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Reliability on calibration of CMM

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Abstract

This paper presents a method which is able to describe the reliability of the parametric error as the calibration result of a CMM. The reliability range may provide uncertainty indication of the calibration. Emphasis is placed on description of expansion of the propagation of error on the linear system expressing the parametric errors of a CMM, and confirmation of the method through the simulation on the calibration performed only by linear displacement measurements.

Keywords: CMM; Calibration; Parametric error; Reliability

1. Introduction

Since the numerical compensation technique was adopted on the coordinate measuring machine (CMM), the ratio of cost and performance has been remarkably improved. It is now recognized as one of the key technology for commercially available CMMs [1–3]. With expansion of the application field of CMMs, the importance of the calibration of CMMs is increasing. However, a smart way to quantify reliability of calibration measurement itself seems not to be established. Typical calibration measurement requires the following verification measurement which partly verifies the validity of the calibration by adopting an independent observation scheme.

The widely used ISO10360-2 standard [13] is applied to acceptance or re-verification of CMM. The standard aims to extract index of the performance from a variety of error sources of the CMM by performing directly traceable size measurement. We may see a partial silhouette of the complex error figure of the CMM through the test.

The authors propose a new approach to investigate statistically predicted reliability of the calibration result [9–12]. The approach applies and expands the propagation of error on a linear system to calibration of CMM. Adopting the approach, the reliability of the calibration activity in the respective parametric errors or even in any combination of them can be statistically predicted.

The integrated model is introduced firstly. The basic concept of the model and practical implementation is given. Next, a result from a simulation experiment is presented to confirm the functionality of the model. The brute force method [4,5] with the laser interferometer is chosen as a simulation example. The method typically consists of a combination

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of simple linear displacement measurements which contribute toward the corresponding parametric error in an indirect manner. Though the simple accumulation of variance contribution as former studies do hardly work in this case, it is shown that the proposed integrated model is able to predict the reliability range of the estimated parametric error.

2. Integrated model

The importance of estimating the uncertainty of geometric calibration activity is widely recognized. Considering the state of calibration of CMMs, only limited possibilities have been realized. The fact can be due to the complex structure, too many contributions to be included, and so on. Furthermore, it should be noted that uncertainty of the calibration result is affected by variance in raw observation and also both the measurement and evaluation strategy actually adopted. The fact enlarges the difficulty to analyze the uncertainty within a complex measurement system, such as a CMM.

The integrated model is proposed to overcome the difficulty. The schematic diagram is shown in Fig. 1. The functionality of the model partly resembles that described in former studies on the point of estimation of the parametric errors (e.g. Refs. [6,7]). The linear parameter model is built and is solved conventionally. A unique characteristic of the model is integration of the error propagation model which predicts the propagated reliability by being transferred the design matrix holding whole the attribute of the calibration measurement from the kinematic error model. Adding to variance within the raw observation data, contribution made by the measurement strategy practically adopted can be statistically evaluated too.

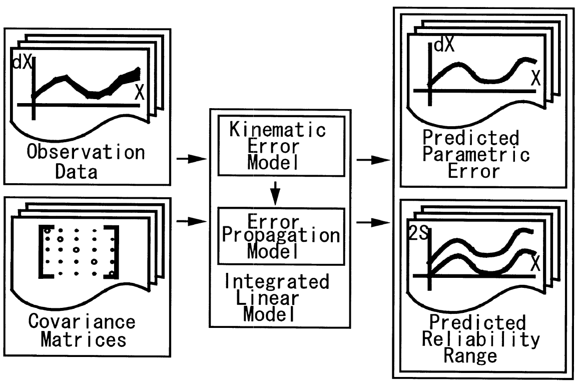

Fig. 1. Schematic of integrated model.

2.1. Propagation of error

Geometric calibration of CMMs is typically performed in the sequential manner. Each of the components to be calibrated is assessed, more or less, independently and serially in sequence. Quality indication of the calibration measurement can be known in fragments as the estimation residual on each of the calculation steps. However, the overall indication is not.

Building a set of the linear equations the parametric errors can be estimated by one step calculation [6]. Former studies adopting this approach focus on realization of the self-calibration method by the linear equations including unknown parameters for the standard to be referred to through the calibration [7,8].

With the characteristic explained as clear and evident process of the propagation of the error in the linear equations, the major benefit of the one step calculation is fully utilized.

2.2. Effect of measurement strategy

One dimensional displacement measurement has been a typical one within various techniques developed for CMM calibration [1]. Linear displacement measurement by the laser interferometer is an example. Conventionally, measurement strategy by the laser interferometer was designed in the component by component manner. A performed measurement corresponds to respective parametric errors. Calculation of the parametric errors can be done by the component by component manner, that is rather simple to implement.

The indirect method was considered mainly for automation possibility. The method was often called the brute force method [4]. Adopting it, the measurement strategy applied does not have to correspond to the respective parametric errors directly, but enough numbers of independent measurements are performed to keep full rank on the design matrix in case of the linear model solution. Since the measurement data are indirect to the parametric errors, adoption of the one step calculation is required. Simple accumulation of variance contributions seems hardly to work for knowing reliability of parametric errors.

The above one step calculation method holds characteristic of the measurement strategy and contribution of the raw observation error in the design matrix in case of the linear solution, and transfers them to the calibration result. Handing the design matrix from the kinematic error model to the error propagation model reliability of the calibration result can be statistically predicted.

2.3. Statistical information in observation

The integrated model expects statistical information from the observation data in the form of a covariance matrix formulated by variance and correlation information. Uncertainty of a raw observation is quantified in the form of standard deviation. Sufficient information for the integrated model is prepared if correlation information is known.

It is usual for calibration bodies on today to state raw uncertainty in calibration measurement under commercial situations. However the uncertainty is normally concluded under assumption of the worst permissible environmental condition. Actual uncertainty can be better than that of claimed in the quality documents, and therefore it is typically unknown. Simply believing the claimed uncertainty the integrated model provides over estimated reliability on the parametric errors as the calibration result. The situation is not a convenient one for the simulation experiment in this study. Furthermore correlation information of observation data is hardly available in practice although dependence between respective observation points can not be bypassed.

Numerical generation of simulated measurement data is adopted to avoid these problems. The method consists of the eigenvalues decomposition and the following re-composition based on the linear combination of the independent vectors built by the corresponding eigenvectors and the eigenvalues [12]. The schematic is shown in Fig. 2. Extended uncertainty with 95% probability of an observation data by the laser interferometer is typically formulated by an expression shown in Eq. (1).

$$S_{\mathrm{Meas}} = a + b \times l_{\mathrm{Observ}} \tag{1}$$

Where, a expresses the random term independent to the observation length l_{Observ}, and b does the length dependant term. Probability distribution of the error function is assumed to show the normal distribution. Variance information for the integrated model is directly derived from the extended uncertainty. On the other hand it is desirable to know reasonable values as correlation information in practical situa-

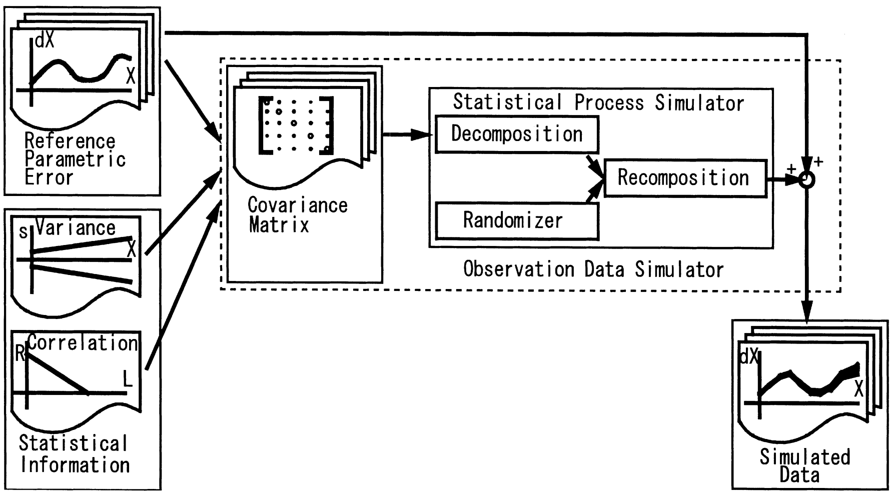

Fig. 2. Schematic of simulation data generation based on decomposition method.

tions. An assumed value is to be adopted in this report. Further consideration on this point is left to a succeeding study.

An observation data following assigned statistical process is numerically generated by algebraic operation. Where, variance and correlation represents a characteristic of the statistical process.

3. Modeling implementation

3.1. Linear model for geometric error

Geometric error of CMM can be described by a linear combination of so-called 21 parametric errors under rigid body kinematics [6,7]. Suppose a parametric error expressed by a linear combination of a basis function f and unknown parameter β, the ith parametric error E_i as a function of longitudinal location x can be written as Eq. (2).

$$E_{(i)(x)} = \sum_j f_{(i,j)(x)} \beta_{(i,j)} \tag{2}$$

Considering the kth linear guide way sub-system on CMM with six degrees of kinematic freedom form Eq. (3).

$$\begin{pmatrix} E_{(1)(x)} \\ E_{(2)(x)} \\ \vdots \\ E_{(6)(x)} \end{pmatrix} = E_{k(x)} = F_k B_k \tag{3}$$

Where, F_k is the basis function matrix and B_k is the unknown parameter vector, respectively, corresponding to the kth sub-system. Composing it in sequence, Eq. (4) yields whole parametric errors on CMM.

$$\begin{aligned} \begin{pmatrix} E_{1(x)} \\ E_{2(x)} \\ E_{3(x)} \end{pmatrix} &= \begin{pmatrix} F_{1(x)} & & 0 \\ & F_{2(x)} & \\ 0 & & F_{3(x)} \end{pmatrix} \begin{pmatrix} B_1 \\ B_2 \\ B_3 \end{pmatrix} \\ &= FB \end{aligned} \tag{4}$$

Introducing the response transform matrix Tr_{CMM} which indicates sensitivity of the parametric error appeared at the measuring spindle of CMM E_{Spindle}, Eq. (5) is given.

$$E_{\mathrm{Spindle}} = Tr_{\mathrm{CMM}} FB \tag{5}$$

Where,

$$Tr_{\mathrm{CMM}} = \begin{pmatrix} I_3 & R_{(P1)} & I_3 & R_{(P2)} & I_3 & R_{(P3)} \\ 0 & I_3 & 0 & I_3 & 0 & I_3 \end{pmatrix} \tag{6}$$

$$R_{(P)} = \begin{pmatrix} 0 & -P_z & P_y \\ P_z & 0 & -P_x \\ -P_y & P_x & 0 \end{pmatrix} \tag{7}$$

The above $P = (P_x \quad P_y \quad P_z)$ indicates effective arm length used to calculate the Abbe error observed at the measuring spindle and I_3 is a 3 by 3 unit matrix.

Unfortunately the error at the spindle E_{Spindle} is not always observable since practical measuring instruments applicable for CMM calibration typically have strictly defined measurand, such as one-dimensional displacement, for example. Introducing one more response transform matrix $Tr_{\mathrm{Measurand}}$ which indicates sensitivity and projection characteristic of the calibration instrument associated with the error at the spindle of the CMM, Eq. (8) expressing relation between the observable error and the parametric errors is finally obtained.

$$\begin{aligned} E_{\mathrm{Observed}} &= Tr_{\mathrm{Measurand}}\, E_{\mathrm{Spindle}} \\ &= Tr_{\mathrm{Measurand}}\, Tr_{\mathrm{CMM}}\, FB \end{aligned} \tag{8}$$

Suppose, Eq. (8) is solved by conventional linear algebra with proper observation data, the expected value of the unknown parameter $\hat{B}$ can be obtained as the result of the calibration.

3.2. Reliability of parametric error

Calibration of CMM is one of the important concerns to keep traceability to the national standard in length. However, quite limited possibility has been utilized to indicate the quality of the calibration activity on coordinate measurement. The attention is drawn to propose a smart method to quantify statistically predicted reliability of the calibration result of CMM.

Assuming the observation equation described as Eq. (9) with observation Y, combined Jacobian matrix G, and unknown parameter vector B, variance of estimated unknown parameter $\hat{B}$ is presented by Eq. (10).

$$Y = GB \tag{9}$$

$$S_{\hat{B}} = (G^T S_Y^{-1} G)^{-1} \tag{10}$$

Where S_Y is variance–covariance matrix of observation Y. Variance of the estimated observation $S_{\hat{Y}(x=x_0)}$ at observation location $x = x_0$ is given as Eq. (11) likewise.

$$S_{\hat{Y}(x=x_0)} = G_{(x=x_0)}\, S_{\hat{B}}\, G^T_{(x=x_0)} \tag{11}$$

However, this known propagation law of error in linear system does not make much sense on the calibration of CMM. What we wish to quantify is, for example, the reliability of straightness curve of X axis, but not variance of a single unknown parameter.

Aiming to quantify reliability of the estimated parametric error, the propagation law of error in the linear system is expanded.

In Eq. (8), the right side is split into two terms as one presents translations subscripted by T and the other does rotations done by R.

$$Y = \begin{pmatrix} G_T & 0 \\ 0 & G_R \end{pmatrix} \begin{pmatrix} B_T \\ B_R \end{pmatrix} \tag{12}$$

Where,

$$\begin{aligned} G_T &= (G_{Txx} \quad G_{Tyx} \cdots G_{Tzz}) \\ B_T &= (B^T_{Txx} \quad B^T_{Tyx} \cdots B^T_{Tzz})^T \\ G_R &= (G_{Rxx} \quad G_{Ryx} \cdots G_{Rzz}) \\ B_R &= (B^T_{Rxx} \quad B^T_{Ryx} \cdots B^T_{Rzz})^T \end{aligned} \tag{13}$$

Variance of estimated parameter $S_{\hat{B}}$ can then be described as Eq. (14).

$$S_{\hat{B}} = (G^T S_{\hat{Y}}^{-1} G)^{-1} = \begin{bmatrix} S_{\hat{B}TxxTxx} & S_{\hat{B}TxxTyx} & \cdots & & & \cdots & \\ S_{\hat{B}TyxTxx} & S_{\hat{B}TyxTyx} & & & & & \\ \vdots & & \ddots & & & \vdots & \\ & & & S_{\hat{B}TzzTzz} & & & \\ & & & & S_{\hat{B}RxxRxx} & & \\ \vdots & & & & & S_{\hat{B}RyxRyx} & \\ & & & & & & \ddots \\ & & \cdots & & & & S_{\hat{B}RzzRzz} \end{bmatrix} \tag{14}$$

Where, $S_{\hat{B}TxxTxx}$ expresses, for example, variance–covariance matrix of longitudinal position error along the X axis. With similar operation, variance of the estimated observation $S_{\hat{Y}\,(x=x_0)}$ at the observation location $x = x_0$ is described as Eq. (15).

$$S_{\hat{Y}(x=x_0)} = G_{(x=x_0)}\, S_{\hat{B}}\, G^T_{(x=x_0)} \tag{15}$$

Notifying the linear relation between variance–covariance matrix corresponding to translation error and rotation one on the right side of Eq. (15), the following is obtained.

$$S_{\hat{T}(x=x_0)} = (G_{T(x=x)_0}) \times \begin{pmatrix} S_{\hat{B}TxxTxx} & S_{\hat{B}TxxTyx} & \cdots & \\ S_{\hat{B}TyxTxx} & S_{\hat{B}TyxTyx} & & \vdots \\ \vdots & & \ddots & \\ & \cdots & & S_{\hat{B}TzzTzz} \end{pmatrix} \times (G_{T(x=x_0)})^T \tag{16}$$

$$S_{\hat{R}(x=x_0)} = (G_{R(x=x)_0}) \times \begin{pmatrix} S_{\hat{B}RxxRxx} & S_{\hat{B}RxxRyx} & \cdots & \\ S_{\hat{B}RyxRxx} & S_{\hat{B}RyxRyx} & & \vdots \\ \vdots & & \ddots & \\ & \cdots & & S_{\hat{B}RzzRzz} \end{pmatrix} \times (G_{R(x=x_0)})^T \tag{17}$$

In the same way as above, variance–covariance matrix of the estimated parametric error at the observation location $x = x_0$ can be extracted as follows.

$$S_{\hat{T}xx(x=x_0)} = G_{Txx(x=x_0)} S_{\hat{B}TxxTxx} G^T_{\hat{T}xx(x=x_0)} \tag{18}$$

$$S_{\hat{R}xx(x=x_0)} = G_{Rxx(x=x_0)} S_{\hat{B}RxxRxx} G_{\hat{R}xx(x=x_0)} \tag{19}$$

Eq. (18) shows expression of reliability of the longitudinal position error along the X axis. Eq. (19) does the roll error along the X axis, respectively, as example. Therefore, reliability of the parametric error curve as the calibration result is possible to predict by traversing the observation location through the axial stroke on the above equations.

Qualitative explanation of the integrated model is given in Fig. 3. The model consists of three major parts, the observation specific part, the CMM structure specific part, and the fit method specific part as recognized in Eq. (8) too. All three parts are linked linearly against the value to be handled as well as the dispersion to be propagated. The observable error on

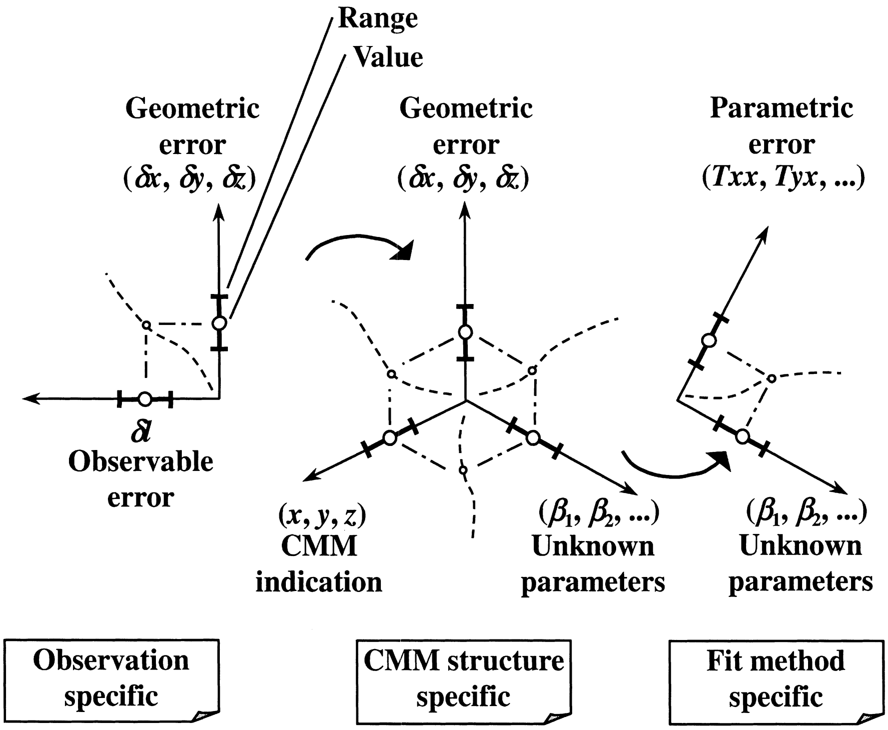

Fig. 3. Conceptual illustration of integrated model space.

the practical measurement as input is located on the left side. The target of the calibration, that is the parametric error of the CMM, is done on the right side. Knowing input information, i.e. observable error data and its reliability range, we may quantify the parametric error curve and the reliability range through geometric error with six degrees of freedom and the discrete unknown parameters.

4. Simulation

The indirect measurement method, often called as the brute force method, is adopted as a simulation example. Measurement strategy of the indirect method is so designed as it is able to derive all the parametric errors from a sufficient number of observation data. Major components are affected by a combination of the plural observation measurements. Direct relation between the observation and the calibration result is not visible. As far as the authors survey, no former study was performed to predict the statistically propagated variance especially on the indirect method.

4.1. Measurement strategy

A measurement strategy for the indirect method is designed to perform the simulation. A typical one is composed by a set of the one dimensional length measurement performed by the laser interferometer. Adding to the longitudinal linear position error components along the axes, straightness error and rotation error components are estimated algebraically. Reliability of the calibration result obtained by this method is not apparent since the error propagation process is indirect.

Xhang et al. reported a possibility of the calibration of a CMM by combining 22 linear displacement measurements in the volume [5]. The calculation method was in component by component manner. The calibration result was verified by the independent inspection measurement. Soons et al. performed a similar calibration with a combination

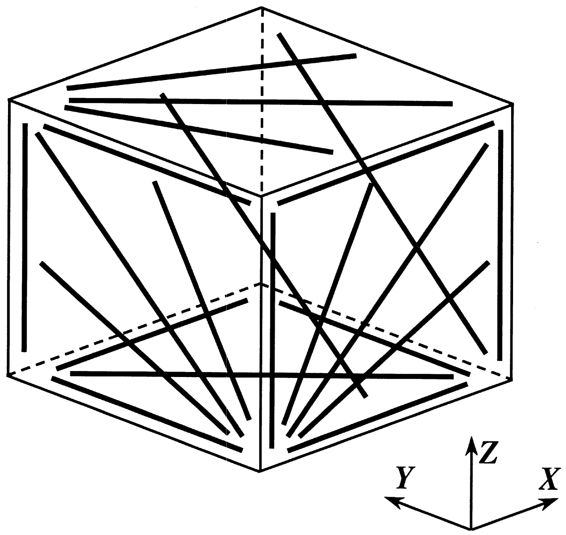

Fig. 4. Measurement strategy adopted for simulation, only by longitudinal displacement.

of length measurement [4]. Estimation was done by the one step calculation method. The reliability of the result was again indicated by the independent inspection measurement. Assessment of the calibration itself seems not to be performed until now.

Though the detail is not described here, a measurement strategy consisting of 21 linear displacements in the volume is designed empirically as shown in Fig. 4. The 21 lines are allocated in the volume as they are algebraically independent. Estimation of 21 parametric errors is possible by adopting the strategy instead of no extra redundancy.

4.2. Numerically generated observation data

The integrated model expects to know variance and correlation information of the observation measurement as mentioned. Introduced extended uncertainty for the simulation $2S$ is as shown in Eq. (19).

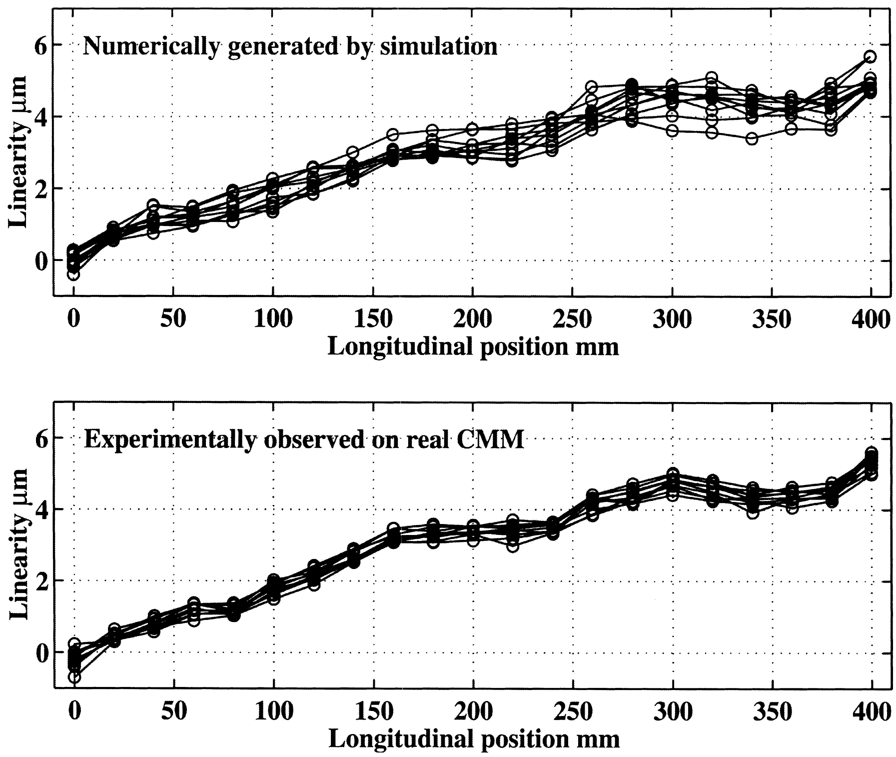

Fig. 5. Example of observation data, numerically generated and experimentally obtained.

$$2S[\mu\text{m}] = 0.2 + 0.7 \times l[\text{m}] \tag{19}$$

This value is adopted as a typical one when the conventional laser interferometer system from Agilent Technologies is applied to CMM calibration. There can be some variation under a practical situation depending on instrument type used and environmental condition assumed. The value in this paper can be an example. Although the next concern is determination of correlation information for the observation measurement, it is not available under the practical situation. Eq. (20) shows an assumed correlation. Future verification may be needed on it.

$$\begin{aligned} r &= 1 - 10 \times l[\text{m}] \quad \text{if } l \leq 0.1[\text{m}] \\ r &= 0 \qquad\qquad\quad\ \ \text{if } l \geq 0.1[\text{m}] \end{aligned} \tag{20}$$

The top plot in Fig. 5 shows an example of the data numerically generated by the decomposition method. The bottom one does an example of the observation measurement obtained by the laser interferometer experimentally according to the calibration procedure conforming to Eqs. (19) and (20). Both plots show linear position error with 10 times repetitive go and back operation along the Y axis. The general trend conforms each other qualitatively. However clear comparison seems to be difficult. This fact is a major reason why the data for the simulation are generated by numerical operation.

In total 21 sets of linear displacement measurement which is just enough to perform the estimation of the 21 sets of the parametric errors of the CMM is generated numerically.

4.3. Estimated result

The observation measurement is composed only by the linear displacement measurements. Reliability range of the estimated parametric errors is derived as to combine all the possible combination of the error contribution from the observation measurement and the measurement strategy.

Two examples are presented here. Fig. 6 shows the simulation result for X axis straightness deviated in the Y axis direction. Fig. 7 does a roll component

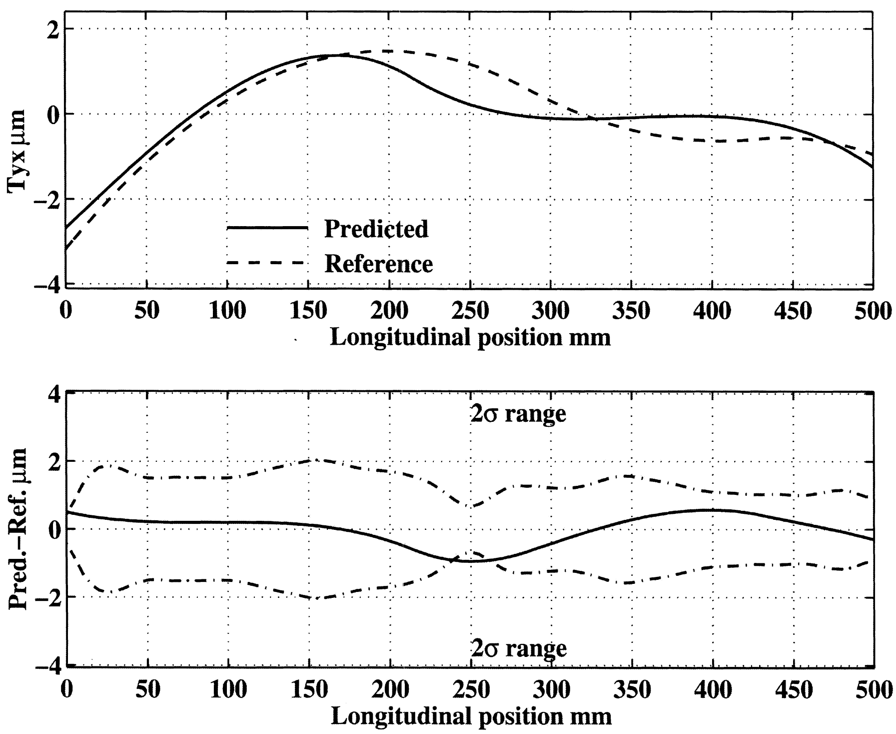

Fig. 6. Example of simulation result by the integrated model, straightness of the X axis.

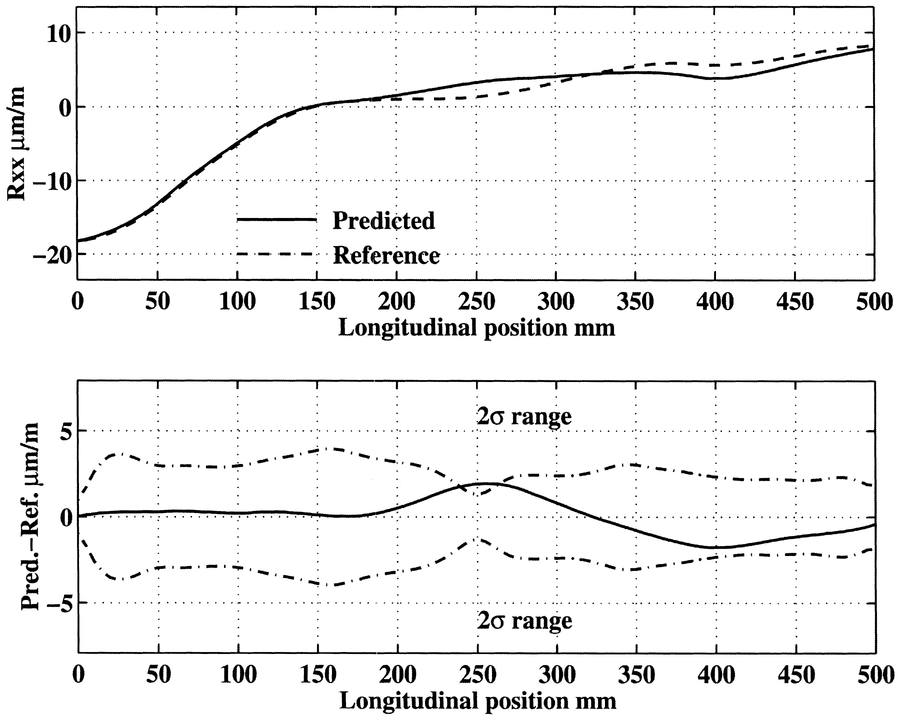

Fig. 7. Example of simulation result by integrated model, roll of the *X* axis.

of the *X* axis. Both components are impossible to detect directly by the linear displacement measurement itself. Performing enough number of the measurements, a combination of plural data with independent relation gives a sufficient response to estimate the parametric error and its reliability on the proposed integrated model.

In these figures, the top plot draws curves over the axis travel for the reference parametric error and the estimated one through the integrated model by a broken line and a solid one, respectively. The bottom plot indicates conformity between these two curves. The integrated model provides predicted reliability of the parametric error by variance. Here twice the predicted standard deviation is defined as the 2σ value and drawn on the bottom plot. Allowing 95% probability on the estimated parametric error curve it is expected for the estimated parametric error curve to lay within the range made by the 2σ value.

It is seen that the predicted reliability range seems to be able to express the practical reliability range of the calibration result. It is believed that this method may provide valuable information to establish the uncertainty indication on the geometric calibration of CMMs.

5. Conclusion

A new approach, adoption of the integrated model, is proposed to predict statistically calculated reliability of the parametric error. The approach deals with all the possible statistical combinations as far as the integrated model expresses. A simulation is performed to confirm the basic functionality of the integrated model. First, observation data for the measurement strategy composed only by 21 of the linear displacements are generated by algebraic emulation. The emulated data are prepared for the simulation as to follow the assumed statistical process expressing uncertainty in the calibration measurement under the practical situation.

Estimation of the parametric errors of a CMM is executed by applying the emulated data on the integrated model. The estimated parametric error shows deviation from the ideal one due to variance in the observation data and is affected by the adopted measurement strategy too. It is shown that the deviation within the predicted parametric error lays within the reliability range predicted by the integrated model, even if the measurement is performed only by 21 linear displacement measurements.

It is concluded that the proposed integrated model can be effective to establish reliability indication on the geometric calibration of CMMs in the practical situation. Since the prediction process is considered algebraically strict as far as the integrated model handles, prediction ability is not limited by the measurement strategy and instruments adopted.

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