ in the case when the two beams travel inside the vacuum cell; and $2d_2$, in the case when the beams travel inside and outside the vacuum cell, where

$$2d_2 = 2d_1 + (n-1)L. \tag{1}$$

Here, n represents the refractive indices of air and L is the length of the cell, which is 99.98 ± 0.05 mm. Finally, we obtain the following equation for the group refractive index of air:

$$n = 1 + \frac{2(d_2 - d_1)}{L}. \tag{2}$$

Here, n is derived as the group refractive index by determining the peak intensity of the envelope of the low-coherence interference fringes. Moreover, the phase refractive index is determined by the center fringe of the low-coherence interference fringes. In this case, the group (n_g) and phase (n_p) refractive indices are related by the following equation at a wavelength of λ:

$$n_g = n_p - \lambda\left(\frac{dn_p}{d\lambda}\right). \tag{3}$$

Therefore, the group refractive index generally has a larger uncertainty than the phase refractive index in the calculation method. However, this vacuum method directly yields the group refractive index of air.

3. Experimental Methods and Results

Experiments were performed on different days using different optical fibers setups. The typical example of the interference fringe signals is shown in Fig. 4, and analysis results are shown in Table I. One experiment was performed in an experimental room using an optical fiber of about 30 km length. Another experiment was performed using an additional optical fiber of 2.6 km length outside the building. The first light source used was generated by ASE (THORLABS SOA240), whose center wavelength and full

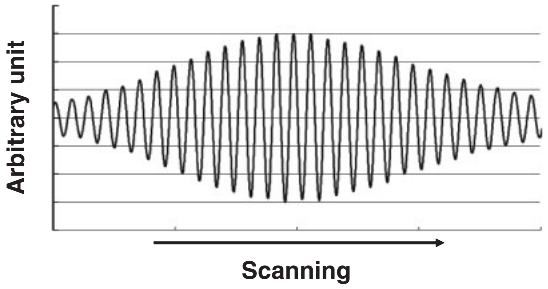

Fig. 4. Typical interference fringes obtained by experiment.

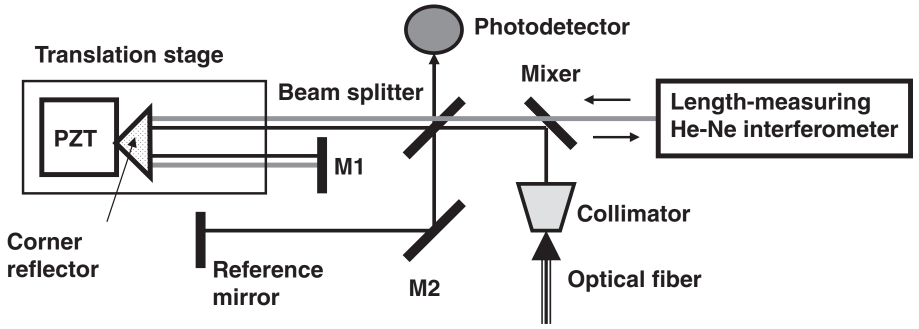

Fig. 3. Reference interferometer of developed tandem interferometer: PZT, piezoelectric transducer; M1 and M2, reflectors.

Table I. Experimental and calculated results and their differences for phase and group refractive indices of air for (a) optical fiber of 30 km length in experimental room and (b) optical fiber of 30 km length in experimental room and optical fiber of 2.6 km length outside building.

(a)

Light source	Day	Phase refractive index $(n-1) \times 10^6$			Group refractive index $(n-1) \times 10^6$		
		Vacuum cell method	Calculation method	Difference	Vacuum cell method	Calculation method	Difference
SLD	Mar.19	268.48	268.14	0.34	269.15	269.41	−0.26
($\lambda = 1550$ nm)	Mar.20a	268.38	268.51	−0.13	269.50	269.78	−0.28
	Mar.20b	268.77	268.70	0.07	270.55	269.97	0.57
	Mar.22a	268.88	268.92	−0.05	270.24	270.19	0.05
ASE	Mar.20	268.48	268.51	−0.03	270.35	269.86	0.48
($\lambda = 1500$ nm)	Mar.22b	268.78	268.80	−0.01	270.48	270.15	0.32
	Mar.22c	267.29	267.72	−0.04	268.33	269.08	−0.75

(b)

Light source	Day	Phase refractive index $(n-1) \times 10^6$			Group refractive index $(n-1) \times 10^6$		
		Vacuum cell method	Calculation method	Difference	Vacuum cell method	Calculation method	Difference
ASE	Mar.26a	266.59	266.56	0.03	268.16	267.91	0.25
($\lambda = 1500$ nm)	Mar.26b	266.37	266.47	−0.10	268.23	267.82	0.41
	Mar.27	266.32	266.45	−0.13	267.09	267.79	−0.70
	Mar.28	265.37	265.26	0.12	266.29	266.60	−0.31
	Mar.28	266.22	265.71	0.51	267.23	267.05	0.18

width at half maximum were 1500 and 100 nm, respectively. The output power was about 5 mW. The other light source used was an SLD (Amonics ASLD-CWDM-3-b-FA). The output power was about 8 mW and its full width at half maximum was about 200 nm with a central wavelength of 1550 nm.

The optical path difference in the reference interferometer was changed by moving the corner reflectors using a translation stage (Sigmatec FS3150) and a piezoelectric transducer (PZT; PI Polytec P-621.1CL). The signal-to-noise (S/N) ratios of the interference fringes were high; thus, the accurate positioning of the peak of the pattern of low-coherence interference fringes was achieved by the accurate positioning of the envelop pattern of interference fringes for the group refractive index of air. Moreover, the phase refractive index of air was obtained from the phase of the center interference fringe with a resolution of 5–10 nm. The displacements of mirrors (d_1 and d_2) were measured using an HP length-measuring interferometer (Agilent 5517B) with a resolution of 1 nm.

The air temperature was measured with an uncertainty of 0.06 °C using a temperature sensor (CUSTOM CT-500P), which was calibrated using higher-level temperature sensor. The air pressure was measured with an uncertainty of 10 Pa (catalog data) using a pressure sensor (GE Sensing DPI142). The humidity of the room was controlled within 10%. However, the distribution variation of the air temperature was 0.2 °C. Therefore, the refractive index of air under evaluation was calculated with a standard uncertainty of 0.22 ppm by the modified Edlén's equation with the sensors.[6)] On the other hand, the proposed method showed standard uncertainties of 0.20 and 0.29 ppm as follows: the uncertainties of the cell were 0.05 mm for its length and 11 nm for the thickness inhomogeneity of its window plates. The variations in repeated interferometric measurements were 15 nm in the phase measurement and 26 nm in the group measurement. The effect of the optical fiber used was negligible, because the fiber acted as a common path.

The experimental results for an optical fiber of 30 km length in the experimental room are shown in Table I(a). The results were obtained using the two different light sources for 4 days. The dispersion of the phase refractive index of air was smaller than that of the group refractive index. The dispersion of data by the SLD was slightly smaller than that by the ASE. The main source was the low-amplitude noise of the SLD, but its effect on the large spectral width was small, though the spectral distribution of the light source was not considered. The average of the experimental data was within 0.1 ppm of that obtained by the modified Edlén's equation. Since the expanded uncertainties (coverage factor $k = 2$, a level of confidence of approximately 95%) of the vacuum cell method proposed and the calculation method were 0.40 and 0.44 ppm, respectively, the combined standard uncertainty was 0.59 ppm. In Table I(a), although the maximum difference is 0.44, the ratio of 0.44 to 0.59 ppm is 0.75;[10)] thus, the obtained results are reasonable. Moreover, these conclusions are obtained for the group refractive index.

Measurements were then performed for 3 days using the 30-km-long optical fiber in the experimental room and the 2.6-km-long optical fiber outside the experimental room. The obtained results are shown in Table I(b). These results are similar to those in Table I(a). Therefore, the effect of the outside optical fiber is very small in this system.

4. Discussion

The measurement of the refractive index of air is required to improve the accuracy of length measurement by optical interferometry. For this purpose, Edlén's empirical equation was analytically improved by Ciddor. We call the latter equation the improved Edlén's equation, which is generally used to calculate the refractive index of air from the atmospheric conditions. However, these conditions are measured using an air temperature sensor, air pressure sensor and air humidity sensor, and their remote calibration traceable to the national reference standards is also required. On the other hand, the vacuum cell method is useful because only a length-measuring technique is required for obtaining the refractive index, although the vacuum pressure should be maintained at values less than about 40 Pa.

In low-coherence interferometry, the group or phase refractive index is utilized depending on the method of analyzing the interference fringes. If the center interference fringe is determined uniquely, its phase can be measured with high accuracy because the peak intensity of the interference fringe pattern is affected by the mechanical vibration and air turbulence in the measuring system, which generally have a smaller effect on the phase than on the intensity. The experimental results are consistent with this theory. The mechanical vibration and air turbulence have a larger effect on the remote measurement using an optical fiber in the field. In particular, such disturbances are very important in tandem interferometry, because such a technique has the signal-to-noise (S/N) ratio of lower than 1/8, although this is 8 times worse than that in general optical interferometry.

A more accurate measurement can be realized using a long vacuum cell. To obtain an uncertainty of 0.05 ppm, a 40-cm-long vacuum cell is reasonable when the phase measured for the interference fringes is about 2% (7.2° in phase).

5. Conclusions

We have measured the phase and group refractive indices of air at the wavelengths of 1.50 and 1.55 μm with standard uncertainties of 0.20 and 0.29 ppm, respectively, using a vacuum cell and a tandem interferometer with a 30-km-long optical fiber. This vacuum cell method developed is applicable to the remote calibration of practical length-measuring devices, such as gauge blocks and ring gauges, using optical-fiber networks. Moreover, this method is useful for the *in situ* measurement of the refractive index of air in optical length metrology.

Acknowledgement

This study was financially supported by the e-trace project of the New Energy and Industrial Technology Development Organization (NEDO).

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