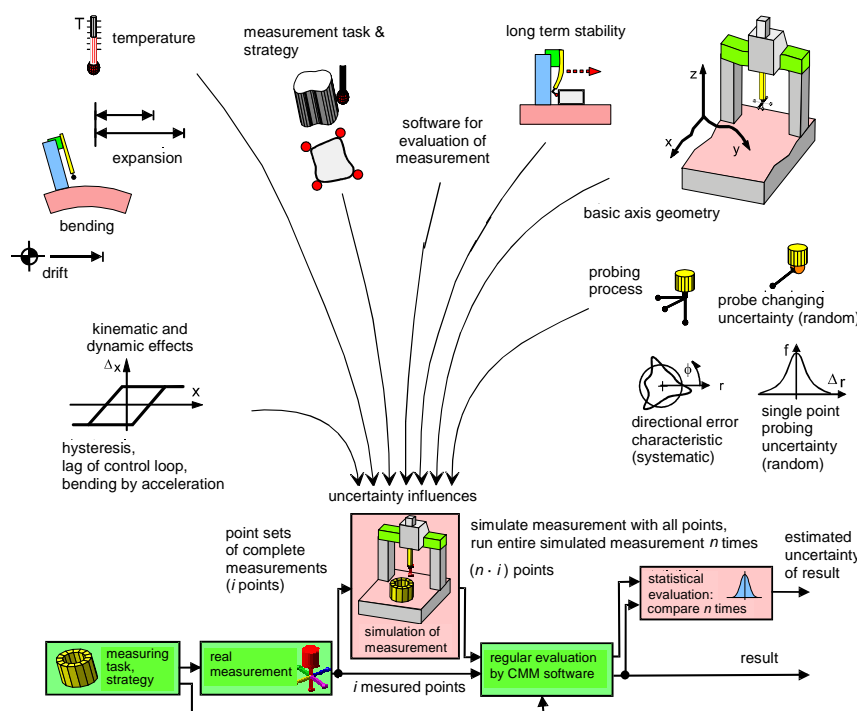


NEDO International Joint Research Project  
(FY 1999 - FY 2001)

# Final Research Report

## International Standard Development of Virtual CMM (Coordinate Measuring Machine)



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May 2002

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## 1. Preface

Recently, CMMs (Coordinate Measuring Machines) are widely used in the mechanical industry to measure three dimensional sizes, positions and forms of machine parts. The CMMs are indispensable instrument specially in the automobile industry for developing new automobiles, evaluation of the mechanical parts and molds, safety tests and environmental tests.

On the other hand, all instruments should be calibrated and traceable to the international standards for corresponding to ISO9000 series and ISO14000 series. However, there is no good calibration method for CMMs. It is mainly because the CMM has complicated constructions and the three-dimensional positions of many measured points have to be used in coordinate metrology.

In this research, the newly calibration method for CMMs and the international traceability system will be developed using the concept of the Virtual CMM. Then, the standard of the Virtual CMM method will be issued as the international standard in ISO/TC 213/WG 10 (Coordinate Measuring Machine). Furthermore, the international calibration system will be established.

In the Virtual CMM method, the geometrical model of the CMM is implemented in the computer system. Using this model, the errors of measurement and the uncertainty of measurements by the CMM are estimated by the Virtual CMM method.

The effects of dissemination such as CMMs diffusion, the international standard development in the measurement of machine parts and so on are expected.

In the NEDO VCMM Project, we have achieved the following results:

- The fundamental concept of VCMM method was established through the collaboration projects and three meetings of VCMM-team.
- VCMM method was widely disseminated to industry through three VCMM workshops and collaboration with many companies.
- The basic concept of ISO 15530 part 4 was decided and the new draft based on VCMM method was completed.
- The round robin measurements of the prototype hall-plate and new hall-plates for evaluating VCMM method were done.
- The round robin measurement of the practical workpiece started for evaluating VCMM method in practical situation.

## 2. Team Members



### ■ Research Coordinator

Kiyoshi Takamasu (UT: The University of Tokyo: Japan)

### ■ Accounting Coordinator

Ryoshu Furutani (TDU: Tokyo Denki University: Japan)

### ■ Research Team Members

Tomizo Kurosawa (NMIJ: National Metrology Institute of Japan: Japan)

Toshiyuki Takatsuji (NMIJ: National Metrology Institute of Japan: Japan)

Franz Wäldele (PTB: Physikalisch-Technische Bundesanstalt: Germany)

Heinrich Schwenke (PTB: Physikalisch-Technische Bundesanstalt: Germany)

Nicholas Brown (NML: CSIRO National Measurement Laboratory: Australia)

Esa Jaatinen (NML: CSIRO National Measurement Laboratory: Australia)

### 3. Summary of Research of NEDO VCMM Team

#### 3.1 Establishment of Virtual CMM Methods

##### Introduction

Importance of coordinate measuring machines (CMM) in the industry is increasing quickly. For example, as for the production system based on "Geometrical Product Specification (GPS)" to advance it with ISO/TC 213 as well, it is the technology which becomes a key as the only coordinate measuring machines to measure the geometrical specifications of the complicated machine parts.

On the other hand, as a result that the machine calibration technology such as the automobile industry becomes global, the way of calibration and evaluation of uncertainty are necessary as to make traceability. It aims at the international standardization of this field through the international joint research to cope with such flow.

##### Purpose of NEDO VCMM Team

1. The theoretical examination of the virtual CMM technique is done for the evaluation of uncertainty of the coordinate measuring machine.
2. It participates in the meeting of ISO/TC 213/WG 10, and work for the international standardization is done.
3. International comparison is carried out about the hall plate with three research organizations of German standard laboratory (PTB) and Australian standard laboratory (NML) and Japanese standard laboratory (NMIJ).
4. One dimensional ball plate is made as a new gauge to calibrate a coordinate measuring machine.
5. The foundation experiments of VCMM are done with PTB, NML and NMIJ in cooperation.

##### Activities Conditions

1. The round robin measurement of the prototype and two new type hall plate has been started after the round robin measurement of PTB hall plate in 1999-2001.

### NEDO VCMM-Team: 3. Summary of Research

2. Many members participated in the five conferences of ISO/TC 213/WG 10 in 1999-2002, and argument for the draft of the ISO 15530 part 4 was done. And, the preparation of the new draft was done as this result.
3. Three workshops of VCMM in Japan and Australia were held in 2000-2002.
4. The sub-group meeting related to the ISO 15530 part 4 of ISO/TC 213/WG 10 was held in Japan and Australia on 2000 and 2002 was held. There were a member of VCMM and participation of NIST (U.S. standard laboratory) and NIM (a standard Chinese laboratory) in the sub-group meeting.
5. Round robin measurement of the new practical workpiece has started to estimate the uncertainty of the workpiece on February, 2002.

### Results

1. The fundamental concept of ISO 15530 part 4 was decided and the new draft which was based on the concept was completed due to the activities of ISO to in 2002. It will be discussed at the conference of ISO/TC 213/WG 10 of Canada (Ottawa) in September, 2002.
2. The conclusion of each laboratory is coming out in the round robin measurement of the new type hall plate which has been done from 2001. As for this result, the benefit as expected does a detailed examination from now on.
3. The round robin measurement of the practical workpiece was started in 2002. This round robin measurement will continue after the project is ended.

### Consideration

1. The international standard for the uncertainty evaluation which is the purpose of this research team of the coordinate measuring machine developed very much. As for the technique of virtual CMM, it found that it was useful enough for uncertainty evaluation of the measurement.
2. Moreover, a result of calibration corresponded very well by the international comparison of the new type hall plate, and each other's calibration ability was confirmed.
3. The round robin measurement of the practical workpice is started, and important results can be expected.

#### Future Schedule

1. It confirmed that our collaborative researches such as the round robin measurements would continued.
2. The prospect when standardization in ISO will be done in 2003 was settled.

## 3.2 Activities of NEDO VCMM Team

We made the following activities of NEDO VCMM Team. Please refer each research report for specified project.

### 1. Kick-off Meeting at PTB on Nov 1999:

(Refer: 3.2.1 Draft Agenda and Resolution on Kick-off Meeting)

- All members joined the kick-off meeting.
- Discussion on targets of the project.
- Discussion on budget and schedule for 1999, 2000 and 2001.

### 2. International Comparison of a Prototype Hole Plate:

(Refer: 5.1 Comparison Measurement on Prototype Hole Plate)

- Measurements of the hole plate at PTB, NML and NMIJ on Dec 1999 to Feb 2000.

### 3. ISO/TC 213/WG 10 Meeting in USA on Jan 2000:

(Refer: 5.7 Activities on ISO/TC 213/WG 10)

- 4 members attended the meeting.
- Discussion on simulation methods for ISO 15530-4
- Start of ISO 15530-4 development.

### 4. NEDO-VCMM Workshop at UT on March 2000:

(Refer: 3.2.4 1st VCMM Workshop at the University of Tokyo)

- 4 members presented their works related to VCMM project.
- 40 attendances from Japanese Universities and Industries.

### 5. Collaborate experiments at NMIJ on March 2000:

(Refer: 5.2 VCMM Installation and Verification at NMIJ)

- 6 members joined the collaborate experiments.
- Install VCMM software.
- Basic experiments using VCMM software.

### 6. Discussion on Ball Plate Measurement at PTB on April 2000:

(Refer: 5.4 Discussion on Ball Plate Measurement at PTB)

- 3 members joined the collaborate experiments.

### 7. Collaborate experiments at NML on July 2000:

(Refer: 5.3 Calculating CMM Measurement Uncertainty with OzSim)

- Install VCMM software



### NEDO VCMM-Team: 3. Summary of Research

- Development and verification of OzSim
8. Annual meeting at NMIJ on 2000-8-28  
(Refer: 3.2.2 Draft Agenda and Resolution of Annual Meeting 2000)
- All members joined the annual meeting.
  - Discussion on budget and schedule for FY2000 and FY2001.
9. NEDO-VCMM 2nd Workshop at NMIJ on 2000-8-28:  
(Refer: 3.2.5 2nd VCMM Workshop at NMIJ)
- 6 members presented their works related to VCMM project.
  - 70 attendances from Japanese Universities and Industries.
10. Laboratories and Factories Visiting by NEDO-VCMM team on 2000-8-29 - 2000-8-31
- Visit to NMIJ laboratories
  - Visit to TSK factory
  - Visit to Mitutoyo factory
11. International Comparison of Two Hole Plates for NMIJ and NML:  
(Refer: 5.5 Collaboration Report on Two New Hole-Plate Measurement)
- Measurements of the hole plate at PTB and NRLM in FY2001.
  - This collaboration will continue in FY2002.
12. ISO/TC 213/WG 10 Meeting  
(Refer: 5.7 Activities on ISO/TC 213/WG 10)
- 2000-9-20 - 9-22 at Milan, Italy
  - 2001-1-15 - 1:17 at Bordeaux, France
  - 2001-9-19 - 9-21 at Vitoria, Spain
  - 2002-2-6 - 2-8 at Madrid, Spain
  - Some members attended the meetings.
  - Discussion on simulation methods for ISO 15530-4
13. Final Meeting at NML on 2002-2-25  
(Refer: 3.2.3 Draft Agenda and Resolution of Annual Meeting 2002)
- All members except Prof. Wäldele joined the annual meeting.
  - Discussion on the research report for FY2001 and final report.

14. NEDO-VCMM 3rd Workshop at Melbourne on 2002-2-26:

(Refer: 3.2.6 VCMM 3rd Workshop)

- 6 members presented their works related to VCMM project.
- 50 attendances from Australian Industries.

15. ISO/TC 213/WG 10, ISO 15530-4 Working Group Meeting at NML on 2002-3-1:

(Refer: 5.7 Activities on ISO/TC 213/WG 10)

- Discussion on new draft of ISO 15530-4

### 3.2.1 Draft Agenda and Resolution on Kick-off Meeting on 1999-11-24/25

NEDO-VCMM team  
Virtual Coordinate  
Measuring Machine

NEDO-VCMM N 1  
1999-11-24

Draft agenda  
for the kickoff meeting of  
NEDO-VCMM team  
1999-11-24/25  
Braunschweig, Germany

Date/Time	Course of events	
1999-11-24 09:00h-09:15h	1.	Opening of the meeting
	2.	Roll call of experts
	3.	Approval of the draft agenda (doc. NEDO-VCMM N 1)
	4.	Appointment of the resolutions editing committee
1999-11-24 09:15h-09:45h	5.	Status report of NEDO-VCMM team (doc. NEDO-VCMM N 2, N 3, N 4 and N )
1999-11-24 09:45h-11:00h	6.	Introduction of research of all members (doc. NEDO-VCMM N 5)
1999-11-24 11:00h-12:00h	7.	Targets of the project and the role of each member (doc. NEDO-VCMM N 6)
1999-11-24 13:00h-15:00h		
1999-11-24 15:00h-16:00h	8.	Budget (doc. NEDO-VCMM N 7)
1999-11-24 16:00h-17:30h	9.	Schedule for 1999, 2000 and 2001 (doc. NEDO-VCMM N 8)
1999-11-25 9:00h-12:00h	10.	Presentations on VCMM
1999-11-25 13:00h-14:00h	11.	Presentations on Online-VCMM
1999-11-25 14:00h-16:00h	12.	Visit to the laboratory related to VCMM
1999-11-25 16:00h-17:30h	13.	Any other business
	14.	Adoption of resolution
	15.	Closure of meeting

**NEDO-VCMM team**

Virtual Coordinate Measuring Machine

**NEDO-VCMM N 15**

1999-12-7

**Resolution in PTB meeting  
(Nov.24-25,1999)**

**Resolution 1-Status Report of NEDO Project(N2-5)**

Dr.Takamasu explained what the NEDO Project is and the purpose of this project.

After a little bit long discussion, it was adopted that the purpose of this project is the "Standardization of Simulation method to estimate the uncertainty budget of CMM"

**Resolution 2-Discussion of Budget(N12)**

Dr.Furutani explained the budget policy and how to use the budget.

The financial report should be sent to Dr.Furutani after March-31,2000, as soon as possible.

The financial plans for 3 fiscal years were shown.

The following items were changed;

1. The meeting site in a fiscal year 2000(FY2000) is changed from NML(Sydney) to NRLM(Tsukuba) and the meeting site in FY2001 from NRLM to NML. Therefore a conference fee 250,000JPY is transferd to NRLM in FY2000 and to NML in FY2001.
2. The materials and supplies expenses 500,000JPY had already been transferred to all members. After that, it was proved that all members could get 600,000JPY as the materials and supplies expenses in FY1999 plan. However, as it is much expensive and boring process to transfer a small mount of budget to abroad, it is adopted that 100,000JPY in FY1999 is not transferred to the members in PTB and NML and additional 100,000JPY in FY2000 is transferred to the members in PTB and NML .

**Resolution 3-Round test of a holeplate**

a holeplate of PTB will be send to NML then to NRLM. The schedule of this test is arranged by Dr.Schwenke. The measurement results of the holeplate will be collected and compared by PTB. This will be a major result in FY1999.

**Resolution 4-Workshop at NRLM**

A small workshop will be held at NRLM. Dr.Schwenke, Dr.Jaatinen and Dr.Takatsuji will join this workshop at least. This workshop will be held in the beginning of March. The exact schedule will be discussed by them.

**Resolution 5-Presentation on VCMM**

Dr.Wäldele reported the status of ISO/TC213/WG10 and the European project.

Dr.Schwenke explained the traceability, uncertainty and VCMM.

Dr.Hätig explained the implementation of the VCMM.

Mr.Busch explained the determination of geometric errors of CMM with 2D-artefacts eloquently.

Mr.Franke explained the interim check with 2D- and 3D-artefacts.

**Resolution 6-Concept for an international standard**

Dr.Schwenke proposed the concept for an international standard

- define the principles of simulation
- give minimum requirements for the software
- demand a checklist:which uncertainty contributor is considered
- demand transparency of the used model
- be open for all technical solutions on the market
- propose a method to check a simulation software.

### NEDO VCMM-Team: 3. Summary of Research

It was adopted that all members should send their opinion to Dr.Furutani([ryo@cck.dendai.ac.jp](mailto:ryo@cck.dendai.ac.jp)) until May-31,2000.

#### **Resolution 7-Next meeting**

The next meeting will be held at NRLM in the end of August,2000.

The exact schedule should be determined as soon as possible.

#### **Resolution 8-Closure of the meeting**

All members expressed their thanks to Dr.Wäldele and Dr.Shwenke for hosting the meeting and their excellent arrangements of city tour, dinners and hotels.

### 3.2.2 Draft Agenda and Resolution of Annual Meeting at NMIJ on 2000-8-28

NEDO-VCMM team  
Virtual Coordinate  
Measuring Machine

NEDO-VCMM N 16  
2000-8-28

Draft agenda  
for the annual meeting of  
NEDO-VCMM team  
2000-8-28  
Tsukuba, Japan

Date/Time	Course of events	
2000-8-28 09:15h-09:30h	1. 2. 3. 4.	Welcome Opening of the meeting Roll call of experts Approval of the draft agenda (doc. NEDO-VCMM N 16) Appointment of the resolutions editing committee
09:30h-10:00h	5.	Status report of NEDO-VCMM team (doc. NEDO-VCMM N 17) <ul style="list-style-type: none"><li>● Kick-off meeting: 1999-11-24,25 (doc. NEDO-VCMM N 9 and N 15)</li><li>● Workshop at UT: 2000-3-6 (doc. NEDO-VCMM N 26)</li><li>● Collaborate research at NRLM: 2000-3-7, 8 (doc. NEDO-VCMM N 18)</li><li>● Collaborate research at NML: 2000-7 (doc. NEDO-VCMM N 19)</li><li>● Research report for FY1999 (doc. NEDO-VCMM N 20)</li><li>● Other report form each member</li></ul>
10:00h-10:30h	6.	Schedule and budget for FY2000 (doc. NEDO-VCMM N 23) <ul style="list-style-type: none"><li>● Renewal application for FY2000 (doc. NEDO-VCMM N 21)</li><li>● Budget plan for FY2000 (doc. NEDO-VCMM N 22)</li></ul>
10:30h-11:00h	7.	Discussion on targets of ISO 15530-4 (also discuss on 2000-8-29)
11:00h-11:30h	8.	Schedule for FY 2001 (doc. NEDO-VCMM N 25)
11:30h-11:45h	9. 10. 11.	Any other business Adoption of resolution Closure of meeting

**NEDO-VCMM team**

Virtual Coordinate Measuring Machine

**NEDO-VCMM N28**

2000-9-18

**Resolution in NRLM meeting  
(Aut.28,2000)**

**Resolution 9-Status Report of NEDO Project(N17,N26,N18,N19,N20)**

Dr.Takamasu explained 1<sup>st</sup> Workshop at UT(N26).

Dr.Takatsuji explained the collaborate research at NRLM on March 7,2000 and at NML in July,2000(N28,N19).

The research report for FY1999 which was submitted to NEDO was delivered.(N20)

**Resolution 10-Round test of a holeplate**

Dr.Schwenke collects the data of round test of a holeplate in FY1999 and reports the result to EUROMET and/or APMP.

**Resolution 11-Discussion of Budget(N12)**

Dr.Furutani explained the renewal application for FY2000(N23) which had already been submitted to NEDO in last March. NEDO decided the amount of budget to this project was same to that of FY1999.

Dr.Furutani explained the budget plan for FY2000.(N22).

In this explanation,

each Japanese member get 1,200,000JPY.

each German and Australian member get 1,300,000JPY.

rest of the cost of VCMM is paid in FY2000. It costs about 1,360,000JPY.

The amount of 4,500,000JPY remains. after discussion what purpose the budget should be used for, as NML and NRLM member hope to get the hole plate respectively and its price was about 2,500,000JPY, we decided following options.

options	products	payment	rest of budget
1	two holeplates	in FY2000	0
2	two holeplates	separate in FY2000 and FY2001	2,000,000
3	two holeplates in FY2000 and FY2001	In FY2000 and FY2001	2,000,000

Dr.Schwenke will check the price of the holeplate and he kindly negotiates which option is possible. After his negotiation, we will decide which option we select.

In option 2 or 3, the rest of budget, which is about 2,000,000JPY, will be used for Dr.Schwenke's plan in FY2000.

In option 1, the amount of 2,000,000JPY will be used for Dr.Schwenke's plan in FY2001.

**Resolution 12-Questionnaire**

Dr.Takamasu informed that JSPMI(Japan Society for the Promotion of Machine Industry) has a plan to send a questionnaire about the uncertainty of CMM to 500 Japanese companies. It was approved that questionnaire would be sent to PTB and NML to put the additional questionnaire.

**Resolution 13-next meeting in FY2001**

The next meeting will be held at NML in March-2002.

The exact schedule should be determined as soon as possible.

**Resolution 14-Closure of the meeting**

All members expressed their thanks to Dr.Kurosawa and Dr.Takatsuji for hosting the meeting.



### 3.2.3 Draft Agenda and Resolution of Final Meeting at Melbourne on 2002-2-25

NEDO-VCMM team  
Virtual Coordinate  
Measuring Machine

NEDO-VCMM N 28  
2002-2-25

Draft agenda for the annual meeting of  
NEDO-VCMM team  
2002-2-25 Melbourne, Australia

Date/Time	Course of events
2002-2-25 09:15h-09:30h	1. Welcome 2. Opening of the meeting 3. Roll call of experts 3. Approval of the draft agenda (doc. NEDO-VCMM N 28)
09:30h-10:00h	4. Status report of NEDO-VCMM team (doc. NEDO-VCMM N 32) <ul style="list-style-type: none"><li>● Annual meeting: 2000-8-28 at NRLM, Tsukuba</li><li>● 2nd Workshop at NRLM, Tsukuba: 2000-8-28 (doc. NEDO-VCMM N 26)</li><li>● Collaborate research for the hole plate measurements started on FY2001</li><li>● Research report for FY2000 (doc. NEDO-VCMM N 29)</li><li>● Renewal application for FY2001 (doc. NEDO-VCMM N 30)</li><li>● Other report form each member</li></ul>
10:00h-10:30h	5. Budget for FY2001 (doc. NEDO-VCMM N 31) <ul style="list-style-type: none"><li>● Budget plan for FY2001</li></ul>
10:30h-12:00h	6. Discussion on final report format and contents (doc. NEDO-VCMM N34) <ul style="list-style-type: none"><li>● Research report for FY2001</li><li>● Final report of NEDO VCMM team</li></ul>
	Lunch
13:30h-15:00h	7. Discussion on future plans <ul style="list-style-type: none"><li>● Collaborate research on workpieces measurements (doc. NEDO-VCMM N 35)</li><li>● Collaborations after NEDO project</li><li>● Asia-Oceania collaboration on coordinate metrology</li></ul>
15:00h-15:30h	8. Discussion on targets of ISO 15530-4 (also discuss on 2002-3-1)
15:30h-16:30h	9. Any other business 10. Closure of meeting

**NEDO-VCMM team**

Virtual Coordinate Measuring Machine

**NEDO-VCMM N37**

25-Feb-2002

**Resolution in Melbourne meeting  
(25-Feb-2002)**

**Resolution 15-Status Report of NEDO Project(N26,29,30,32)**

The research report for FY2000 and the renewal application for FY2001 which were submitted to NEDO were circulated(N29,N30).

Dr.Takamasu reported 2nd workshop at NRLM,Tsukuba, at 28-Aug-2000(N26).

**Resolution 16-Discussion of Budget(N31)**

NEDO initially decided the amount of budget was 18,000,000JPY.

The amount of additional budget was 2,000,000JPY.

Dr.Furutani explained the budget plan for FY2001.(N31).

In this explanation,

all member got 1,400,000JPY.

rest of the cost of two hole-plates was paid in FY2001. It cost about 3,444,000JPY.

Dr.Takatsuji, Dr.Schwenke and Dr.Brown received the expenses for transferring the workpiece.

The amount of 2,000,000JPY remains. After discussion what purpose the budget should be used for, we decided following.

Dr.Schwenke will buy NT-version Quindos(2 licenses) for 5,000EURO.

The cost of transferring a new hole plate from NML to NMIJ is 2,000AUD.

The cost of air fare from Melbourne to Sydney for 8 persons is 1,864AUD.

**Resolution 17-Research report for FY2001 and Final report(N34,N35)**

Dr.Takamasu informed research report for FY2001.

- Research report(FORM 1) shall be sent to Dr.Furutani until 20-March-2002 by air mail.
- List of amount used for research and list of research result(Conference,Journal,academic awards) should be sent to Dr.Furutani until 20-March-2002 by e-mail.
- Evidence of amount used for research shall be sent to Dr.Furutani until 5-April-2002 by air mail.

Dr.Takamasu informed final report for three years NEDO projects.

- Each organization is requested to write summary of research and to send to Dr.Furutani until 30-April-2002 by e-mail.
- Collaboration report on
  - PTB's proto-type hole plate measurement by Dr.Schwenke.
  - VCMM software on March-2000 at NRLM by Dr.Schwenke.
  - VCMM software tests on July-2000 at NML by Dr.Jaatinen.
  - Discussion on Ball Plate Measurement on April-2000 at PTB by Dr.Takatsuji and Dr.Osawa.
  - two new hole plates measurement in FY2001 by Dr.Takatsuji and Dr.Osawa
  - workpiece measurements(just started) by Dr.Takatsuji and Dr.Osawa.
  - Activities on ISO/TC213/WG10 by Dr.Furutani.
- Above items shall be sent to Dr.Furutani until 30-April-2002 by e-mail. These reports shall be written in English.

### NEDO VCMM-Team: 3. Summary of Research

- Copies of Published reports by each member, Journal papers, proceedings, lectures and so on, is permitted in any language, and shall be sent to Dr.Furutani until 30-April-2002 by air mail.

#### **Resolution 18-Closure of the meeting**

All members expressed their thanks to Dr.Brown and Dr.Jaatinen for hosting the meeting.

It was a pity that prof.Waeldele was absent due to his backache. All members hope he will be recovered earlier.

### 3.2.4 1st VCMM Workshop at The University of Tokyo on 2000-3-6

NEDO バーチャル CMM チーム主催

#### 三次元測定機の測定の不確かさに関するワークショップ

NEDO 国際共同研究助成事業のひとつとして 1999 年度から 3 年計画で、バーチャル CMM チームを結成しました。バーチャル CMM チームでは、三次元測定機の測定の不確かさを計算機シミュレーションで評価する手法を研究し、その国際標準化を目指しています。

このたび、研究の一環としてバーチャル CMM チームのメンバーである、ドイツ標準研究所のシュベンケ博士およびオーストラリア標準研究所のヤーティネン博士をお迎えして、簡単なワークショップを計画しました。お二人は、30 歳前半と若い研究者であり、三次元測定機の校正および計算機シミュレーションを用いた三次元測定機の不確かさ推定の一人者です。

測定の不確かさは、全ての測定機において重要なテーマであり、国際規格としても非常に影響力が大きいことが予想されます。この機会に、今後の技術動向を考える意味で、ぜひ参加をお願いします。

また、ワークショップ後に大園・高増研究室の簡単な見学と、簡単な懇親会（参加費無料）を用意しました。こちらのほうもぜひご参加ください。事前の申込は不要ですので、当日会場へお越しください。（お二人の講演は英語で行いますが、質疑などは日本語の通訳を適宜行う予定です。）

なお、このシンポジウムの開催にあたっては、通商産業省工業技術院標準部 知的基盤課のご支援を受けています

日 時： 2000 年 3 月 6 日（月） 午後 2 時より

会 場： 東京大学本郷キャンパス 工学部 14 号館 1 階 142 講義室

#### プログラム

- |       |   |
|-------|---|
|       | 司会 東京電機大学 古谷涼秋                                    |
| 14:00 | 開会のあいさつ、VCMM チームの概要 東京大学工学系研究科 高増潔                |
| 14:30 | ドイツ PTB におけるバーチャル CMM の研究<br>ドイツ標準研究所 シュベンケ博士     |
| 15:30 | コーヒープレイク  |
| 15:50 | オーストラリア NML における CMM の研究<br>オーストラリア標準研究所 ヤーティネン博士 |
| 16:50 | 計量研における研究、ワークショップのまとめ<br>計量研究所力学部 高辻利之            |
| 17:20 | 大園・高増研究室の見学                                       |
| 18:00 | 懇親会（立食パーティ：参加費無料）                                 |

## 講 師 紹 介



Dr. Heinrich Schwenke

ドイツ標準研究所 (Physikalisch-Technische Bundesanstalt)

1995 年 TU Braunschweig で修士 "Design of a Spindelless Instrument for the Roundness Measuring of Ultraprecision Ball"

1999 年 PhD "Assessing Measurement Uncertainties by Simulation in Dimensional Metrology"

1995 年より PTB で CMM キャリブレーション, マイクロマシン用センサの開発, シミュレーション法による CMM の不確かさ解析の研究に従事 現在は Head of Coordinate Metrology Section .



Dr. Esa Jaatinen

オーストラリア標準研究所 (National Metrology Laboratory)

1990 年 University of Queensland を卒業

1994 年 Australian National University で PhD "Nonlinear Optics"

1994 年より NML にて波長標準の開発と研究に従事, 現在は CMM の理論的な研究および CMM の幾何誤差とボールプレートの干渉法による校正を担当 .

### 3.2.5 2nd VCMM Workshop at NMIJ on 2000-8-28

NEDO バーチャル CMM チーム主催

## 第 2 回 三次元測定機の測定の不確かさに関するワークショップ





NEDO 国際共同研究助成事業のひとつとして 1999 年度から 3 年計画で、バーチャル CMM チームを結成しました。バーチャル CMM チームでは、三次元測定機の測定の不確かさを計算機シミュレーションで評価する手法を研究し、その国際標準化を目指しています。

3 月には、研究の一環としてドイツ標準研究所のシュベンケ博士およびオーストラリア標準研究所のヤーティネン博士をお迎えして、第 1 回ワークショップを東京大学で開催しました。今回は、やはりバーチャル CMM チームのメンバーである、ドイツ標準研究所のベルデル博士、シュベンケ博士、オーストラリア標準研究所のブラウン博士および特別に米国標準研究所のシャカルジ博士をお迎えして、第 2 回ワークショップをつくば国際会議場「エポカルつくば」で開催いたします。

講演者は、すべて三次元測定機の校正および計算機シミュレーションを用いた三次元測定機の不確かさ推定の一人者です。計量研究所を含めた 4 つの主要標準研究所から三次元測定機関係の責任者が集まり、今後の国際規格の動向などの紹介と議論を行います。測定の不確かさは、全ての測定機において重要なテーマであり、国際規格としても非常に影響力が大きいことが予想されます。この機会に、今後の技術動向を考える意味で、ぜひ参加をお願いします。事前の申込は不要ですので、当日会場へお越しください。(講演は英語で行いますが、質疑などは日本語の通訳を適宜行う予定です。)

なお、このシンポジウムの開催にあたっては、通商産業省工業技術院標準部 知的基盤課のご支援を受けています。

### 講演者紹介

			
Dr. Franz Wäldele Head of Department “Measuring Instruments Technology” ドイツ標準研究所 (PTB) 長さ測定関係の責任者	Dr. Heinrich Schwenke Head of Coordinate Metrology Section ドイツ標準研究所 (PTB) 三次元測定関係の責任者	Dr. Nickolas Brown Leader, Length Standards Project オーストラリア標準研究 所 (NML) APMP の長さ関係の責任 者	Dr. Craig Shakarji Manufacturing System Integration Division 米国標準研究所 (NIST) ISO 15530-3 (シミュレー ション法) のグループリー ダー

NEDO バーチャル CMM チーム主催

## 第 2 回 三次元測定機の測定の不確かさに関するワークショップ

日 時： 2000 年 8 月 28 日（月） 午後 1 時 30 分より  
会 場： 茨城県つくば市 つくば国際会議場「エポカルつくば」 303 号室  
（高速バスつくばセンターバス停より徒歩 10 分  
交通の詳細は <http://www.epochal.or.jp/> 参照）  
連 絡 先： 計量研究所力学部 高辻利之  
（Tel: 0298-61-4041, Fax: 0298-61-4042, E-mail: [takat@nrlm.go.jp](mailto:takat@nrlm.go.jp)）  
参 加 費： 無料

### プログラム

司会 東京電機大学 古谷涼秋

13:30	歓迎のあいさつ	計量研究所所長 今井秀孝
13:35	ワークショップの趣旨，NEDO VCMM チームの紹介	東京大学 高増潔
13:45	計量研究所の研究紹介	計量研究所 高辻利之
14:05	アジア太平洋計量計画（APMP）の活動	オーストラリア標準研究所 ブラウン博士
14:25	ドイツにおけるバーチャル CMM の状況	ドイツ標準研究所 ベルデル博士
14:55	コーヒーブレイク	
15:15	米国におけるシミュレーション法の状況	米国標準研究所 シャカルジ博士
15:45	ISO 標準化の状況	ドイツ標準研究所 シュベンケ博士
16:05	議論（軽食および歓談）	

### 3.2.6 3rd VCMM Workshop at Melbourne on 2002-2-26

#### **PROGRAM FOR CMM WORKSHOP – MELBOURNE: 26 FEBRUARY 2002**

***2:00 pm – Introduction:***

**Prof. Kiyoshi Takamasu**, The University of Tokyo  
“The NEDO-VCMM project – an explanation”

***2:10 pm***

**Dr Heinrich Schwenke**, PTB Germany  
“Research and Development in Coordinate Metrology at PTB -  
Innovation and Collaboration with Industry”  
  
“The VCMM - Bringing Traceability to Industry”

***2:50 pm***

**Prof. Ryoshu Furutani**, Tokyo Denki University  
“A simulation software of uncertainty of CMM based on Matlab”

***3:10 pm***

**Dr Toshiyuki Takatsuji**, National Metrology Institute of Japan (NMIJ)  
“e-measure project for co-ordinate measurement metrology- remote calibration  
system based on information technology”

***3:30 pm Afternoon tea break***

***4:00 pm***

**Dr Makoto Abbe**, Mitutoyo Japan  
“Recent progress in coordinate measurement at Mitutoyo”

***4:20 pm***

**Prof. Kiyoshi Takamasu**, The University of Tokyo  
“Development of novel CMM (Nano-CMM and Parallel-CMM)”

***4:40 pm***

**Dr Nick Brown**, NML Sydney  
“The NML team and what we can provide for Industry”

***5:00 pm Drinks & further discussion..***



## 4. Research Report by Members

### 4.1 Research Report by the University of Tokyo

Kiyoshi Takamasu, The University of Tokyo

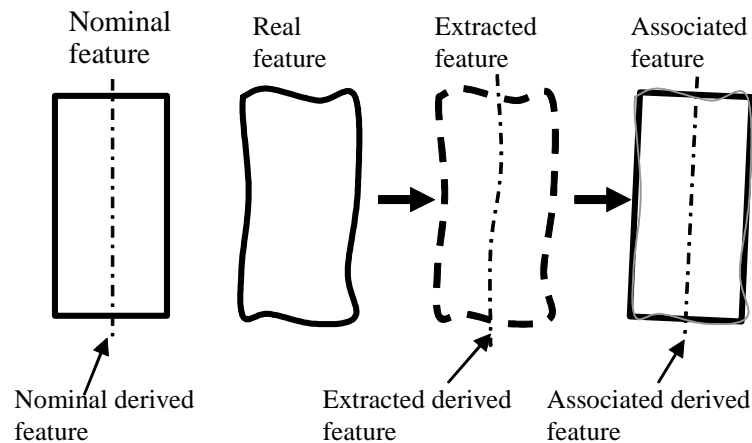
During the NEDO VCMM project (time from 1999-2002), the theoretically study and research for Virtual CMM method has been done at the University of Tokyo. We established the basic theories for estimating the uncertainty of measurement in coordinate metrology. The main work items have been:

- Establishment of concept of feature-based metrology for estimating the uncertainty of measurements in coordinate metrology.
- Establishment of evaluation method for effect of unknown systematic errors

#### 1. Establishment of concept of feature-based metrology

##### 1.1 Introduction

In coordinate metrology, an associated feature is calculated from a measured data set on a real feature by CMM (Coordinate Measuring Machine). Then, the associated features are compared with the nominal features which are indicated on a drawing (see figure 1). In this data processing, the features are primal targets to calculate, to evaluate and to process. Consequently, this process is called as “Feature-Based Metrology”.



**Figure 1.** Data processing flow in feature based metrology

##### 1.2 Uncertainty of feature

The uncertainty of each measured point is defined by error analysis of CMM and probing system, and the results of profile measurement on each feature. From the uncertainty of measured point, the uncertainty of measured feature can be calculated statistically using following equations.

Equation (1) shows an observation equation, a regular equation and a least squares solution, where  $\mathbf{A}$  is Jacobian matrix,  $\mathbf{p}$  is a parameter vector and  $\mathbf{S}$  is an error matrix.

observation equation :  $\mathbf{d} = \mathbf{A}\mathbf{p}$

regular equation :  $\mathbf{A}'\mathbf{S}^{-1}\mathbf{A}\mathbf{p} = \mathbf{A}'\mathbf{S}^{-1}\mathbf{d}$  (1)

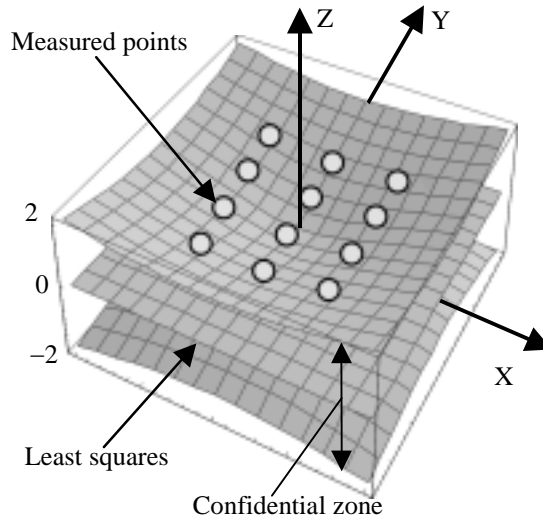
least squares solution :  $\mathbf{p} = (\mathbf{A}'\mathbf{S}^{-1}\mathbf{A})\mathbf{A}'\mathbf{S}^{-1}\mathbf{d}$

Using the propagation law of error to least squares method, the error matrix of parameter  $\mathbf{S}_p$ , and the error matrix of observation  $\mathbf{S}_m$  are calculated as equations (2) and (3) respectively. The matrices  $\mathbf{S}_p$  and  $\mathbf{S}_m$  indicate the variations of the parameters and the values of observation equations at each position.

$$\mathbf{S}_p = (\mathbf{A}'\mathbf{S}^{-1}\mathbf{A})^{-1} \quad (2)$$

$$\mathbf{S}_m = \mathbf{A}\mathbf{S}_p\mathbf{A}' \quad (3)$$

Figure 2 shows an example of error analysis form twelve measured points on a flat plane. Middle plane is least squares plane, the upper and the lower planes are the upper and the lower limits of confidential zone of feature respectively. We note that the upper and the lower limits of confidential zone is equal to the range of the uncertainty of measured feature. Using equation (3), the uncertainty at the position out of measuring range also can be calculated.



**Figure 2.** Confidential zone of measure plane; number of measured points is 12 and standard deviation of each measured point is 1.0.

### 1.3 Conclusions and future works

In this research, we have placed basic concept of feature based metrology which is used in coordinate metrology and constructed the data processing flow of it using CMM. From theoretical analysis, we reach the following conclusions:

1. least squares method is suited to calculate the geometric parameters and the uncertainty of features,
2. simple (low degree) model is fitted to the model of feature in the condition of feature based metrology,

3. calculation method of the uncertainty of feature is presented using least squares method and statistical evaluation,
4. calculation method of the uncertainty of related feature is also presented.

The future works in feature based metrology as follows:

1. how to define uncertainty of each measured point in CMM,
2. how to select the model of each feature; evaluating function of selection,
3. how to evaluate of the results of measurement; how to compare geometric parameters and tolerancing,
4. how to decide the strategy of measurement using feature based metrology.

## **2. Establishment of evaluation method for effect of unknown systematic errors**

### **2.1 Introduction**

In this research, the effects of systematic errors are theoretically analyzed to estimate the uncertainties in feature-based metrology. The center position error and the diameter error of the ball probe are taken up for the examples of the effects of systematic errors. These errors are occurred from the random errors of probing in calibration process and propagate as unknown systematic errors to the uncertainties of measured parameters such as the center position and the diameter of a measured circle.

Figure 3 shows the model for the theoretical analysis. Firstly, diameter and center position of a probe ball are calibrated by measuring a reference circle. The diameter of the reference circle is calibrated with the uncertainty  $s_c$ . After the calibration, a measured circle is measured by the ball probe with random measured error  $s_p$ .

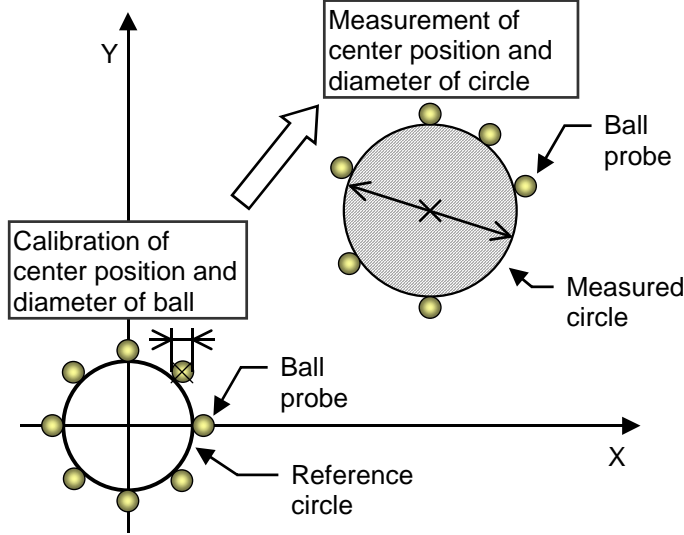
When the only random errors are put in the consideration and  $n$  measured points are probed uniformly on the measured circle, the uncertainties of measured diameter and center position of the measured circle can be calculated as equations (4), (5), (6) and (7) using equations (2) and (3). Where the position of probed point is displayed by  $t_i$  and  $r_i$  in figure 4.

$$\mathbf{P} = \begin{pmatrix} s_x^2 & s_{xy} & s_{xd} \\ s_{xy} & s_y^2 & s_{yd} \\ s_{xd} & s_{yd} & s_d^2 \end{pmatrix} = (\mathbf{A}'\mathbf{S}^{-1}\mathbf{A})^{-1} \quad (4)$$

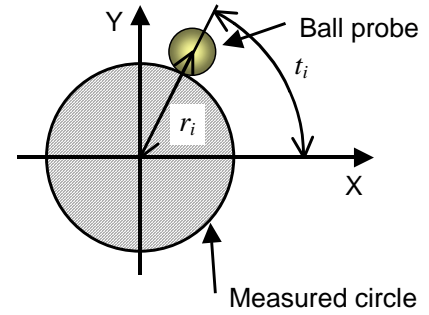
$$\mathbf{S} = \begin{pmatrix} s_p^2 & & 0 \\ & \ddots & \\ 0 & & s_p^2 \end{pmatrix} = s_p^2 \begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix} \quad (5)$$

$$\mathbf{A} = \begin{pmatrix} -\cos t_1 & -\sin t_1 & -\frac{1}{2} \\ \vdots & \vdots & \vdots \\ -\cos t_n & -\sin t_n & -\frac{1}{2} \end{pmatrix} \quad (6)$$

$$\mathbf{P} = \begin{pmatrix} \frac{2}{n}s_p^2 & 0 & 0 \\ 0 & \frac{2}{n}s_p^2 & 0 \\ 0 & 0 & \frac{4}{n}s_p^2 \end{pmatrix} \quad (7)$$



**Figure 3.** Model for calibration of ball probe and measurement of circle



**Figure 4.** Measured positions by angle  $t_i$

## 2.2 Unknown systematic errors

From the calibration process of the ball probe, the unknown systematic errors of diameter and center position of the ball probe are occurred. The effect of these unknown systematic errors is not same as the effect of the random errors.

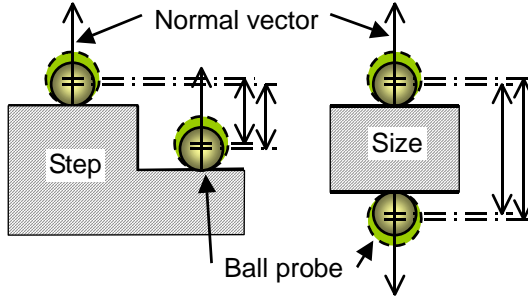
### 2.2.1 Unknown systematic errors from diameter of ball probe

Figure 5 displays the influences of uncertainty of the diameter of the ball probe for the step measurement and size measurement. The uncertainty of diameter effects the only size measurement. Figure 6 and equation (8) show the measuring errors  $dr_1$  and  $dr_2$  from diameter errors on the measured circle. The variance and the covariance from diameter errors are shown in equation (9), where  $c_d$  is diameter error. In this case the error matrix of the parameters is calculated as equation (10).

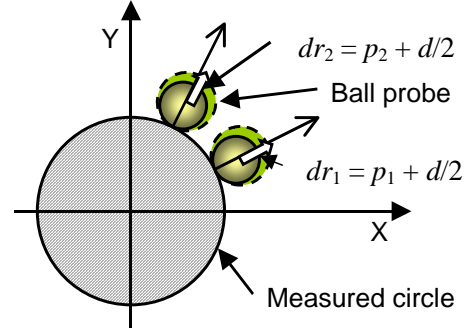
$$\begin{aligned} dr_1 &= p_1 + \frac{d}{2} \\ dr_2 &= p_2 + \frac{d}{2} \end{aligned} \quad (8)$$

$$\begin{aligned} s_1^2 &= s_2^2 = s_p^2 + \frac{c_d^2}{4} \\ s_{12}^2 &= \frac{c_d^2}{4} \end{aligned} \quad (9)$$

$$\mathbf{P} = \begin{pmatrix} \frac{2}{n} s_p^2 & 0 & 0 \\ 0 & \frac{2}{n} s_p^2 & 0 \\ 0 & 0 & \frac{4}{n} s_p^2 + c_d^2 \end{pmatrix} \quad (10)$$



**Figure 5.** Effect of diameter errors of ball probe on step dimension and size dimension



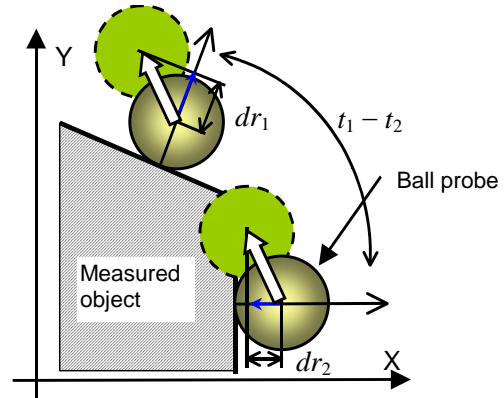
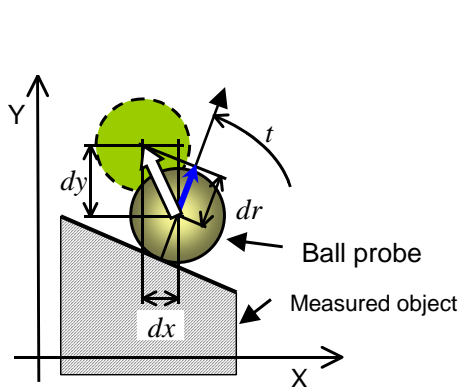
**Figure 6.** Correlation of two measured points by effects from diameter error

### 2.2.2 Unknown systematic errors from center position of ball probe

Figure 7 displays the influences of uncertainty of center position of the ball probe. Figure 8 and equation (11) show the measuring errors  $dr_1$  and  $dr_2$  from center position errors on the measured circle. The variance and the covariance from center position errors are shown in equation (12), where  $c_x$  is center position error.

$$\begin{aligned} dr_1 &= dx \cos t_1 + dy \sin t_1 \\ dr_2 &= dx \cos t_2 + dy \sin t_2 \end{aligned} \quad (11)$$

$$\begin{aligned} s_1^2 &= s_2^2 = c_x^2 \cos^2 t_1 + c_y^2 \sin^2 t_1 = c_x^2 \\ s_{12}^2 &= c_x^2 \cos t_1 \cos t_2 + c_y^2 \sin t_1 \sin t_2 \\ &= c_x^2 \cos(t_1 - t_2) \end{aligned} \quad (12)$$



**Figure 7.** Effect of center position errors of ball probe

**Figure 6.** Correlation of two measured points by center position errors

### 2.3 Conclusions

In this research, we theoretically analyzed the effects of the unknown systematic errors in feature-based metrology. The center position error and the diameter error of the ball probe are occurred from the random errors of probing in calibration process. These errors propagate as unknown systematic errors to the uncertainties of measured parameters such as the center position and the diameter of a measured circle. The method to calculate the error matrix was derived when the center position and the diameter of the circle are measured.

Using this method, the uncertainties of the measured parameters can also be calculated in the complex measuring strategy. The series of simulations for this method in statistical way directly implies that the concept and the basic data processing method in this paper are useful to the feature based metrology.

## 4.2 Research Report by Tokyo Denki University

Ryoshu FURUTANI, Tokyo Denki University

### 1 Introduction

Virtual CMM was a hot topics in a working group 10 in ISO/TC213 in 1998. At that moment, ISO/TC213/WG10 concentrated to finalize the document ISO 10360 series, which described verification and reverification test of CMM. In WG10, we discussed how to estimate the uncertainty of CMM measurement and determined the following document series.

15530-1: Terms

15530-2: Expert Judgment

15530-3: Substitution Method

15330-4: Simulation Method

15330-5: Historical Estimation

15530-6: Estimation using Uncalibrated objects.

In these documents, only 15530-3 and 15530-4 seemed to estimate the uncertainty of CMM measurement. This was our feeling. When the simulation method would be on the table in WG10, we would have to decide to support it or not. However nobody knew the details of simulation method. So we had selected the VCMM as one of possible simulation method and proposed the collaborated research work to PTB.

Now ISO/TC213/WG10 continues the process to issue the ISO 15530-4.

### 2 Purpose

Our purpose was

- 1) understanding the details of VCMM.
- 2) understanding How VCMM estimate the uncertainty of measurement.

### 3 Result

#### 3.1 Translation of document of VCMM

We translated the document “Traceability of Coordinate Measurements According to the Method of the Virtual Measuring Machine” to Japanese. In the process of translation, we discussed the technical details and we could understand the details of VCMM.

After understanding the details of VCMM, we had two new questions.

1) The estimated uncertainty by VCMM is adequate or not ?

2) It is difficult to build simulation software(VCMM) or not?

To make these questions clear, I had built VCMM software by myself.

### 3.2 Building VCMM Software

In Simulation software, it is most import to determine what type of the model can be handled. Generally, the model of CMM are following,

- 1) including geometrical errors
- 2) including dynamic behaviors
- 3) including temperature effects
- 4) including probing effects.

In our simulation method, we did not want to add any additional sensors to CMM. So we could not handle 3). The model 2) requires more the calibrated artifacts and information of errors. Finally we could handle only geometrical errors and probing effects.

In this simulation software, we had implemented two kind of probing effects, which are the random probing effects and the spherical harmonics probing effects. The traditional probing system of CMM has the spherical harmonics probing effects.

The geometrical errors were extracted by KALKOM, which is a software for extracting the geometrical errors from measurement result of the ball/hole plate. It can extract only the geometrical errors and can not extract any uncertainty of geometrical errors.

The simulation software was described in Matlab script language. The implementation of simulation software was not more difficult than understanding the details.

### 3.3 Execution of simulation software

The simulation software requires the definition of CMM, measuring program and measured points.

The definition of CMM are kinematical model of CMM, geometrical errors of CMM and probing effects as above section. The measuring program are dependent on the workpiece. The measured points are dependent on the

```
% Plane
100 100 100 0 0 -1
200 200 100 0 0 -1
0 200 100 0 0 -1
100 300 100 0 0 -1
% Circle
100 100 100 0 -1 0
200 200 100 1 0 0
```

Figure 1 Example of measured points



actual measurements.

In this implementation, the measured points, the measuring program are stored in the files.

```
cmm_circle(64,[[0,0,0];[0,0,1]])  
;
```

Figure 2 Example of measuring program

Figure 1 shows an example of measured points. Each line means one measured point. First three values mean x,y,z coordinate of measured points. Next three points mean the probing vector of measured points. This information is used to distinguish the inner circle and the outer circle and so on. The beginning of % means the comment line.

Figure 2 shows an example of measuring program. This line means measuring circle by

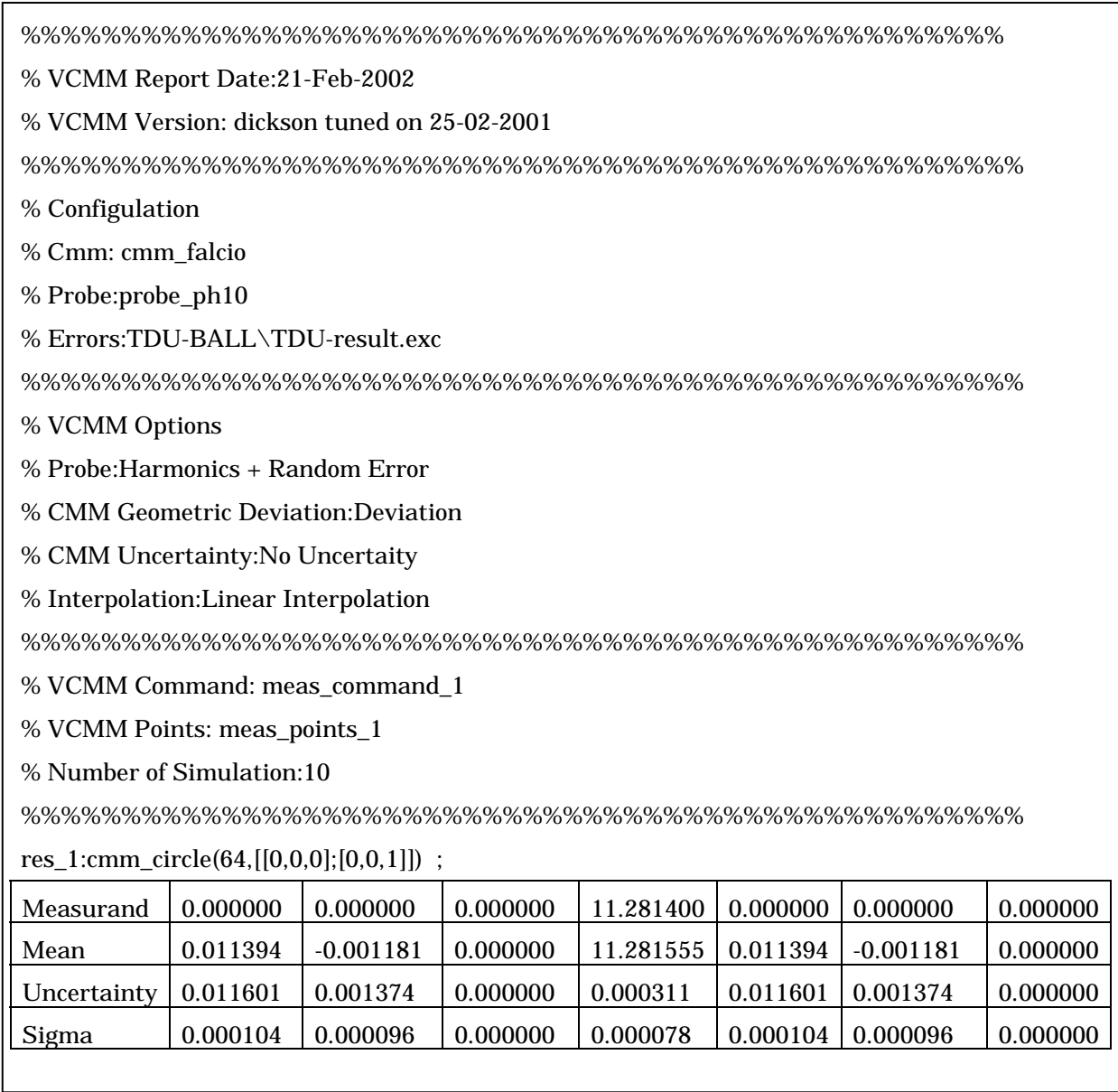


Figure 3 Example of output from simulation software

64 points and measuring result(circle) should be projected on the plane of which the normal vector is (0,0,1) and run through the origin.

Now the simulation software can measure the circle, line, plane, cylinder and sphere.

The measuring results are automatically stored special memories and can be easily used in Matlab script.

Figure 3 shows an example of output from simulation software. The output for circle measurement has four lines, which are measurand, mean of measurand, uncertainty of measurand and standard deviation of measurand. As the uncertainty includes the geometrical errors, it is larger than the standard deviation. The first three fields mean x,y,z coordinates in machine coordinate system, the 4th value is diameter, the last three fields mean x,y,z coordinates in workpiece coordinate system. In this example, the workpiece coordinate system fits the machine coordinate system.

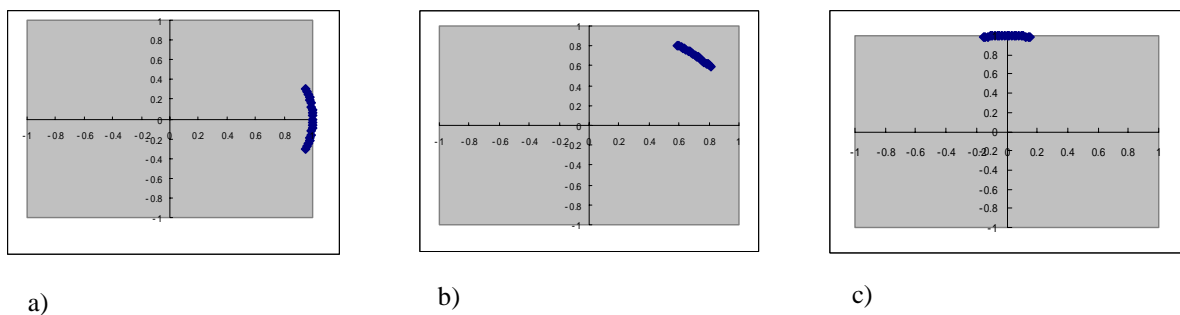


Figure 4 Measured points are located in a part of workpiece

### 3.4 Application of simulation software

Figure 4 shows three ultimate situation in measurement that measured points are located in a part of workpiece. The simulation software can estimate the measurement result and uncertainty of these measurements. The estimated result is shown in Table 1. The uncertainty of the direction where the measured points are located is smaller than the others.

## 4 Result

In the process to build the simulation software,

- 1) Simulation software looks well to estimate Uncertainty of CMM dependent on probing strategy.
- 2) The primary measured elements(eg. origin, primary axis etc.) should be carefully measured, because the measurement results in the workpiece coordinate system are severely affected by the result of primary elements. The geometrical errors look not so large effects on the measurement result in workpiece coordinate

system, because the output in workpiece coordinate system is calculated from the difference between the coordinates of two objects and the errors are localized.

Table 1 Estimated result when the measured points are located in a part of workpiece

		x	y	diameter
a)	measurand	0.783095	0	1.716762
	unc.	0.019993	0.004527	0.026785
b)	measurand	0.280714	-0.341425	2.333673
	unc.	0.075418	0.079618	0.101738
c)	measurand	0	-0.966708	3.027374
	unc.	0.018935	0.155764	0.153357

### 4.3 Research Report by National Metrology Institute of Japan

AIST, NMIJ, T. Kurosawa and T. Takatsuji

#### 1. Introduction

In order to assess the uncertainty of the coordinate measurement based on simulation methods, the characteristics of the target coordinate measuring machine (CMM) should be evaluated with an aid of a calibrated gauge in advance. The uncertainty of the calibration of the gauge contributes to the uncertainty of the coordinate measurement directly; hence it is important to calibrate the gauge accurately and with traceability.

Taking advantage of the fact that the AIST is a national metrology institute responsible for national standard, we were going to take part in this international project and contribute to the standardization of the simulation method mainly by calibrating the gauges.

Main research items are analysis of calibration uncertainty of a ball plate using gauge blocks, development and calibration of a ball step gauge, international comparison of gauges, international comparison of the measurement of workpieces, and so on. In the following sections, these research items will be explained.

#### 2. Analysis of calibration uncertainty of a ball plate using gauge blocks

The characteristic of the target CMM, which is used in the simulation method, is normally evaluated using a ball plate or a hole-plate. The ball plate is a steel plate in which steel or ceramic spheres are buried on grid positions as shown in figure 1, and the positions of the spheres should be calibrated accurately.

The ball plate is calibrated using a CMM. At the last step of the calibration procedure, the measurement values are compared with the measurement values of the gauge blocks and compensated to keep the traceability to the length standard.



Figure 1 Ball plate

We found that the probing error of the CMM is involved in the calibration

result in this compensation procedure; therefore we proposed a method of using multiple gauge blocks to avoid this problem. This method was theoretically investigated to show its validity, and its efficiency in the calibration of the ball plate was demonstrated. Detail of this method is explained in the references.

### 3. Development and calibration of a ball step gauge

Although the ball plate can be calibrated with the aid of gauge blocks as explained above, it is ideal to transfer the length standard from a subsidiary gauge, which has the same shape as the ball plate, to the ball plate.

Hence a ball step gauge was developed as shown in figure 2, which has one-dimensional shape of the ball plate. The ball step gauge has H-shaped cross-section and the spheres are aligned in its neutral plane. Owing to this structure the ball step gauge is rigid against deformation due to heat or mechanical stress.

Additionally the subsidiary gauge should be directly calibrated from the laser, which is the length standard. An interferometric stepper enables to do so; hence it is one of the ideal gauges as a subsidiary gauge used for the calibration of the ball plate. Detail of the ball step gauge is explained in the references.

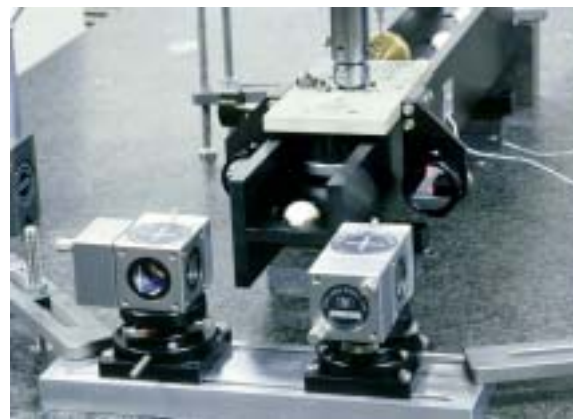


Figure 2 The ball step gauge and the interferometric stepper

### 4. International comparison of the calibration of the gauges

Only one way of checking the validity of the calibration of the gauges is an international comparison. Not only the AIST as a Japanese national metrology institute (NMI) but the PTB as a German NMI and the NML as an Australian NMI took part in this project. All of these laboratories are top-level NMIs in the world. The result of the international comparison carried out by these three NMIs is sufficient to prove the validity of the calibration of the gauges.

Three laboratories have their own hole-plates as shown in figure 3. These three hole-

plates were measured by three laboratories; i.e. each hole-plate was measured three times. The result of this comparison will be reported in sections c and g. Good results have been obtained in all measurements, and it indicates the capability of the calibration of the three laboratories. Additionally the uncertainty of the calibrated values of the hole-plates is small enough to use for characterizing the geometrical error of the CMM.

#### 5. International comparison of the measurement of the workpieces

Although the title of this project is VCMM, we have been researching on simulation methods for assessing the uncertainty of coordinate measurement. Many simulation methods including VCMM are currently available. In this project, Prof. Furutani and Dr. Jaatinen made their original simulation methods.

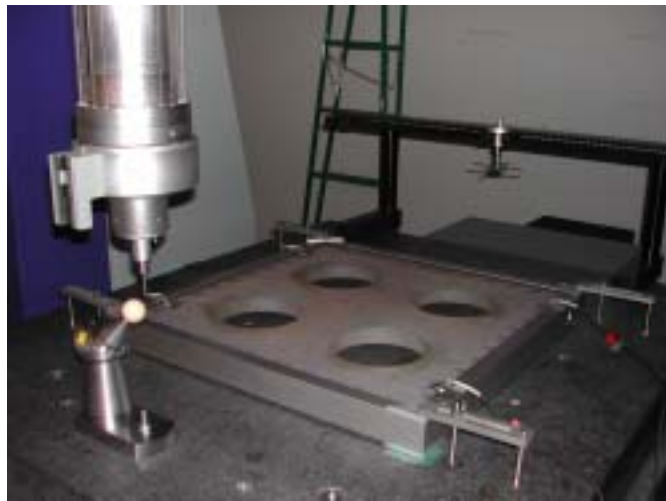


Figure 3 The hole-plate

We made two sample workpieces and planned an international comparison using the workpieces to observe the difference of various simulation methods. Details of the international comparison will be explained in section h.

Currently the sample workpieces are being circulated in participant laboratories.

#### 6. Other activities

On March 2000, a part of the members gathered in National Research Laboratory of Metrology (NRLM, former institute of AIST) in Japan, and performed a joint experiment on the characterization of the geometrical errors of the CMM and VCMM.

Dr. Takatsuji visited PTB on April 2000 and exchanged the information about the calibration of geometrical gauges.

Dr. Kurosawa organized the second project meeting held in Japan on July 2000.

Taking advantage of this opportunity, he also organized the workshop for Japanese industries to disseminate the simulation methods and advertise the project. Dr. Takatsuji played a role of an assistant.

Dr. Takatsuji has attended the ISO meeting several times and put an effort to standardize the simulation method.

Both Dr. Kurosawa and Dr. Takatsuji attend monthly project meeting held in Japan to have discussions and exchange information on the simulation methods.

Both of them attended many international and domestic conferences.

## 7. Conclusions

With the support of NEDO, the simulation methods have been more popular in these three years and its standardization is progressed. It is a matter of time to become a formal ISO standard. We would like to express our greatest gratitude to NEDO.

## 4.4 Research Report by Physikalisch-Technische Bundesanstalt

Franz Waldele, Heinrich Schwenke, PTB, Germany

During the time from 1999-2002 the development and dissemination of the Virtual CMM has been one of the key activities of the laboratory for CMM at PTB. In this time, we made significant progress both in the technical development and the proliferation to industry. The following report summarizes the main activities within and outside the NEDO project and lists the main achievements. The main work items have been:

### *Work items within the NEDO project:*

- Standardization of the VCMM
- Establishment of an International Network on CMM research
- Installation of the VCMM in Japan and Australia
- Comparison of Hole Plate calibrations
- Manufacturing of primary Hole Plate standards for NMIJ and CSIRO
- Dissemination of VCMM methods to the scientific and industrial community

### *Work items concerning the Virtual CMM outside the NEDO project:*

- Establishment of a calibration service for prismatic parts based on the VCMM
- Improvement of VCMM software routines
- Development of new software tools for the VCMM
- Preparation of Quality Manuals and procedures for the Virtual CMM

In the following these work items are shortly summarized.

### **Standardization of the VCMM**

During the project, a draft for an international standard for uncertainty calculation based on simulation has been prepared and made a priority work item for the ISO group TC213/WG10. The task force consists of two NEDO



members and a delegate from the US. In several productive meetings the general concept was developed and a comprehensive draft was produced and extensively discussed in ISO. Additionally, an national German guideline VDI 2617-7 was produced and already published as a draft.

### **Establishment of an International Network on CMM research**

In several meetings in Germany, Japan and Australia a close international network for CMM research has been established. Workshops, presentations and discussions not only resulted in an effective professional know-how transfer, but also in a very good personal relationship between the NEDO members, which will be a solid base for future collaborations. This network already produced important contacts on the field of laser trackers for CMM calibration and on internet based monitoring of CMMs.

### **Installation of the VCMM in Japan and Australia**

Calibration of the CMMs in NMIJ and CSIRO were performed and the VCMM was installed at both laboratories. First measurements have been performed which delivered promising results. The staff in both laboratories was trained in the calibration of CMM using ball plates, the operation of the Virtual CMM and in the scientific background.

### **Comparison of Hole Plate calibrations**

Comparison of Hole Plate calibrations have been performed between NMIJ, CSIRO and PTB. The results document the ability of all laboratories to perform calibration of Hole Plates within their stated uncertainties. During the project, NMIJ and CSIRO have optimized their calibration procedures to a very high degree of confidence. The results of the comparison were documented in a report.

### **Manufacturing of primary Hole Plate standards for NMIJ and CSIRO**

In collaboration of the NEDO members with the German company SCHOTT two state-of-the-art Hole Plate Standards have been manufactured. These Hole Plates have and unprecedented surface quality and long term stability. They will serve as primary standards for NMIJ and CSIRO.

### **Dissemination of VCMM methods to the scientific and industrial community**

During a number of seminars and workshops on three continents the VCMM method was disseminated to the scientific and industrial community. Many publications and conference contributions have been made concerning the VCMM. The publications are documented and supplied as an addendum to this final report.

### **Establishment of an calibration service for prismatic parts based on the VCMM**

One of the major achievements in the national context is the establishment of a calibration service in the German industry based on the VCMM. Eight private laboratories have calibrated their CMMs according to this method and have installed the VCMM to produce task related uncertainty statements. Not less than 10 000 measurements of different parameters on different objects employing different probing strategies have been performed to validate the method. The results confirm the feasibility and correctness of the approach. For the end of 2002, the accreditation of 6 laboratories within the DKD (German Calibration Service) is scheduled.

### **Improvement of VCMM software routines**

The simulation routines have been improved concerning completeness, handling, and transparency. Two new uncertainty contributors have been included and verified: Relative probe calibration uncertainty and surface roughness contribution. The simulation core now produces detailed debug information files. The interfaces to the manufacturers software have been extended. The manufacturers have improved the integration of the VCMM routines significantly. The operation is simplified.

### **Development of a new software tool for the VCMM**

A new software tool has been developed to improve the generation and the management of the VCMM input data, the so called "VCMM Tool". It can visualize the input data including all geometric uncertainty contributors and

helps the user to generate input data by specialized “assistants”. Furthermore, this tool can perform simulations of length measurements on a specific CMM to generate a global “accuracy parameter” of the machine. The new tool contributes to an increased transparency of the method and makes the generation of new uncertainty scenarios a lot easier.

### **Production of Quality Manuals and procedures for the Virtual CMM**

As an important prerequisite for the implementation of the virtual CMM in quality systems the procedures and quality manuals have been prepared and discussed with our industrial partners. These procedures are now compulsory for workpiece calibrations in PTB and have been a template for the documentation in the DKD laboratories.

## 4.5 Research Report by National Measurement Laboratory, CSIRO

Nicholas Brown, Esa Jaatinen, CSIRO, NML, Australia

### **Functions and responsibilities**

CSIRO's responsibility for maintaining the nation's primary physical standards is identified in two Acts of Parliament, the Science and Industry Research Act 1949 and the National Measurement Act 1960.

Under the Science and Industry Research Act 1949, CSIRO is required to establish, develop and maintain standards of measurement of physical quantities and, in relation to those standards: (i) to promote their use; (ii) to promote, and participate in, the development of calibration with respect to them; and (iii) to take any other action with respect to them that the Executive thinks fit.

Under the National Measurement Act 1960, the Organisation shall maintain, or cause to be maintained: (i) such standards of measurement as are necessary to provide means by which measurement of physical quantities for which there are Australian legal units of measurement may be made in terms of those units; and (ii) such standards of measurement (not being Australian primary standards of measurement) as it considered desirable to maintain as Australian secondary standards.

The Organisation's responsibilities are discharged through NML.

### **Current Research Activities**

1. Development of a Virtual Coordinate Measuring Machine with NMIJ (Japan) and PTB (Germany).
2. New Microwave frequency standard based on Laser-Cooled  $^{171}\text{Yb}^+$  Ions
3. New high-accuracy deadweight pressure standard
4. Development of a new current Quantized Hall Resistance

**Development of a Virtual Coordinate Measuring Machine/ new measurement techniques.**

Software has been developed at NML to determine CMM uncertainties using a different approach to the Virtual CMM software developed at PTB in Germany. This is called OzSim and uses a simpler approach to the VCMM software but can be applied to more complex situations. It has shown good agreement with the VCMM results.

New probing techniques are being investigated to compare mechanical probing with laser interferometry. This is important for traceability. The probe position is monitored with a laser interferometer as the probe approaches the contact surface and after contact. Elastic changes are observed and are being investigated to allow “zero force” probing.

A web page has been established to provide information on the VCMM work and can be found at (<http://www.metrologu.asn.au/cmmgroup.htm>)

**Recent publications:**

- "Temperature of Laser-Cooled  $^{171}\text{Yb}^+$  Ions and Application as a Microwave Frequency Standard" - R.B. Warrington, P.T.H. Fisk, M.J. Wouters and M.A. Lawn

**MSA Conference, Gold Coast, 2-4 October 2001**

- “A New Facility at NML for the Development of Reference Gas Mixtures as a Part of the Australian Metrology Infrastructure” – G de Groot, M Arnautovich, S Rennie and L Besley
- “International Metrology – Recent Developments in Mutual Recognition of National Metrology Institutes” – G Sandars
- “Optimizing CMM Measurement Processes by Calculating the Uncertainty” – E Jaatinen, R Yin, M Ghaffari and N Brown
- “Uncertainty Estimation for a Novel Pendulum” – R Cook and W Giardini

#### NEDO VCMM-Team: 4. Research Reports by Members

- “Static Torque Instrument Calibration” – I Bentley (NATA) and J Man
- “*Algorithm and Uncertainty of AC-DC Transfer Measurements*” – I Budovsky
- “*Development of a Cryogenic Comparator System at NML*” – B Pritchard
- “*Measurement of Temperature, Humidity and Pressure Coefficients of Zener-Based Voltage Standards*” – R Frenkel
- “*Automation of a Pressure Controlled Heatpipe for Use as a Thermometer Calibration Enclosure*” – M Ballico
- “*Calibration of Thermometers for Hygrometry in a Temperature-Controlled Chamber*” – K Chahine, N Bignell and E Morris
- “*A Miniature Copper Point Crucible for Thermocouple Calibration*” – M Ballico, F Jahan and S Meszaros
- “*The NML-Australian Definition of the Kelvin*” – M Ballico and K Nguyen
- “*The Piston Cylinder Pressure Gauge*” – W Giardini and B Triiwat
- “*A New Solid Density Standard with a Relative Uncertainty of 1 in 10<sup>7</sup>*” – K Fen, E Jaatinen, B Ward and M Kenny
- “*A Comparison of Roundness Measurements Between Australia and Indonesia*” – A Baker
- “*How Well Can We Provide Uncertainties for CMM Measurements?*” – R Yin, E Jaatinen, M Ghaffari and N Brown
- “*Uncertainty Analysis of the Chemical Composition of Reference Gas Mixtures Created by the Gravimetric Method*” – M Arnautovich, G de Groot and L Besley
- “*Facility at NML to Provide Traceability for Low-Frequency EMC Tests*” – G Hammond and I Budovsky
- “*The Measurement of RF-DC Difference 1 MHz to 100 MHz*” – S Grady
- “*Propagation of Correlations Down the Traceability Chain*” – J Gardner
- “*Thermal Effects in Small Sonic Nozzles*” – N Bignell, Y Choi (KRISS, Korea)
- “*Automation of Calibration in Hygrometry*” – K Chahine
- Bruce Warrington presented a paper entitled “*A Microwave Frequency Standard based on Laser-cooled <sup>171</sup>Yb<sup>+</sup> Ions*” at the 6<sup>th</sup> Symposium on

Frequency Standards and Metrology, St Andrews, Scotland, during the week 10-14 Sept.

- Yi Li presented two papers at the 12<sup>th</sup> International Symposium on High Voltage Engineering in Bangalore, India (20-24 August), entitled ,  
*"Frequency Composition of Lightning Impulses and K-Factor Filtering"* –
- Y Li and J Rungis, and *"The Contribution of Software to the Uncertainty of Calibrations of Calibration Pulse Generators"* – T McComb and Y Li.

### **Seminars / Workshops**

- Calculation of Measurement Uncertainty using the Monte Carlo Method:  
Seminar at PTB, Germany, 17-18 June 2001
- N. Brown "The NML team and what we can provide for industry", CMM traceability & the Virtual CMM- A workshop for industry held in Melbourne on 26 February 2002

## 5. Collaboration Reports

### 5.1 Comparison measurement on prototype Hole Plate

Heinrich Schwenke, PTB

#### Scope

PTB provided a calibrated prototype Zerodur Hole Plate as part of the NEDO project to allow both NRLM and NML to determine their CMM measuring capabilities. The plate was first sent to NML, where it was measured over a period of two weeks from the 28th of January 2000 to the 9th of February 2000 before being shipped to Japan, where it was measured during the following month.

The plate is 450 mm x 450 mm x 31.1 mm and its serial number is PTB 5.32 3/97. The last PTB calibration sticker is 3319. There are fifty-six holes in the plate and all of them have a nominal diameter of 30 mm. The holes are arranged in rows and columns that are parallel to defined 'U' and 'V' orthogonal axes on the plate. The spacing of the holes in the two outer rows and two outer columns is 40 mm but twice that (80 mm) for the holes in the inner rows and columns. No calibration values were supplied with the plate.



Photo Hole Plate



## Results

The measured coordinates of NML and NRML were send to PTB and a comparison of calibration values was performed at PTB. Figure 2 and 3 show the results.

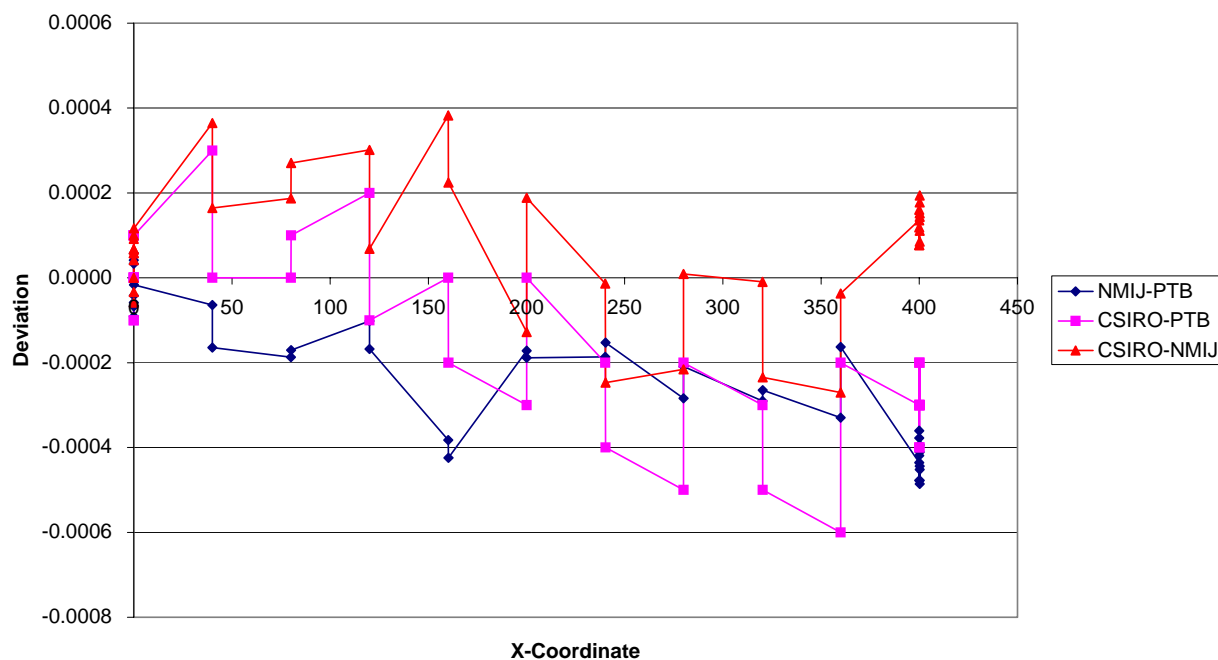


Figure 1: Direct comparison X-coordinate

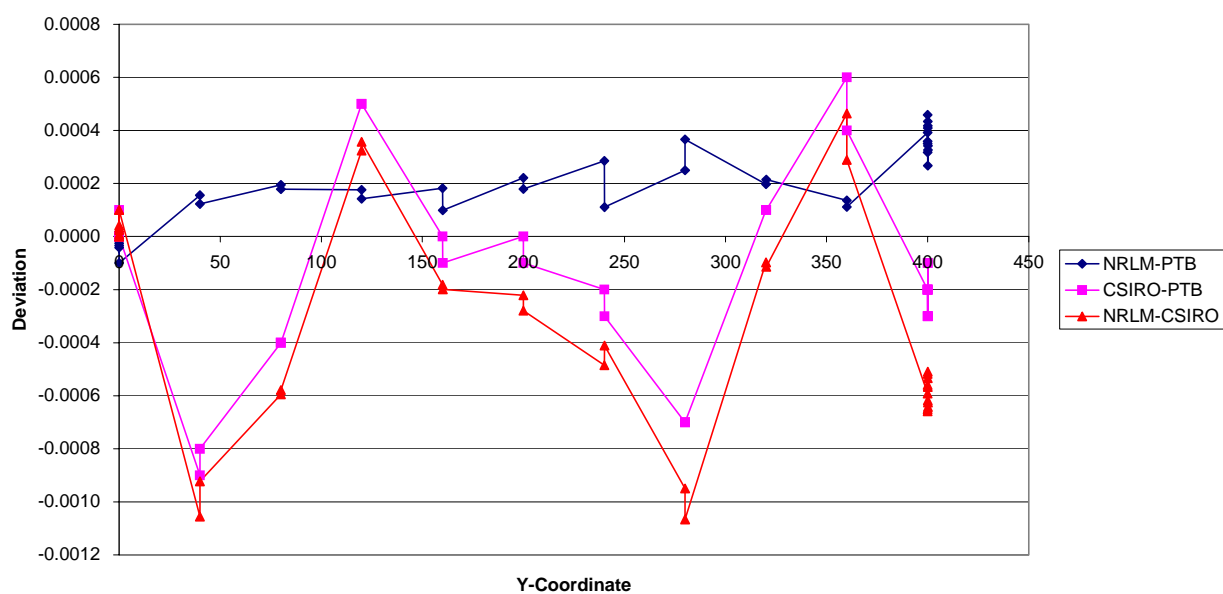
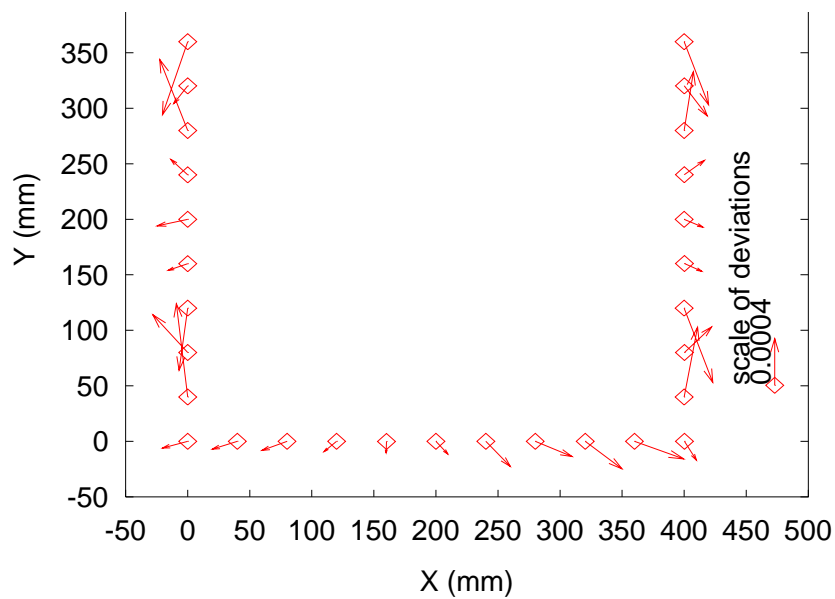
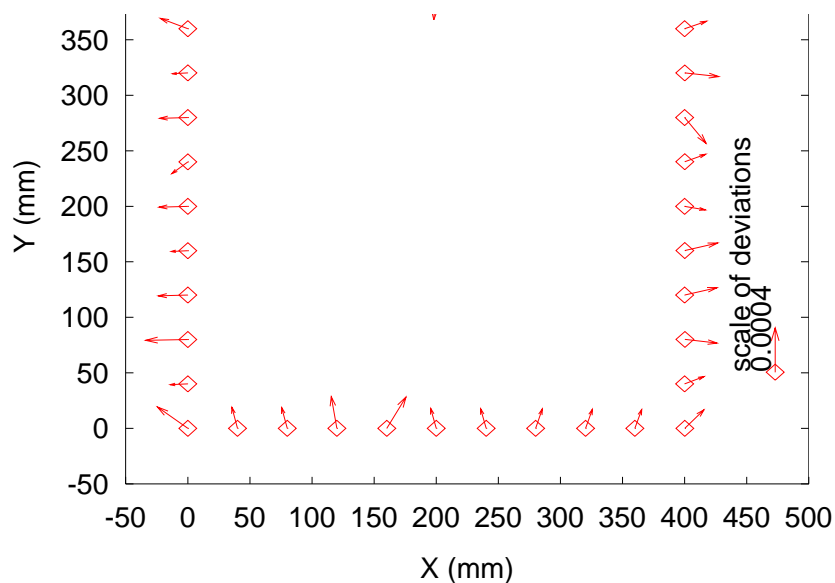


Figure 2: Direct comparison Y-coordinate



14. May. 2002 16:31 Programm TRANSFORM

Figure 3: Comparison CSIRO-PTB after best fit in orientation



15. May. 2002 13:16 Programm TRANSFORM

Figure 4: Comparison NMIJ-PTB after best fit in orientation

Comparing the data, the following observations have been made:

1. The difference between CSIRO and PTB in coordinates was within a range of  $1.2\ \mu\text{m}$
2. The difference between NMIJ and PTB in coordinates was within a range of  $0.5\ \mu\text{m}$
3. The difference between NMIJ and PTB partly is due to a scale error in the X and Y axis

To verify observation Nr. 3, a comparison between the NMIJ and PTB data was made with a scalefactor in X and Y separately optimized by a best fit algorithm. Figure 5 shows the residual errors. The maximum error after scale adjustment is  $0.22\ \mu\text{m}$  !

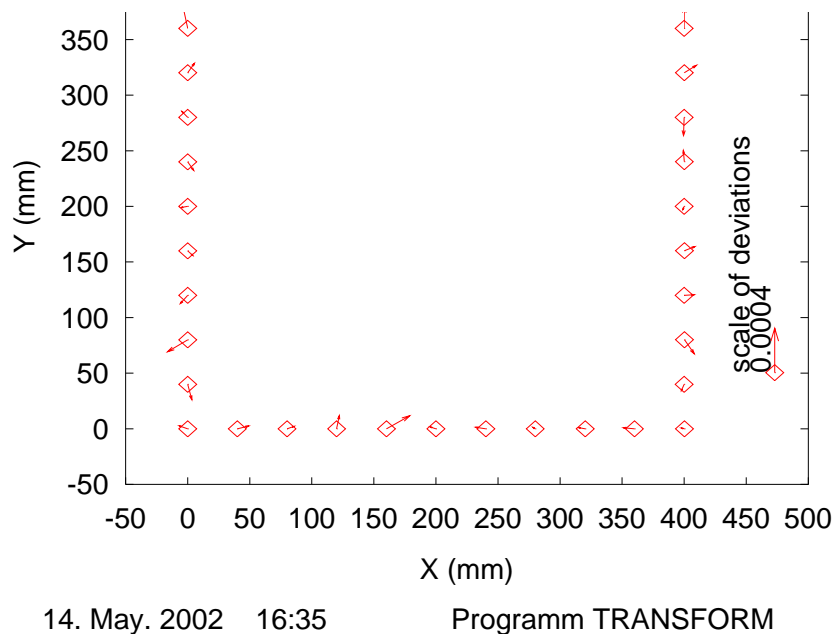
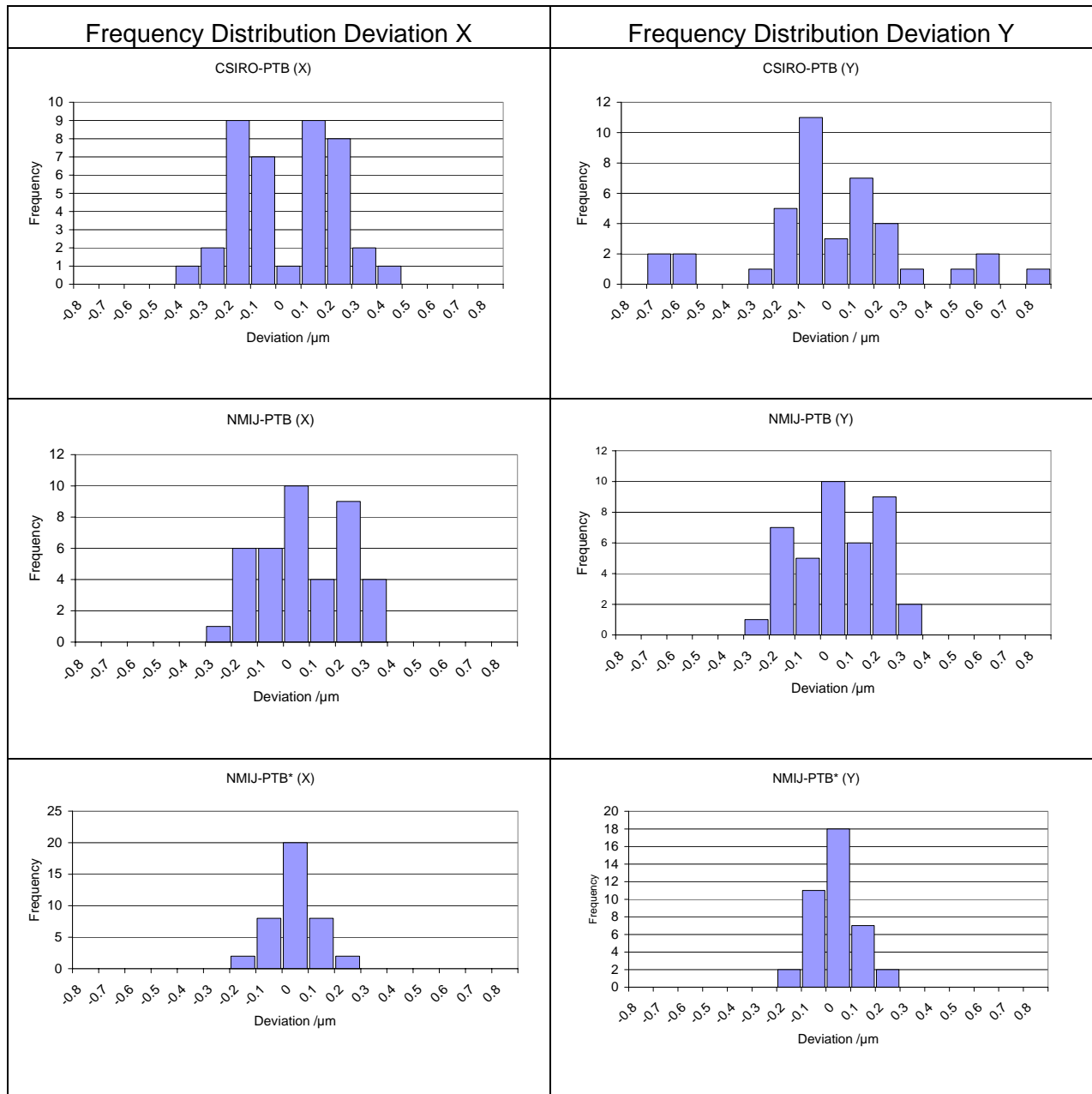


Figure 5: Comparison NMIJ-PTB after best fit in orientation and scalefaktor in X and Y

## Conclusion

The deviation observed are within the expected range. Especially the good agreement between NMIJ and PTB after adjustment of the scalefactors shows the general potential of Zerodur Hole Plates and the respective calibration procedure.

## Annex 1: Frequency distribution of observed deviations after best fit of data in orientation



The evaluation marked with \* was performed with a best fit optimization of the scalefactors in X and Y separately (see Figure 5).

## 5.2 VCMM Installation and Verification at NMIJ

Heinrich Schwenke, PTB

**Time:** March 6<sup>th</sup> –10<sup>th</sup> , 2000

### **Target**

Calibration of Coordinate Measuring Machine Leitz PMM 866 at NMIJ, installation and verification of the VCMM software on a Quindos VMS system.

### **Participating NEDO project members**

Dr. Kurosawa, NMIJ; Prof. Takamasu, Tokyo University; Prof. Furutani, Tokyo Denki University; Dr. Takatsuji, NMIJ; Dr. Osawa, NMIJ; Dr. Jaatinen, CSIRO; Dr. Schwenke, PTB.



Research team at NMIJ in March 2000

### **Guests**

Mr. Enomoto, NIDEK TOSOK Corp.; Dr. Abbe, Mitutoyo Corp.

### **Work items**

1. Calibration of CMM using a Hole Plate and the software KALKOM
2. Installation of VCMM Software
3. Comparison measurements on simple artifacts
4. Scientific exchange on calibration of CMM using lasertrackers

### **Calibration of CMM using a Hole Plate**

Using a PTB Zerodur Hole Plate the PMM 866 was fully calibrated. We determined all 21 kinematic parameters and generated an input file for the VCMM with the residual errors of the CMM. The residual errors that we measured are documented in Annex 1. While the observed linear errors can be contributed to the non-ideal temperature compensation, some rotational errors were found to be significant:

- Pitch of the X-axis (XRY): 8  $\mu$ rad
- Yaw of X-axis (XRZ): 5  $\mu$ rad
- Pitch of Y-axis: 6  $\mu$ rad
- Roll of z-axis: 8  $\mu$ rad

All other kinematic errors have been found to be well compensated.

Additionally, the environmental conditions were estimated by temperature data recorded by NMIJ. Probe tests have been performed to determine the appropriate base characteristic for simulation.

### **Installation of VCMM Software**

The installation of the VCMM software was performed in cooperation with Mr. Enomoto from NIDEK TOSOK Cooperation (Japan), who was trained for that task by the Brown&Sharpe Company in Wetzlar (Germany). After first simulations with synthetic test files have been successful, we generated specific input-files for the CMM at NMIJ.

### **Comparison measurements on simple artifacts**

After minor problems concerning the VCMM software installation had been solved, we performed tests on simple artifacts like gauge blocks and ring gauges. We varied measurement strategy and probe configuration to see the impact on the measurement uncertainty statement produced by the VCMM software. The result showed good agreements with the expectations. E.g. the uncertainty of diameter measurement increased dramatically, when decreasing the probed sector on the ring gauge. Annex 2 shows the results for the coordinates and the diameter of a ring gauge.

This verification showed the general functioning of the installed software, even if the operation of the software was very difficult at that time and many careful considerations had to be made by the operator. In this context it has to be mentioned, that the current version (Last update 20.4.2002) of Quindos XP using the Virtual CMM driver significantly improved the simplicity of operation.

The installation and the first test have been the prerequisite for further experiments concerning the VCMM at NMIJ.

## Scientific exchange on calibration of CMM using lasertrackers

Since NMIJ and PTB both work on using lasertrackers for CMM calibration, some interesting discussion concerning this matter took place. Both sides benefited from the ideas and solutions presented.

## Annex 1: Results of CMM calibration with Hole Plates (Residual geometric errors)

Titelpage (Secured)

First test of holeplate-method in japan  
corrected cmm at NRLM

CMM-Type	Serial Number	Customer	Date	Operator	Certification-No.
FIM 600	00	NRLM	8.2.2000	Toshi	

Constants			
Orbitality (mm)	40	Interpolation Spacing Position (mm)	20
Enduser Position (µm)	0.3	Interpolation Spacing Straightness (mm)	20
Enduser Straightness (µm)	0.3	Interpolation Spacing Rotation (mm)	20
Enduser Rotation (µrad)	0.3	Elasticity Factor for machines with horizontal arm	0

Probe Lengths					
Position \$11 (mm)	50	Position \$21 (mm)	-100	Position \$31 (mm)	-100
Position \$12 (mm)	50	Position \$22 (mm)	100	Position \$32 (mm)	100

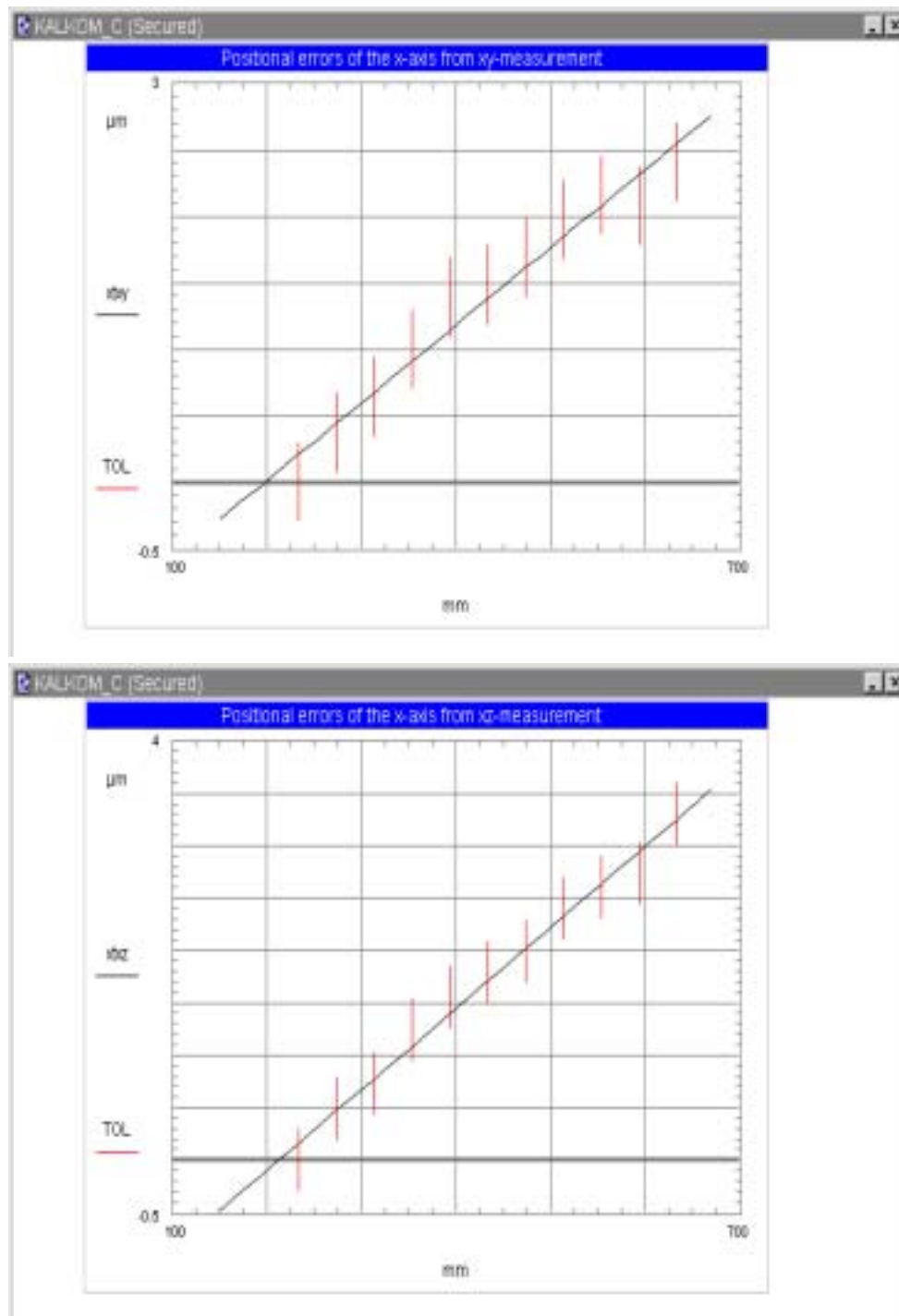
Shift of reference point in Z (mm)

0

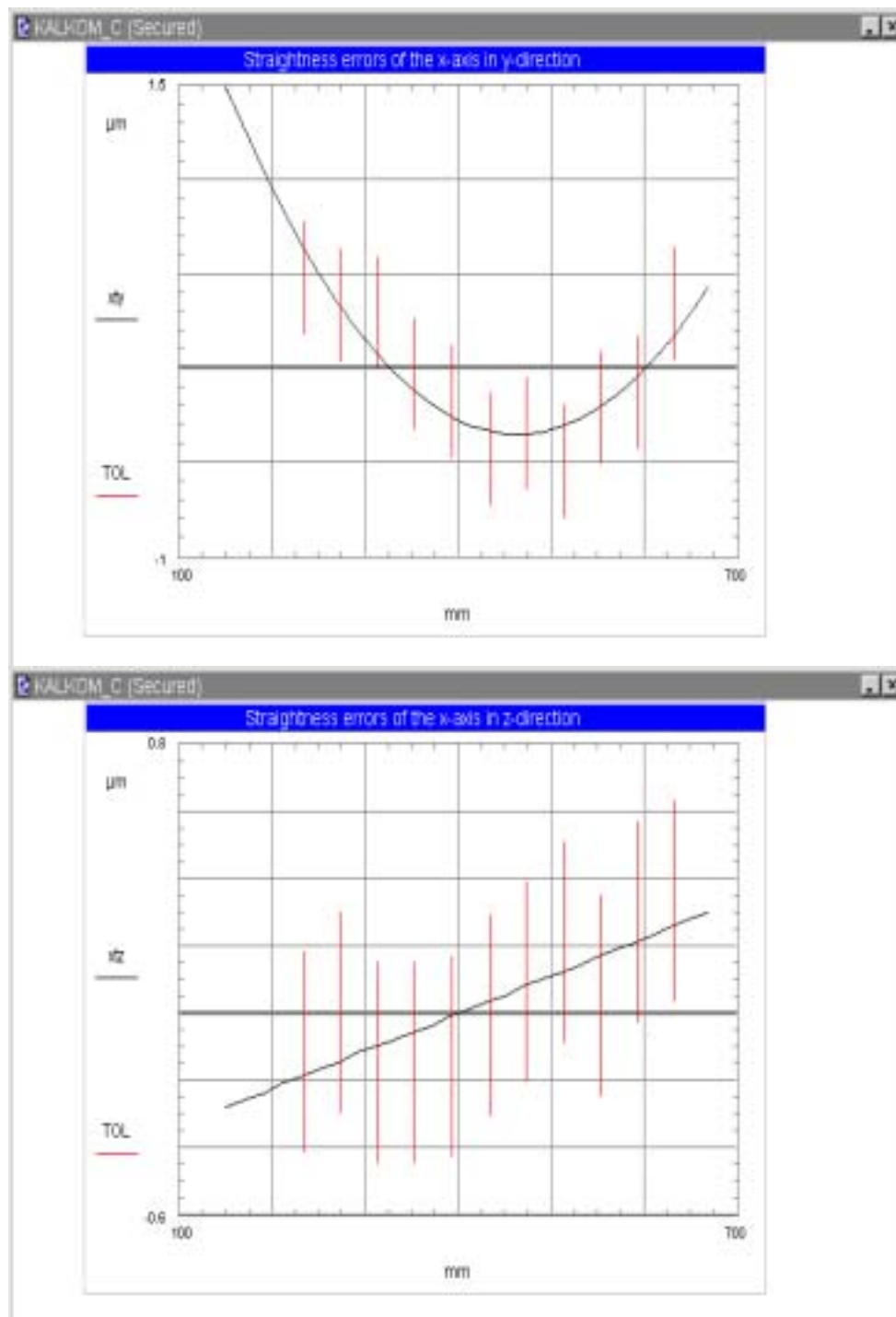
Plate Positions			
	X	Y	Z
Position \$11 (mm)	233	105	507
Position \$12 (mm)	233	105	156
21\$	0	0	
31\$	0	0	
41\$	0	0	
51\$	0	0	
Position \$121 (mm)	235	320	169
Position \$122 (mm)	234	320	169
22\$	0		0
32\$	0		0
42\$	0		0
52\$	0		0
Position \$131 (mm)	438	105	169
Position \$132 (mm)	438	105	169
23\$		0	0
33\$		0	0
43\$		0	0
53\$		0	0

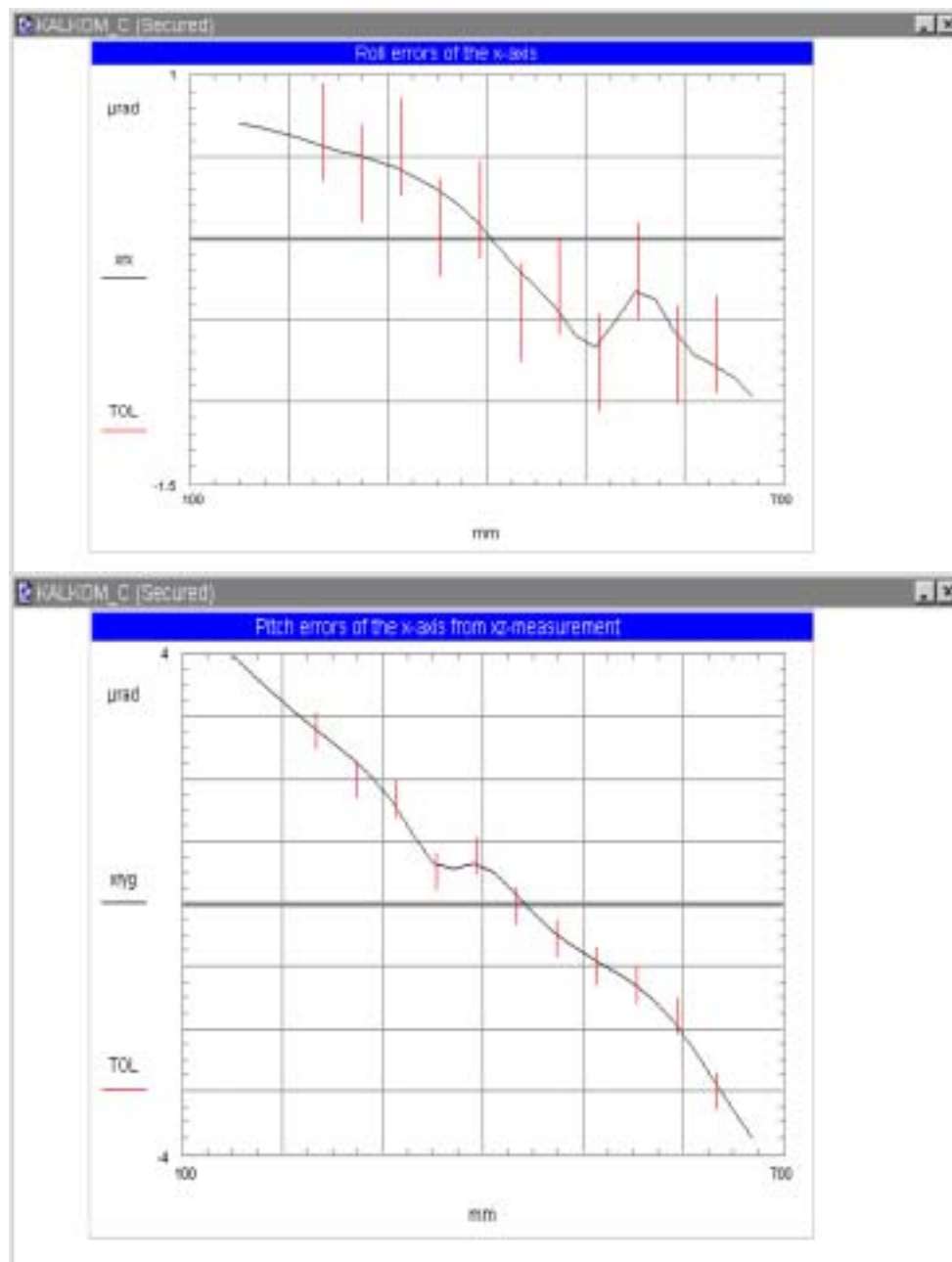
Orthogonalities			
xyz (µrad)	-3.21	xyz (µrad)	-1.76
xyz (µrad)	-5.21		

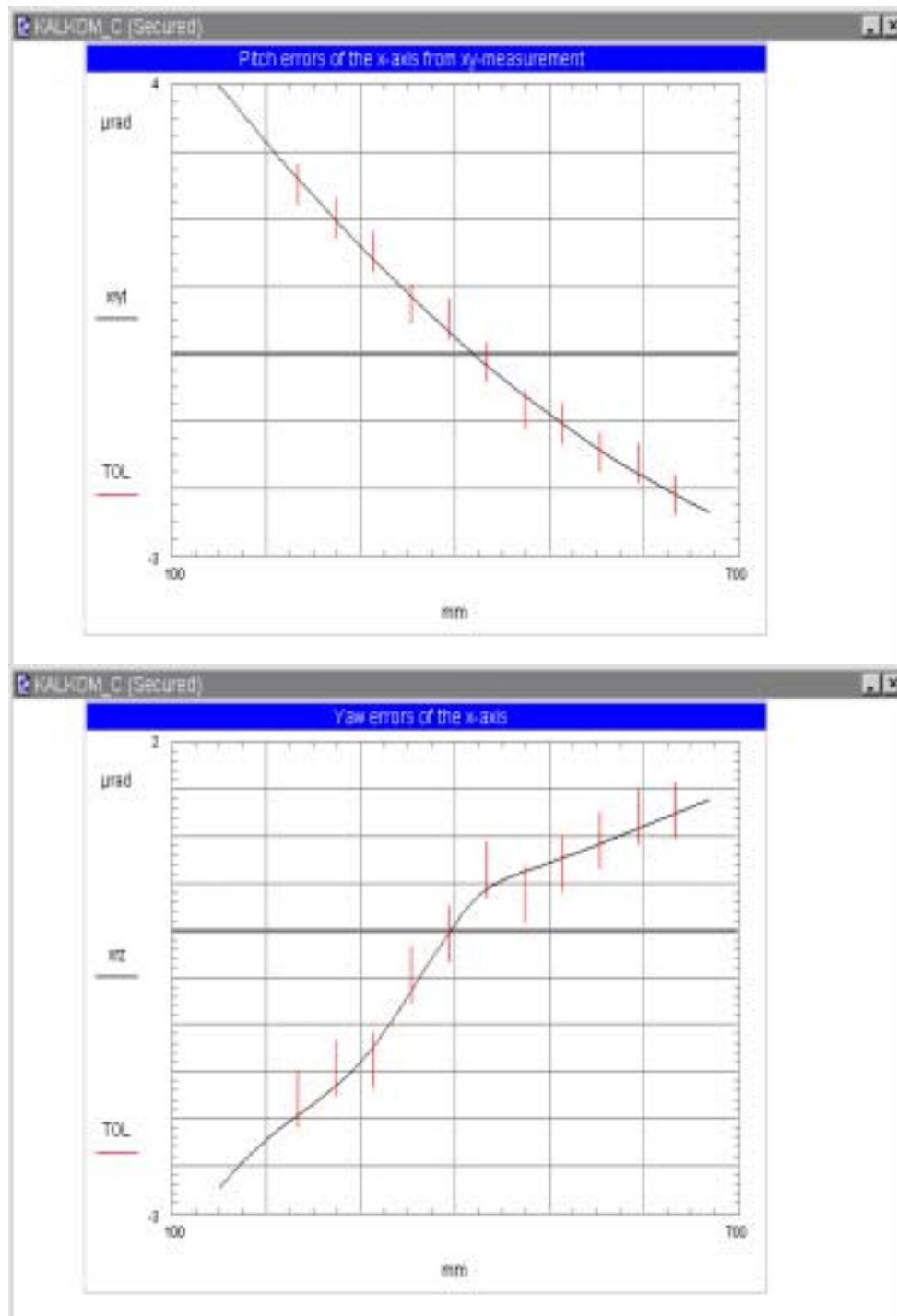
PTB Evaluation by Kalikom 4.0, (c) Physikalisch-Technische Bundesanstalt Braunschweig PTB

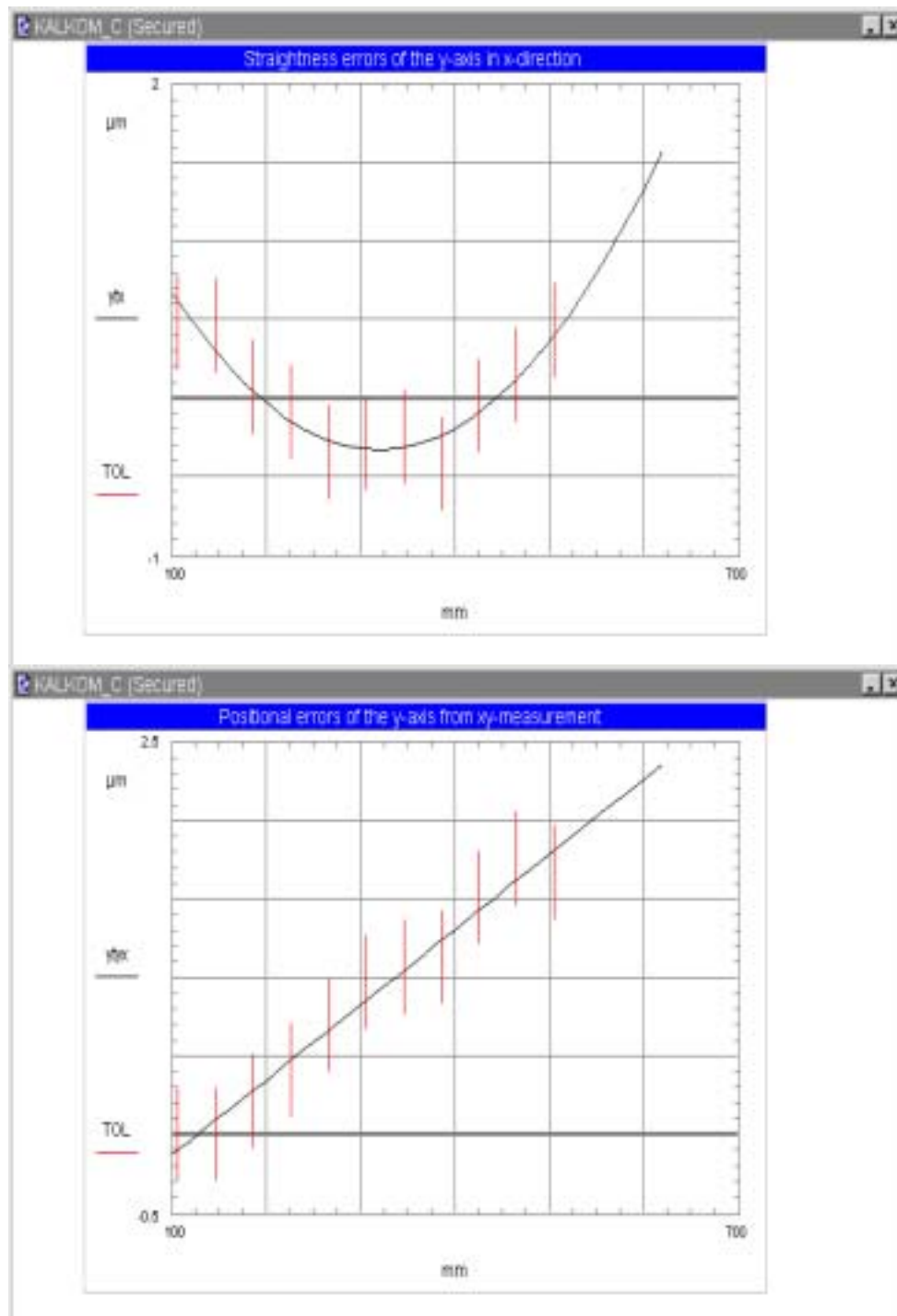


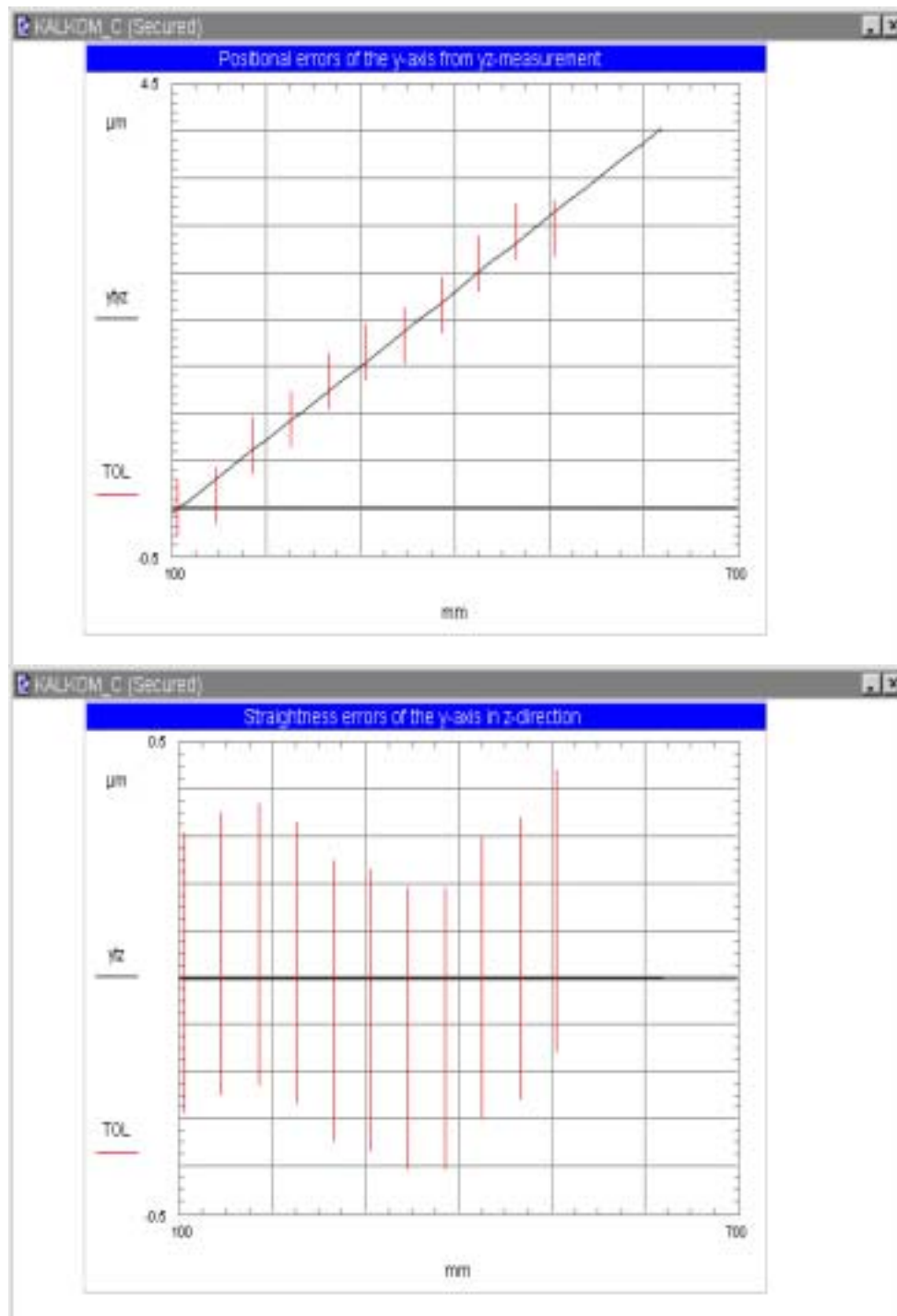


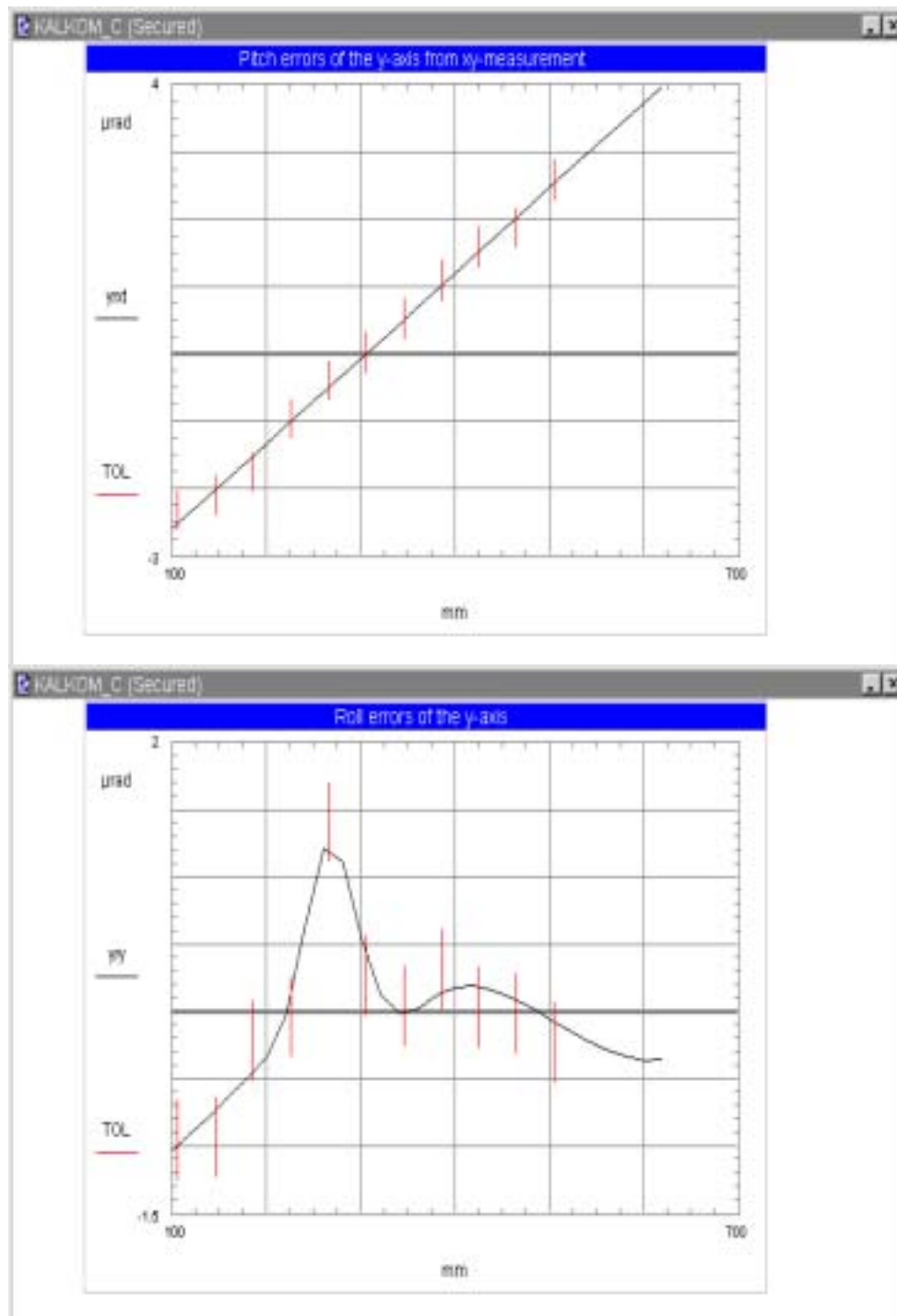


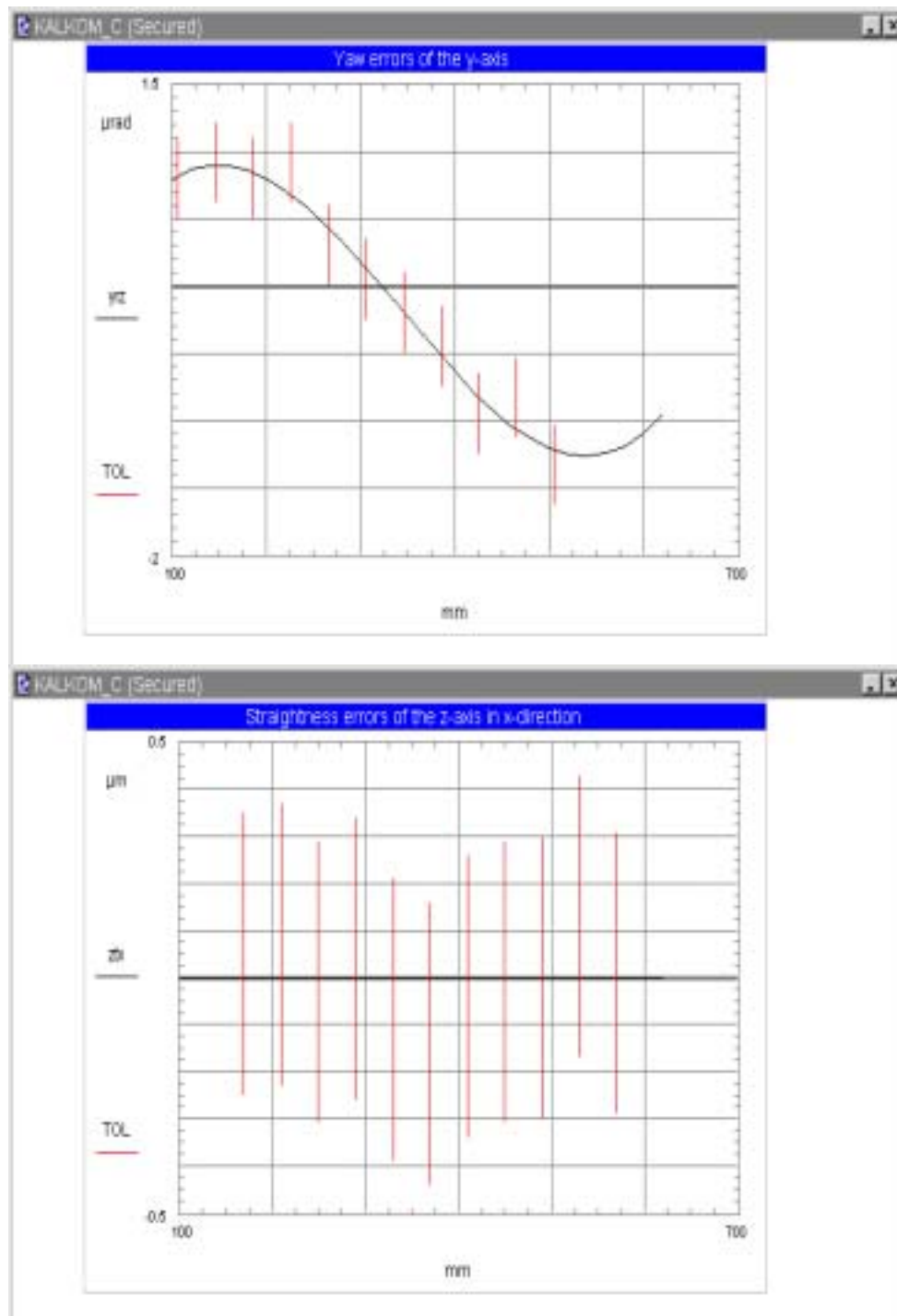


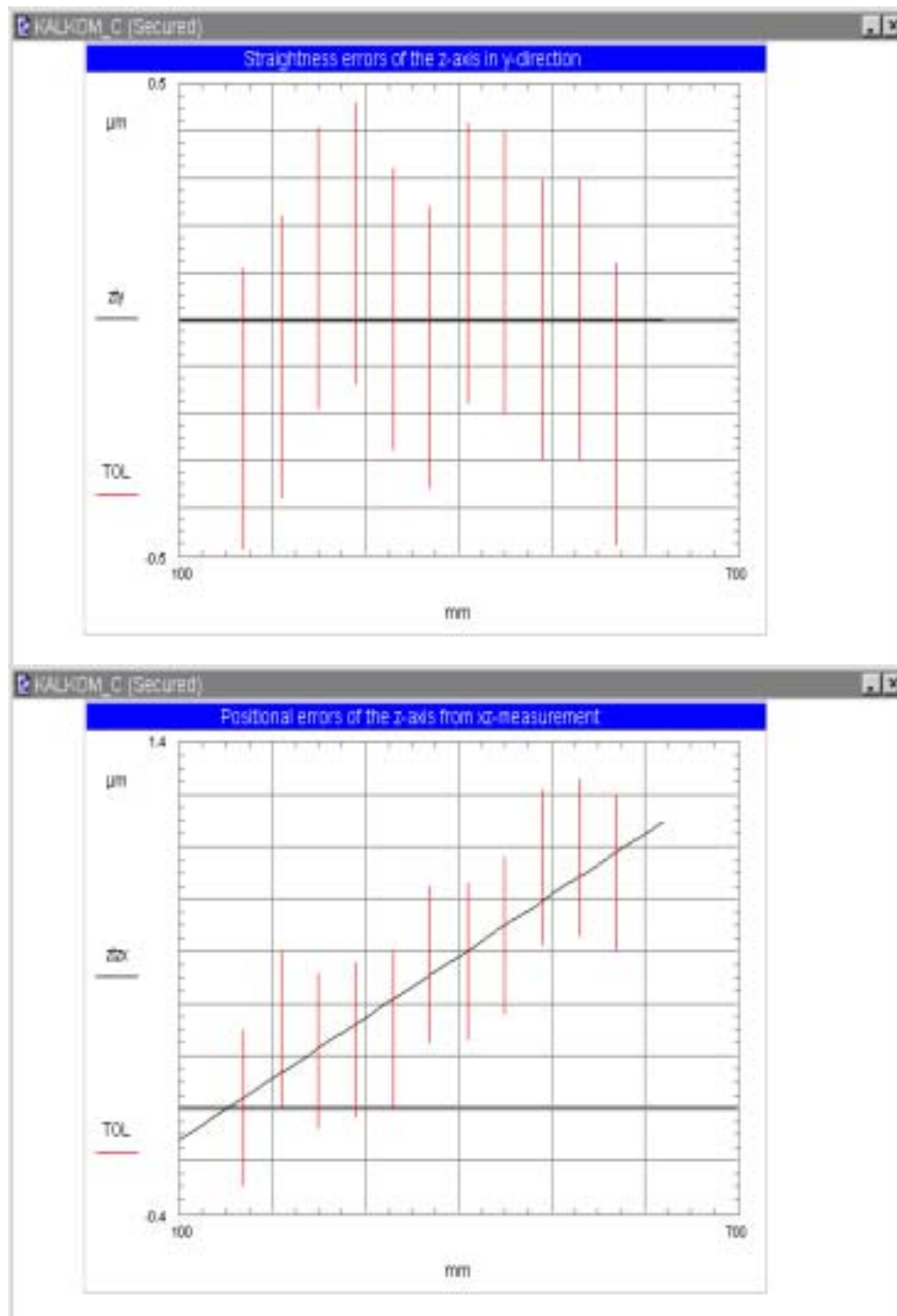




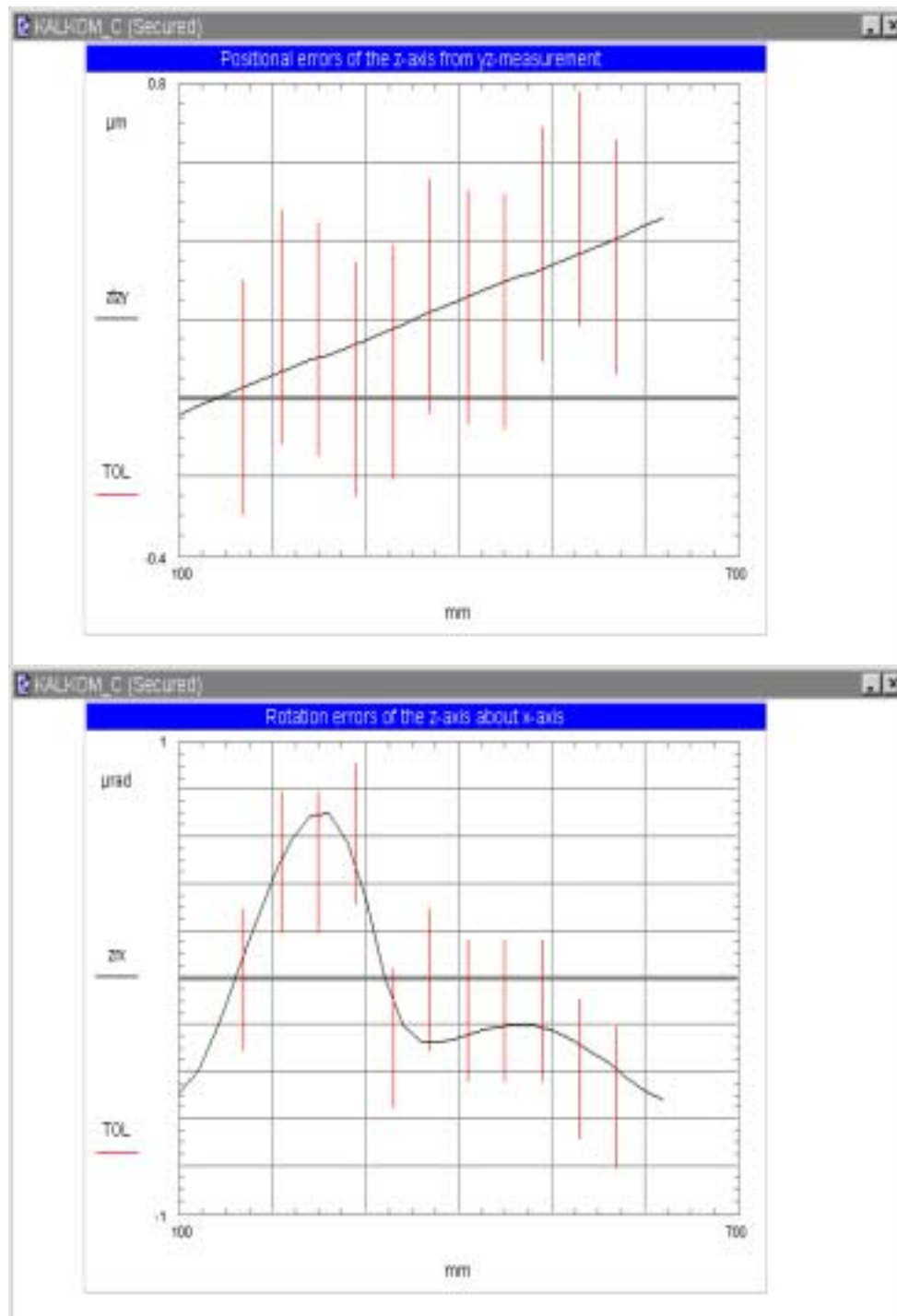


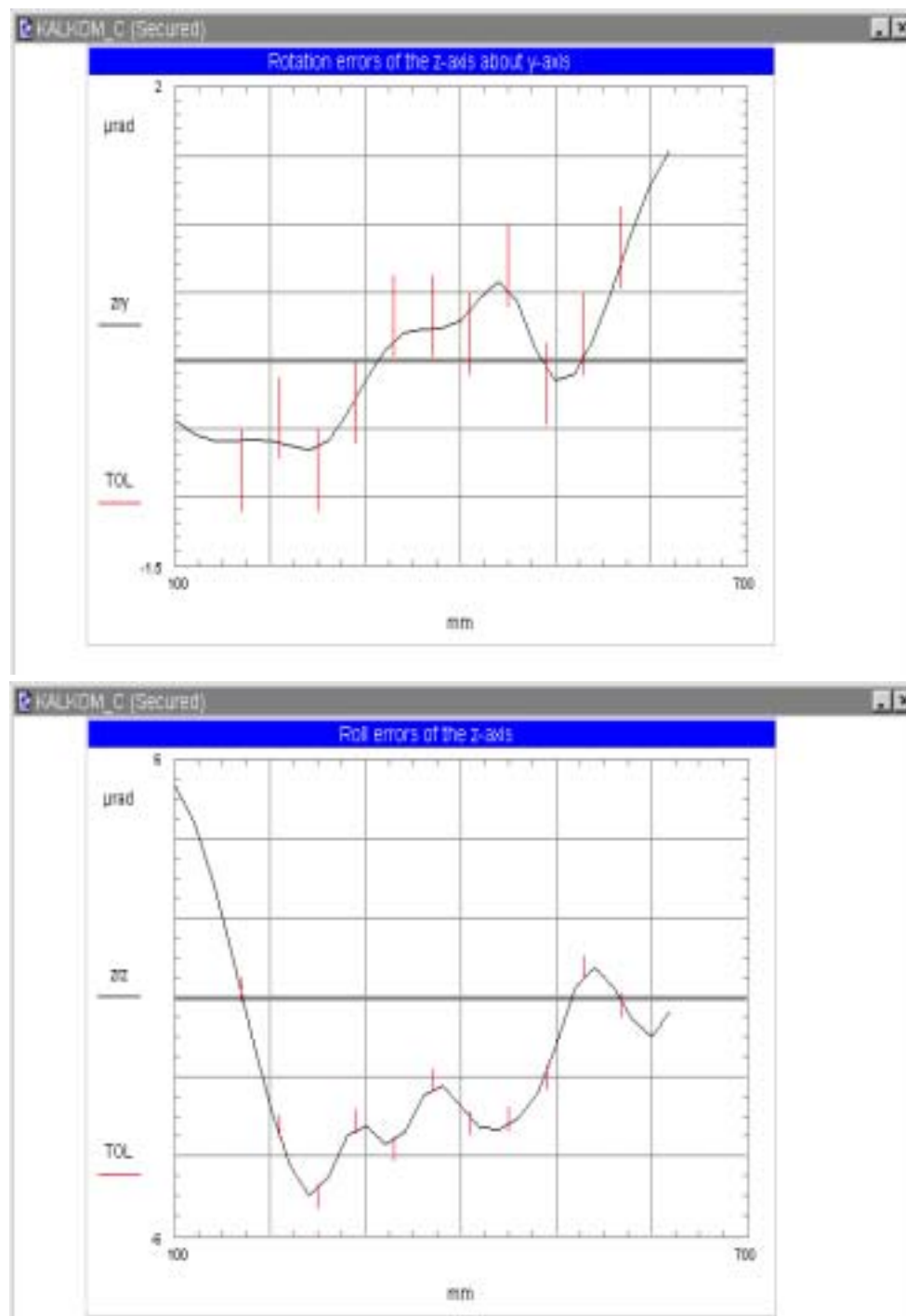




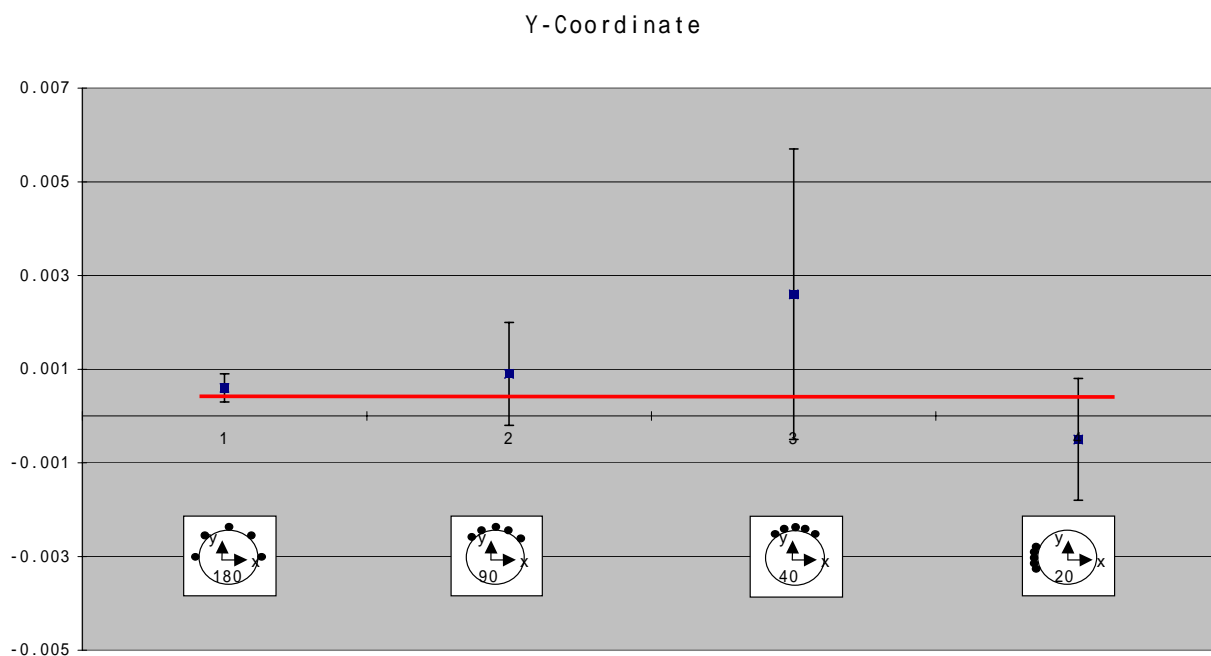
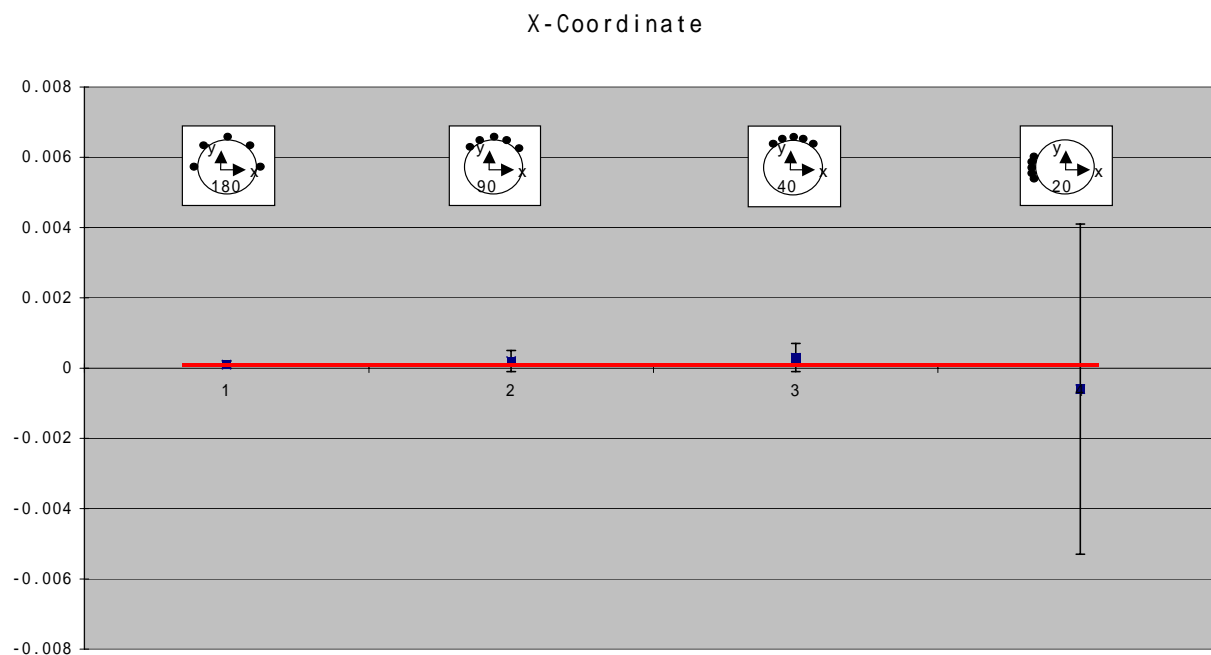


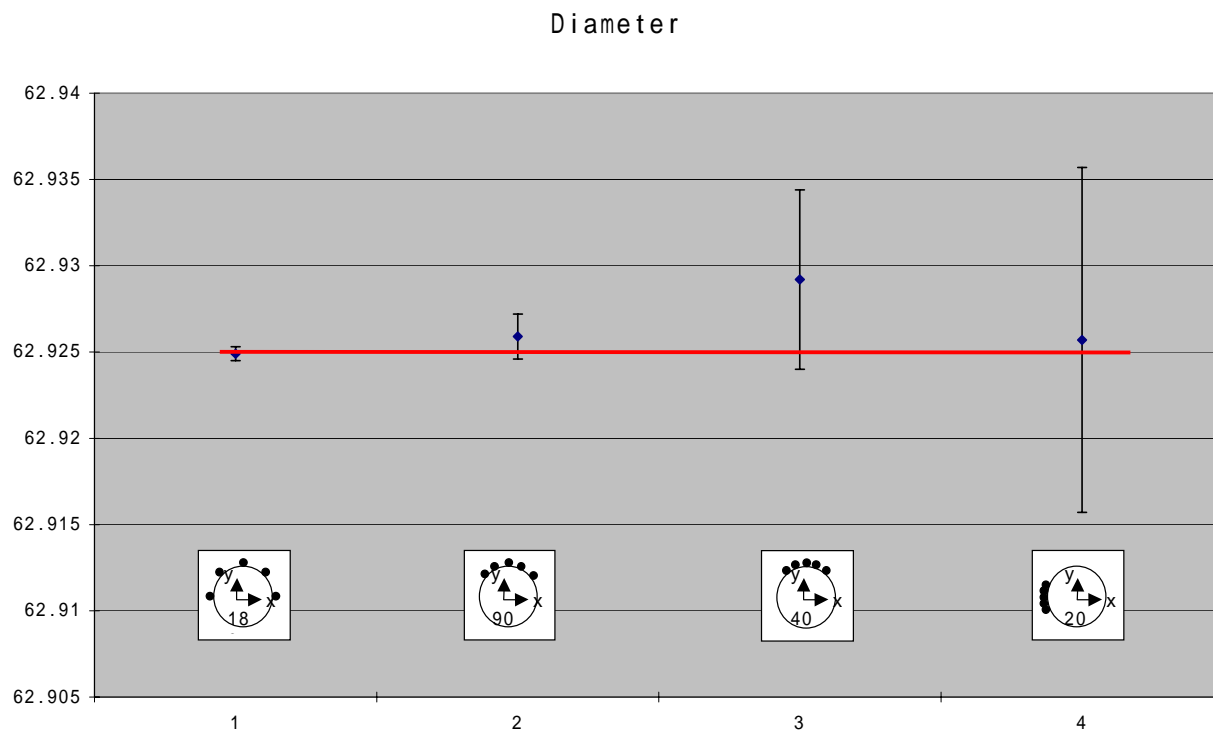






## Annex 2: Results of first verification measurements on ring gauges





## 5.3 Calculating CMM Measurement Uncertainty with OzSim

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from CTIP Technical Memorandum TIPP1312, March 2001

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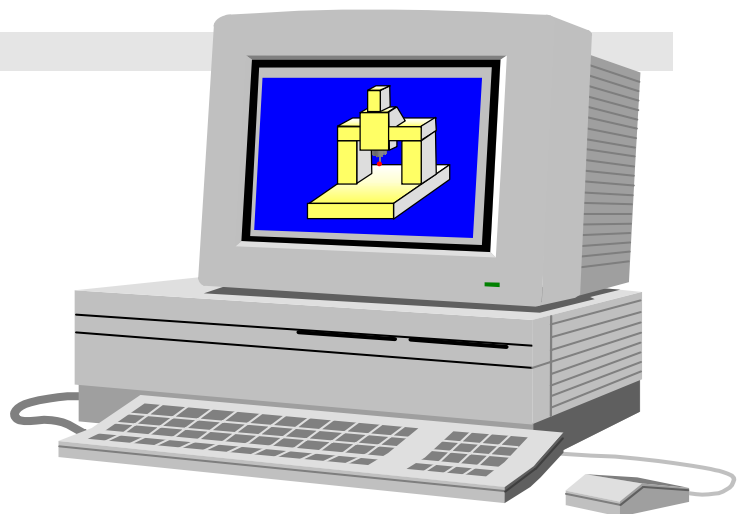
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## **Introduction**

A CMM does not directly measure geometric features such as distances, angles or diameters. Instead it measures the coordinates of a set of points on the surface of an artefact and then combines them to evaluate the desired geometric feature. So unlike Vernier calipers which directly measures the diameter of a cylinder, a CMM measurement requires some processing of the combination of measured values to produce an estimate of the diameter. Clearly then, the outputs from a CMM measurement will depend on the way in which this processing is performed (the algorithm), and the properties of the points chosen to sample the surface of the artefact.

In this report we discuss how the probing strategy used to sample the artefact's surface affect the results of a CMM measurement, and how the measurement uncertainty is a powerful tool for reflecting this. This we do by using a simple simulation method, called OzSim, to estimate the CMM's measurement uncertainty for tasks with different sampling strategies. The measurement of four different tasks are discussed, a plane, circle, sphere and a helical gear. For the first three, the measurement uncertainty could be determined by alternative methods allowing for a comparison with those generated by OzSim.

## **Determining CMM measurement uncertainty**

Estimating the uncertainty for a CMM measurement is a difficult task. Just how do you determine the uncertainty in the tooth angle of a helical gear, or the co-linearity of the axes of two partial cylinders, from the coordinates of the points measured on the artefacts surface? While there is no one method that works for all CMM tasks, there are specific situations in which an uncertainty can be provided. A CMM can be used as a comparator, where corrections and an uncertainty for a test gauge are obtained from the measurement of a master gauge. The weakness of this method is the requirement of a calibrated master gauge - an artefact that exists for a limited number of tasks. Another approach is to identify all major sources of uncertainty and calculate the overall uncertainty by combining the contributions. However, this can only be done for relatively simple tasks and even then requires a high level of expert knowledge to assess the contributions and prepare the uncertainty budget.

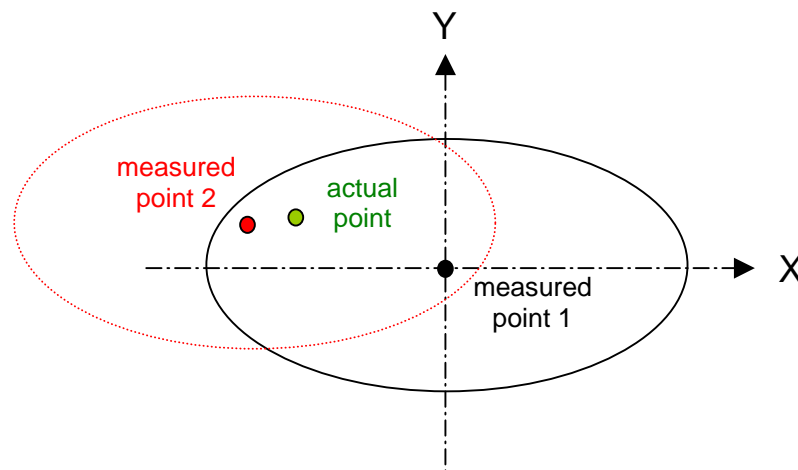
A new promising alternative has emerged that calculates uncertainties for CMM measurements that does not depend on task. Leading European and American national measurement institutes have developed simulation methods that can be used to estimate the uncertainty of any CMM measurement. Put simply, the entire measurement process is theoretically modeled to produce a computer simulation of the real CMM. Given the same input data, this simulator will mimic the real CMM and deliver similar outputs. This allows the effect that uncertainties in the input parameters, such as the work-piece temperature, have on the measurement to be evaluated. And because the technique is independent of the type of measurement, it works for even the most complicated tasks.

## **OzSim - The Australian simulation method**

OzSim is NML's simulation method for determining CMM uncertainty. Its operating philosophy is similar to other simulation methods but some assumptions are incorporated that simplify its application. The price paid for this simplicity is an increase in the measurement uncertainty making OzSim unreliable for applications where uncertainties at the tenth of a micrometer level are desired. So while the approach cannot be used for some of the high accuracy tasks encountered at national measurement institutes, it is more than adequate for tasks requiring uncertainties at the 0.5  $\mu\text{m}$  level or larger.

### Measurement uncertainty

A CMM measurement occurs when a probing event triggers the CMM to take readings of its three orthogonal scales. Errors in probing, the scale reading or the CMM itself, lead to each measured coordinate departing from the true coordinate of the actual point. Therefore, the measurement of each coordinate is characterized by a region of uncertainty that has a particular likelihood of encompassing the true value. Expanding this to three dimensions, produces an ellipsoid of likelihood centred on the measured point. This is shown graphically for the two dimensional case in figure 1.



*Figure 1: The region around a measured point where there is particular likelihood of finding the actual point.*

If a second measurement of the same point is made with the CMM a different estimate of the coordinates of the actual point is obtained. Repeating measurements provides additional information and combining them with the first measurement gives a better estimate of the coordinates of the actual point.

A CMM produces a value for a desired geometric feature by combining a number of measured points. For example, consider the simple two dimensional situation of deducing the diameter of a circle from a number of probed points on its 'surface'. This is achieved by fitting an appropriate ideal surface (ie circle) to the measured points. The diameter of the circle is then simply that of the fitted circle. This scenario is shown in figure 2.

If the measurement of the points were repeated, a different diameter will be obtained because of the uncertainty in the coordinates of the measured points. Ultimately, if the measurements were performed many times then a distribution of diameter values would be produced. This distribution gives the probability that a particular diameter is measured, and is characterized by a mean value and distribution width or variance. Figure 3 shows an example of a distribution that can occur.

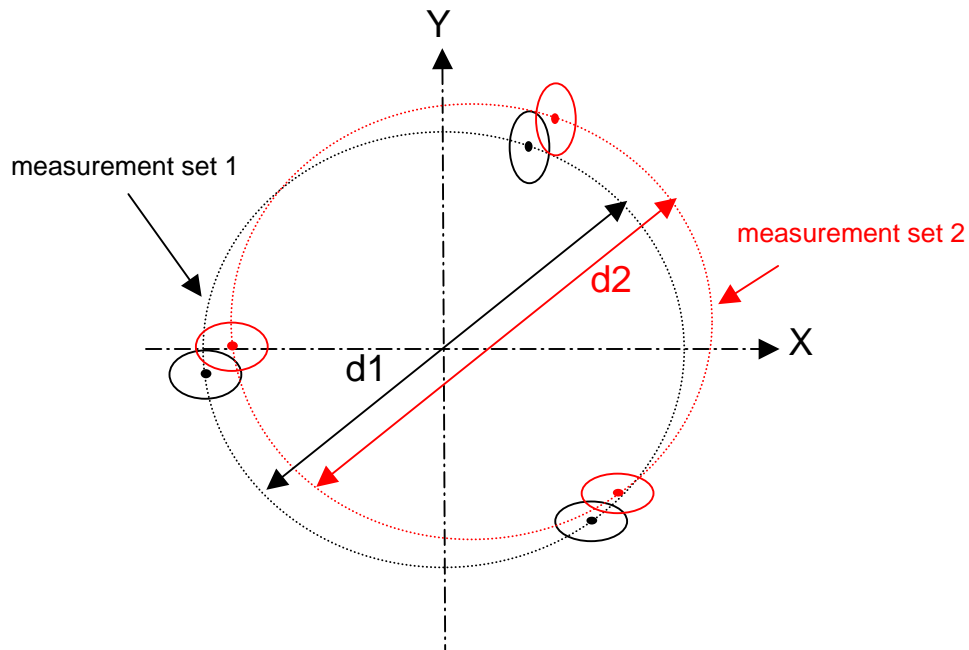


Figure 2: Finding the diameter of a circle by fitting a theoretical circle to a set of measured points on its surface.

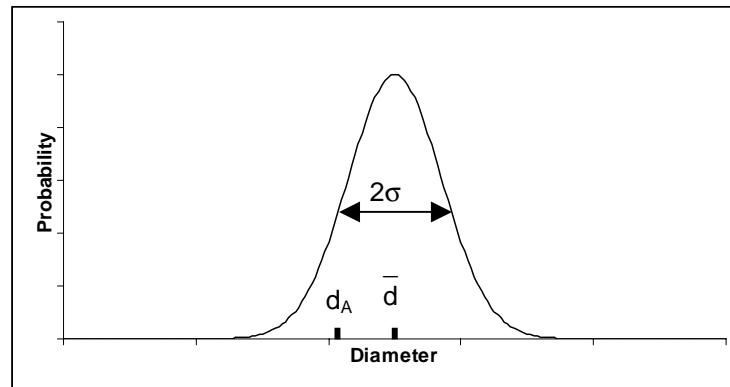


Figure 3: Probability distribution produced by multiple measurements of the diameter. The parameter  $d_A$  is the actual diameter.

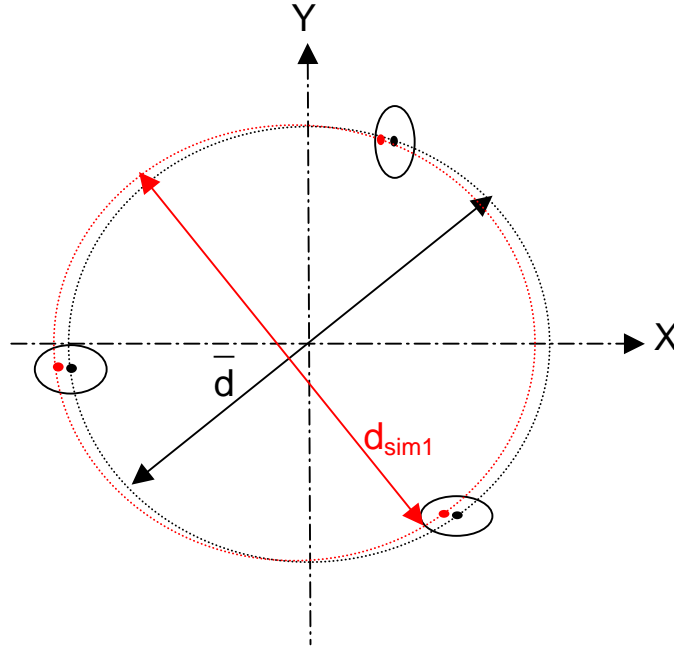
The mean value of the probability distribution gives an estimate of the diameter and the width of the distribution provides an estimate of the uncertainty. Therefore, by simply repeating the measurement many times an estimate of the uncertainty can be obtained. Note however, this assumes that all systematic effects have been corrected for.

### Pros and cons of simulating measurements

One problem with performing multiple repeats of a measurement is the time involved. A single CMM measurement run can take many hours to complete, so clearly, it would take far too long to do the 100 or so runs required for good statistics. This is true of even relatively simple CMM tasks.



A much quicker alternative is to model the entire CMM measurement procedure and simulate the many repeat measurements required on a computer. The artefact is measured once and the measured points and their uncertainties are used as the input parameters to the model. Each simulated measurement run generates new coordinates for each of the points by randomly selecting values from the probability distributions defined by the measured points and their uncertainties. These new point coordinates are then used to evaluate a simulated value for the geometric feature of interest. Figure 4 shows this process for the simple two-dimensional example of the circle.



*Figure 4: Multiple measurements of the circle's diameter simulated by randomly varying the measured point values throughout their uncertainty range.*

The simulation is repeated until the required number of iterations have been performed to sufficiently map out the probability distribution for the geometric feature (eg. the diameter of the circle), and it is the width of this distribution that gives an estimate of the uncertainty in the parameter.

There is one important difference between the results generated by repeating the actual measurements and those by simulating them. When repeat measurements are made the mean value for a geometric feature is obtained from the resulting distribution, whereas when the measurements are simulated, the mean value is fixed to be equal to the value given by the initial measurement. For example, the mean diameter of the circle in figure 2 comes from calculating the mean of the diameters from all the measurements.

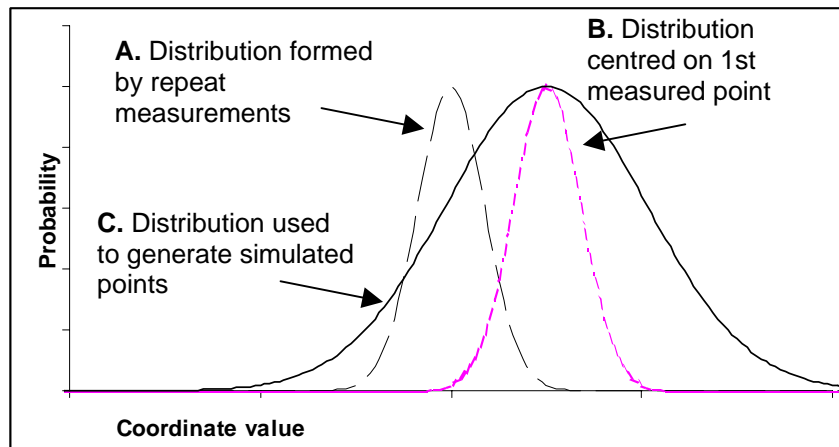
$$\bar{d} = \sum_i d_i$$

But when the repeat measurements are simulated, we perform one actual measurement and assume that the measured diameter is the mean diameter. The difference can be seen by comparing figures 2 and 4. For both figures, assume that the ellipsoids around the measured points encompass the regions where there is a 95% likelihood of finding the true coordinates of the points. From figure 4 we see that the ellipsoids around the initial measured points also have a 95% likelihood of encompassing the coordinates of all the simulated points. But this is

not true for the repeated measurement runs shown in figure 2. Here the ellipsoid around each measured point has a 95% chance of encompassing the true coordinate values but does not necessarily have a 95% chance of containing a repeat measurement of the point.

One consequence of this is that while a simulation method can give information about the shape and width of the probability distribution of a feature, it cannot reveal anything about the distribution's mean value. Information about the mean value can only be obtained from further actual measurements.

The problem with assuming the mean of the simulated point distribution to be equal to the initially measured value is that it leads to an underestimate of the uncertainty. The initial measurement is just one of a distribution of possible outcomes and could be a poor estimate of the actual value. In a worse case scenario, the measured point could deviate from the actual point by its uncertainty. This would then mean that the half of simulated points would also be bad estimates of the actual point as they depart by more than the uncertainty. The OzSim simulation method avoids this by expanding the uncertainties of the initial measured points by a factor of 3. This ensures that there is at least a 95% chance of any repeat measured point being contained within the distribution used to generate the simulated points. Figure 5 shows this graphically.



*Figure 5: A. is the distribution obtained if the measurement were repeated many times. B. is the distribution centred on the first measurement value. C. is the distribution of B. expanded so that at least 95% of all distribution A. lies within it.*

### **Estimating the uncertainty in the coordinates of a measured point**

Among the important parameters required by a simulation method are the uncertainties of the coordinates of a measured point. These uncertainties result from errors in the probing of the CMM, errors in reading of scales and departures of the CMM measurement frame from a truly orthogonal Cartesian frame. In general these errors are not constant and result in coordinate uncertainties that depend on the location of the point within the measurement volume. For high accuracy requirements, where uncertainties less than 0.5  $\mu\text{m}$  are desired, then a full characterization of all error sources must be done. This is a complicated procedure taking a high level of understanding of the CMM usually only obtained through many years experience experimenting with CMM error sources. The PTB have developed procedures to do this by devoting many years of research into this specific area. However, if these ultra-high levels of accuracies are not required then a significant time saving can be gained with some simplifying assumptions.

One of these is to assume a simple relationship for the coordinate uncertainty. One common form of such a relationship is,

$$\Delta x = \sqrt{a_x^2 + (a_{xl} \cdot X)^2}$$

Where X is the distance from a reference point and  $a_x$  and  $a_{xl}$  are constants. As a starting point the constants in this relationship can be obtained from the manufacturers specification sheet that is issued with each CMM. Of course if subsequent work has been performed with the CMM to give a more accurate picture of the way in which the coordinate uncertainty varies throughout the volume then that should be used.

### **Modeling the CMM measurement**

The measured points and their uncertainties are the input parameters to the simulation model that mimics the behaviour of the CMM. In order for the model to produce reliable uncertainty estimates it is vital that the model processes the measured coordinates in exactly the same way as what the CMM does. Therefore, the model must use exactly the same fitting algorithm as the one used by the CMM even if a superior one exists. In most cases the fitting algorithm used by the CMM is a least squares fit, where the sum of the square of the deviation of the points from a best fitted theoretical surface is minimized.

Verification of the model is relatively straightforward as the outputs from the model should be exactly the same as that obtained from the CMM, when both use the same measured points as inputs.

### **Implementing OzSim**

Once a CMM measurement task has been modeled and estimates for the uncertainties for the measured points are obtained a simulation can take place. All the input data is placed in an Excel™ spreadsheet as are the fitting equations required to produce the values for the desired geometric features. At this stage the input parameters to the equations are the coordinates of the points initially measured with the CMM and the outputs are the same as that given by the CMM. At this stage the uncertainty in the measured points has no effect.

This same spreadsheet is then opened under Risk™, an Excel™ add-on application. Risk™ is a simulation package that allows a spreadsheet cell to contain a distribution of values rather than a single fixed value. Therefore, in our spreadsheet we replace the coordinates of the measured points with a distribution of values centred on the measured value, and that has a width given by three times the uncertainty. When the simulation is run, every calculation on the sheet is performed 100 times and on each cycle, Risk™ selects a value for the measurement points from their defined probability distributions.

Once the simulation is completed a distribution of output values is obtained for each geometric feature of interest. These distributions have mean values determined by the initial measured points, and have widths that can be used to calculate the uncertainty in those values.

## **Examples**

Four examples were performed to demonstrate the concept. The first three are of simple geometric objects, a circle, a sphere and a plane. The results from OzSim in these three instances were compared against the more sophisticated VCMM developed by the PTB. The fourth example is for a helical gear. No comparison was possible for this object due to its complexity.

### Uncertainty of the origin and radius of a circle

The CMM was used to measure the radius and origin of a circle on the inside of a 35 mm cylindrical gauge in a plane perpendicular to its axis. Two different measurement strategies were used, both sampling the surface at 10 points. The first sampling strategy probes the surface within 2 mm of the reference point on the defined 'x' axis, and therefore, only covers a limited fraction of the available surface. The second strategy has all 10 points evenly spaced around the entire circle. Both strategies are shown in figure 6.

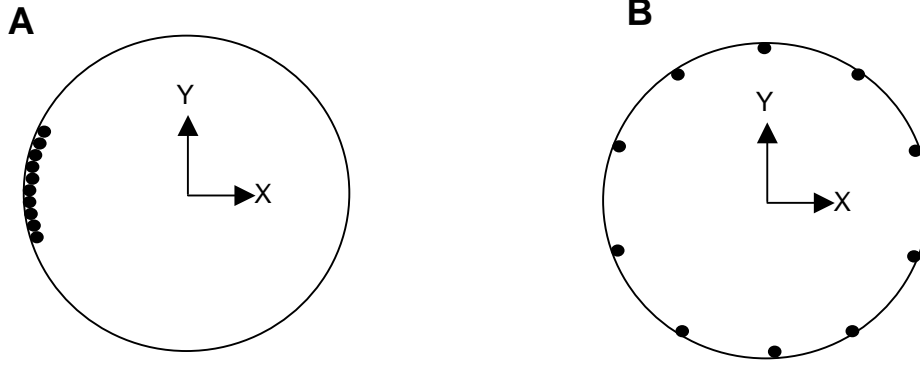


Figure 6: The two sampling strategies used to measure the circle.

The fitting algorithm used in OzSim was based on performing a least squares fit of the measured points to a circle as given by:

$$\Delta = \sum_{i=1}^N (R^2 - (x_i - X_0)^2 - (y_i - Y_0)^2)^2$$

Here R is the radius of the cyle,  $X_0$  is the x coordinate of the origin,  $Y_0$  is the y coordinate of the origin. These three parameters are then chosen so that  $\Delta$  is a minimum. It can be shown that  $\Delta$  is a minimum when:

$X_0 =$

$$\begin{aligned} & \left( N \left( \sum_{i=1}^N x_i^2 \right) \sum_{i=1}^N x_i y_i - \left( \sum_{i=1}^N x_i^2 \right) \left( \sum_{i=1}^N y_i \right) \sum_{i=1}^N x_i y_i - \left( \sum_{i=1}^N y_i \right) \left( \sum_{i=1}^N x_i y_i \right) \sum_{i=1}^N y_i^2 + \left( \sum_{i=1}^N x_i^3 \right) \left( \left( \sum_{i=1}^N y_i \right)^2 - N \sum_{i=1}^N y_i^2 \right) + \right. \\ & \left. \left( \sum_{i=1}^N y_i \right)^2 \sum_{i=1}^N x_i y_i^2 - N \left( \sum_{i=1}^N y_i^2 \right) \sum_{i=1}^N x_i y_i^2 + N \left( \sum_{i=1}^N x_i y_i \right) \sum_{i=1}^N y_i^3 + \right. \\ & \left. \left( \sum_{i=1}^N x_i \right) \left( - \sum_{i=1}^N y_i x_i^2 \sum_{i=1}^N y_i + \left( \sum_{i=1}^N x_i^2 \right) \sum_{i=1}^N y_i^2 + \left( \sum_{i=1}^N y_i^2 \right)^2 - \left( \sum_{i=1}^N y_i \right) \sum_{i=1}^N y_i^3 \right) \right) / \\ & \left( 2 \left( -2 \left( \sum_{i=1}^N x_i \right) \left( \sum_{i=1}^N y_i \right) \sum_{i=1}^N x_i y_i + N \left( \sum_{i=1}^N x_i y_i \right)^2 + \left( \sum_{i=1}^N x_i \right)^2 \sum_{i=1}^N y_i^2 + \left( \sum_{i=1}^N x_i^2 \right) \left( \left( \sum_{i=1}^N y_i \right)^2 - N \sum_{i=1}^N y_i^2 \right) \right) \right) \end{aligned}$$

$$\begin{aligned}
 Y_0 = & - \left( 2 \left( \left( \sum_{i=1}^N x_i \right) \sum_{i=1}^N y_i - N \sum_{i=1}^N x_i y_i \right) \left( \left( \sum_{i=1}^N x_i \right) \left( \sum_{i=1}^N x_i^2 + \sum_{i=1}^N y_i^2 \right) - N \left( \sum_{i=1}^N x_i^3 + \sum_{i=1}^N x_i y_i^2 \right) \right) + \right. \\
 & 2 \left( \left( \sum_{i=1}^N x_i \right)^2 - N \sum_{i=1}^N x_i^2 \right) \left( N \sum_{i=1}^N y_i x_i^2 - \left( \sum_{i=1}^N x_i^2 \right) \sum_{i=1}^N y_i - \left( \sum_{i=1}^N y_i \right) \sum_{i=1}^N y_i^2 + N \sum_{i=1}^N y_i^3 \right) \Bigg) / \\
 & \left( -4 \left( \left( \sum_{i=1}^N x_i \right) \sum_{i=1}^N y_i - N \sum_{i=1}^N x_i y_i \right)^2 - 4 \left( \left( \sum_{i=1}^N x_i \right)^2 - N \sum_{i=1}^N x_i^2 \right) \left( - \left( \sum_{i=1}^N y_i \right)^2 + N \sum_{i=1}^N y_i^2 \right) \right) \\
 R = & x_0^2 + y_0^2 - \frac{2 x_0 \sum_{i=1}^N x_i}{N} + \frac{\sum_{i=1}^N x_i^2}{N} - \frac{2 y_0 \sum_{i=1}^N y_i}{N} + \frac{\sum_{i=1}^N y_i^2}{N}
 \end{aligned}$$

Mathematical software packages such as Mathematica™ and MathCad™ can greatly assist in obtaining these expressions. These equations are then be placed into the Excel™ spreadsheet to obtain values for R, X<sub>0</sub> and Y<sub>0</sub> for the two data sets that came from the two different sampling strategies. Each data value is entered as the mean value of a normal distribution with a standard deviation equal to three times the standard uncertainty in the coordinate. Our knowledge of the CMM enables us to express this as

$$\Delta x_i = \sqrt{0.2^2 + (0.6 \cdot D)^2} \text{ } \mu\text{m (where D is in m)} \quad (1)$$

For this example D is always less than 50 mm so the length dependent term is negligible. The Risk™ simulation was then run with these parameters. The results of the simulation are shown in figures 7 and 8. Also shown in these figures are the results obtained from the more sophisticated simulation model, the VCMM, developed by the PTB. The VCMM is much more detailed in its determination of the coordinate uncertainty but other than that the general philosophy is the same.

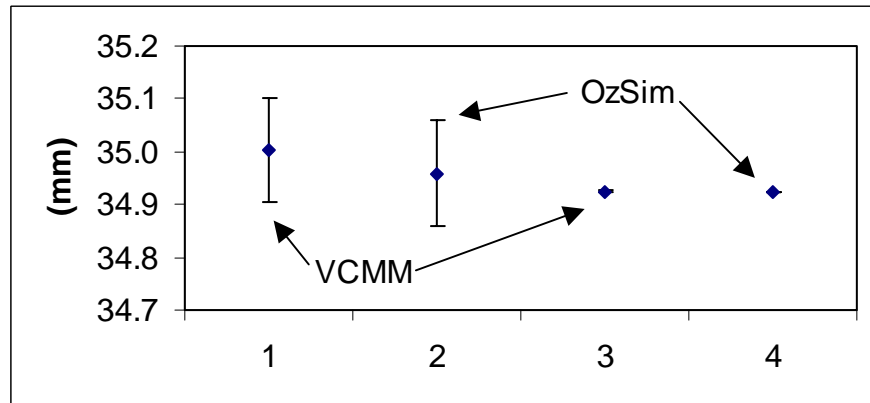


Figure 7: The diameter of the circle. Cases 1 and 2 are the results when sampling strategy A are used, while cases 3 and 4 show the results with sampling strategy B.

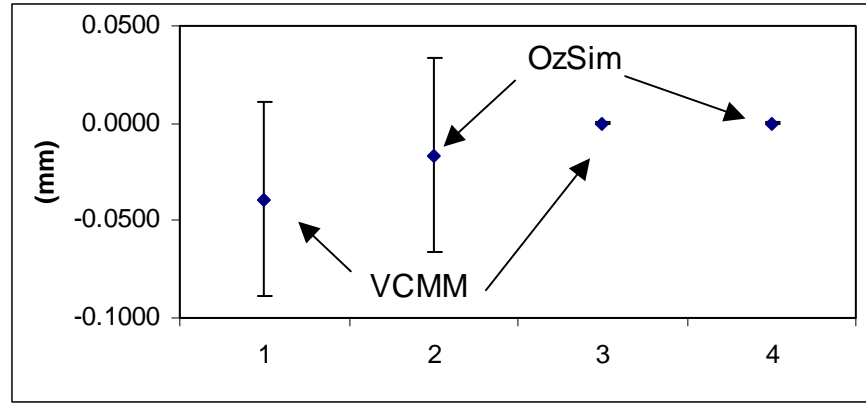


Figure 8: The x coordinate of the circle's origin. Cases 1 and 2 are the results when sampling strategy A are used, while cases 3 and 4 show the results with sampling strategy B.

The results show that OzSim is producing similar mean values to that calculated by the CMM which implies that the algorithm used by OzSim is correct. Also, the magnitude of the uncertainties produced by OzSim are virtually the same as those obtained with the VCMM, giving confidence that the simulations are operating as they should. Note that the values and uncertainties for the y coordinate of the origin are not shown as both the VCMM and OzSim gave similar results for both sampling strategies. This is expected as both sampling strategy A and B give a good determination of  $Y_0$ .

#### Uncertainty in the angles that characterize a plane

The CMM was used to measure the angles that define the plane on top of the 35 mm cylindrical gauge that is perpendicular to the axis. Two different measurement strategies were used, both sampling the surface at 10 points. The first sampling strategy probes the surface within 2 mm of the reference point. The second strategy has all 10 points evenly spaced around the entire surface. Both strategies are shown in figure 9.

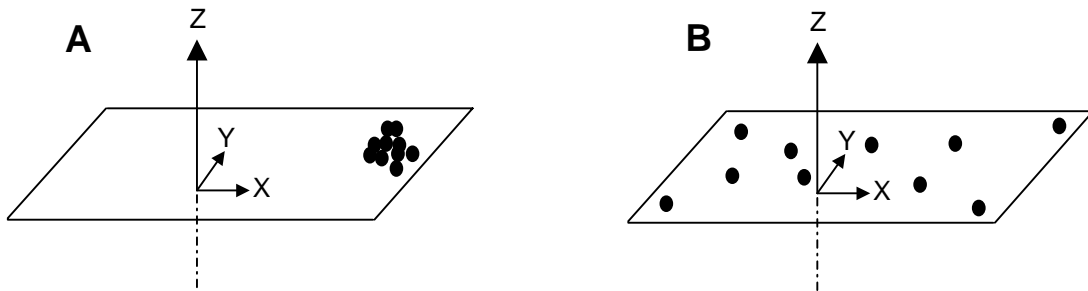


Figure 9: The two sampling strategies used to measure the plane.

The fitting algorithm used in OzSim was based on performing a least squares fit of the measured points to a plane as given by:

$$\Delta = \sum_{i=1}^N (1 - (x_i \cdot U + y_i \cdot V + z_i \cdot W))^2$$

Here  $U$  is the cosine of the angle between the normal of the plane and the x axis,  $V$  is the cosine of the angle between the normal of the plane and the y axis,  $W$  is the cosine of the angle between the normal of the plane and the z axis. These three parameters are chosen so that  $\Delta$  is a minimum. It can be shown that  $\Delta$  is a minimum when:

$$\begin{aligned}
 U = & - \left( \left( \sum_{i=1}^N Y_i^2 \right) \left( \sum_{i=1}^N Z_i \right)^2 - 2 \left( \sum_{i=1}^N Y_i \right) \left( \sum_{i=1}^N Z_i \right) \sum_{i=1}^N Y_i Z_i + N \left( \sum_{i=1}^N Y_i Z_i \right)^2 + \left( \sum_{i=1}^N Y_i \right)^2 \sum_{i=1}^N Z_i^2 - N \left( \sum_{i=1}^N Y_i^2 \right) \sum_{i=1}^N Z_i^2 \right) / \\
 & \left( - \sum_{i=1}^N Y_i^2 \left( \sum_{i=1}^N Z_i \right) \sum_{i=1}^N X_i Z_i + \left( \sum_{i=1}^N X_i Y_i \right) \left( \sum_{i=1}^N Z_i \right) \sum_{i=1}^N Y_i Z_i + \left( \sum_{i=1}^N Y_i \right) \left( \sum_{i=1}^N X_i Z_i \right) \sum_{i=1}^N Y_i Z_i - \left( \sum_{i=1}^N X_i \right) \left( \sum_{i=1}^N Y_i Z_i \right)^2 - \right. \\
 & \left. \left( \sum_{i=1}^N Y_i \right) \left( \sum_{i=1}^N X_i Y_i \right) \sum_{i=1}^N Z_i^2 + \left( \sum_{i=1}^N X_i \right) \left( \sum_{i=1}^N Y_i^2 \right) \sum_{i=1}^N Z_i^2 \right) \\
 V = & \left( \left( \left( \sum_{i=1}^N X_i \right) \sum_{i=1}^N Z_i - N \sum_{i=1}^N X_i Z_i \right) \sum_{i=1}^N Y_i Z_i + \left( \sum_{i=1}^N X_i Y_i \right) \left( - \left( \sum_{i=1}^N Z_i \right)^2 + N \sum_{i=1}^N Z_i^2 \right) + \right. \\
 & \left. \left( \sum_{i=1}^N Y_i \right) \left( \left( \sum_{i=1}^N Z_i \right) \sum_{i=1}^N X_i Z_i - \left( \sum_{i=1}^N X_i \right) \sum_{i=1}^N Z_i^2 \right) \right) / \\
 & \left( \left( \sum_{i=1}^N Y_i Z_i \right) \left( - \sum_{i=1}^N Y_i \sum_{i=1}^N X_i Z_i + \left( \sum_{i=1}^N X_i \right) \sum_{i=1}^N Y_i Z_i \right) + \left( \sum_{i=1}^N Y_i^2 \right) \left( \left( \sum_{i=1}^N Z_i \right) \sum_{i=1}^N X_i Z_i - \left( \sum_{i=1}^N X_i \right) \sum_{i=1}^N Z_i^2 \right) + \right. \\
 & \left. \left( \sum_{i=1}^N X_i Y_i \right) \left( - \sum_{i=1}^N Z_i \sum_{i=1}^N Y_i Z_i + \left( \sum_{i=1}^N Y_i \right) \sum_{i=1}^N Z_i^2 \right) \right) \\
 W = & \left( \left( \sum_{i=1}^N X_i \right) \left( \sum_{i=1}^N Y_i^2 \right) \sum_{i=1}^N Z_i + \left( \sum_{i=1}^N Y_i \right)^2 \sum_{i=1}^N X_i Z_i - N \left( \sum_{i=1}^N Y_i^2 \right) \sum_{i=1}^N X_i Z_i + N \left( \sum_{i=1}^N X_i Y_i \right) \sum_{i=1}^N Y_i Z_i - \right. \\
 & \left. \left( \sum_{i=1}^N Y_i \right) \left( \left( \sum_{i=1}^N X_i Y_i \right) \sum_{i=1}^N Z_i + \left( \sum_{i=1}^N X_i \right) \sum_{i=1}^N Y_i Z_i \right) \right) / \\
 & \left( \left( \sum_{i=1}^N Y_i Z_i \right) \left( \left( \sum_{i=1}^N Y_i \right) \sum_{i=1}^N X_i Z_i - \left( \sum_{i=1}^N X_i \right) \sum_{i=1}^N Y_i Z_i \right) + \left( \sum_{i=1}^N Y_i^2 \right) \left( - \sum_{i=1}^N Z_i \sum_{i=1}^N X_i Z_i + \left( \sum_{i=1}^N X_i \right) \sum_{i=1}^N Z_i^2 \right) + \right. \\
 & \left. \left( \sum_{i=1}^N X_i Y_i \right) \left( \left( \sum_{i=1}^N Z_i \right) \sum_{i=1}^N Y_i Z_i - \left( \sum_{i=1}^N Y_i \right) \sum_{i=1}^N Z_i^2 \right) \right)
 \end{aligned}$$

The expression for the coordinate uncertainty is the same as that for the circle given in equation 1. Again the length dependent term is negligible because the distances involved are less than 100 mm. The results of the Risk™ simulation are shown in figures 10 and 11. Also shown in these figures are the results obtained with the VCMM.

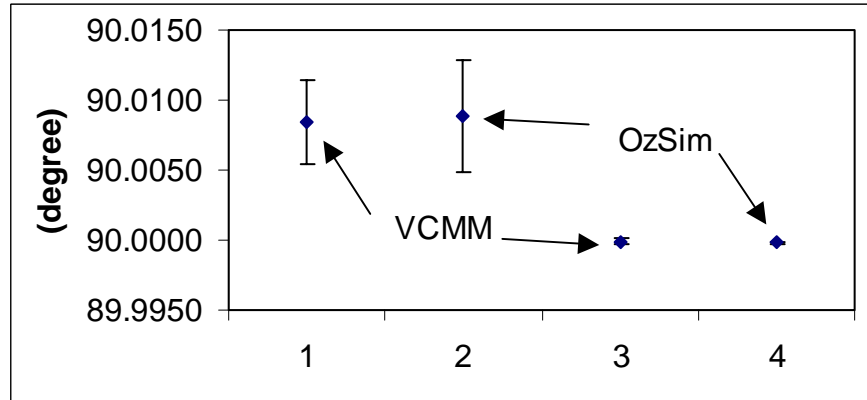


Figure 10: The angle of the plane normal with the x axis. Cases 1 and 2 are the results when sampling strategy A are used, while cases 3 and 4 show the results with sampling strategy B.

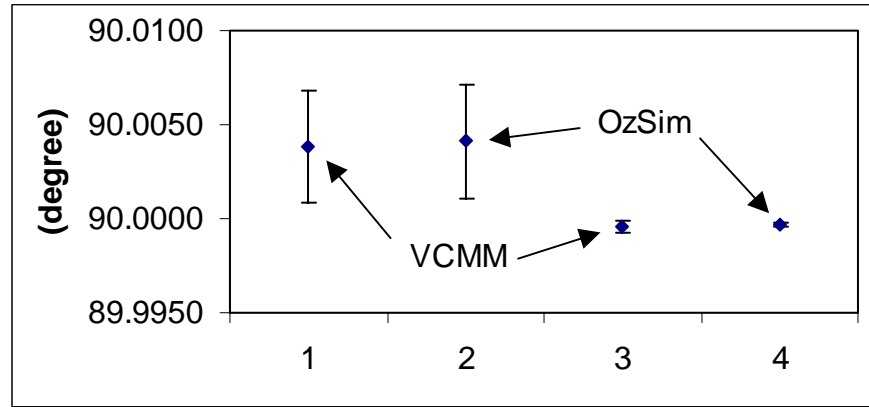


Figure 11: The angle of the plane normal with the y axis. Cases 1 and 2 are the results when sampling strategy A are used, while cases 3 and 4 show the results with sampling strategy B.

The results again show that OzSim is producing similar mean values as calculated by the CMM which implies that the algorithm used in OzSim is correct. Also, the magnitude of the uncertainties produced by OzSim are virtually the same as that obtained with the VCMM, giving confidence that the simulations are operating as they should.

#### Uncertainty of the origin and radius of a sphere

The CMM was used to measure the radius and origin of the 25 mm diameter calibration sphere. Two different measurement strategies were used, both sampling the surface at 10 points. The first sampling strategy probes the surface within 2 mm of the reference point on the defined 'z' axis, and therefore, only covers a limited fraction of the available surface. The second strategy has all 10 points evenly spaced around the entire sphere. Both strategies are shown in figure 12.

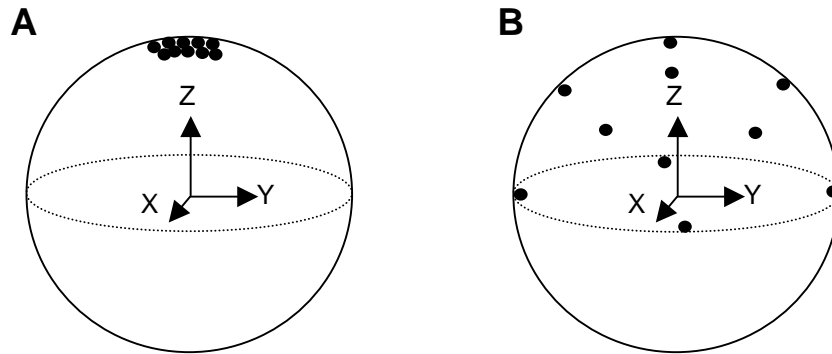


Figure 12: The two sampling strategies used to measure the plane.

The fitting algorithm used in OzSim was based on performing a least squares fit of the measured points to a sphere as given by:

$$\Delta = \sum_{i=1}^N (R^2 - (x_i - X_0)^2 - (y_i - Y_0)^2 - (z_i - Z_0)^2)^2$$



Here  $R$  is the radius of the sphere,  $X_0$  is the x coordinate of the origin,  $Y_0$  is the y coordinate of the origin, and  $Z_0$  is the z coordinate of the origin. These three parameters are chosen so that  $\Delta$  is a minimum. It can be shown that  $\Delta$  is a minimum when:

$$\begin{aligned}
 X_0 \rightarrow & - \left( \left( -2 \left( \sum_{i=1}^N Y_i \right) \sum_{i=1}^N z_i - 2N \sum_{i=1}^N Y_i z_i \right)^2 + \left( 2 \left( \sum_{i=1}^N Y_i \right)^2 - 2N \sum_{i=1}^N Y_i^2 \right) \left( 2 \left( \sum_{i=1}^N z_i \right)^2 - 2N \sum_{i=1}^N z_i^2 \right) \right) \\
 & \left( \left( 2 \left( \sum_{i=1}^N z_i \right)^2 - 2N \sum_{i=1}^N z_i^2 \right) \left( -\sum_{i=1}^N x_i \sum_{i=1}^N x_i^2 + N \sum_{i=1}^N x_i^3 - \left( \sum_{i=1}^N x_i \right) \sum_{i=1}^N Y_i^2 + N \sum_{i=1}^N x_i Y_i^2 - \left( \sum_{i=1}^N x_i \right) \sum_{i=1}^N z_i^2 + N \sum_{i=1}^N x_i z_i^2 \right) - \right. \\
 & \left( 2 \left( \sum_{i=1}^N x_i \right) \sum_{i=1}^N z_i - 2N \sum_{i=1}^N x_i z_i \right) \\
 & \left( N \sum_{i=1}^N z_i x_i^2 + N \sum_{i=1}^N z_i Y_i^2 - \left( \sum_{i=1}^N x_i^2 \right) \sum_{i=1}^N z_i - \left( \sum_{i=1}^N Y_i^2 \right) \sum_{i=1}^N z_i - \left( \sum_{i=1}^N z_i \right) \sum_{i=1}^N z_i^2 + N \sum_{i=1}^N z_i^3 \right) \Bigg) - \\
 & \left( -2 \left( \sum_{i=1}^N x_i \right) \sum_{i=1}^N z_i - 2N \sum_{i=1}^N x_i z_i \right) \left( 2 \left( \sum_{i=1}^N Y_i \right) \sum_{i=1}^N z_i - 2N \sum_{i=1}^N Y_i z_i \right) + \\
 & \left( 2 \left( \sum_{i=1}^N x_i \right) \sum_{i=1}^N Y_i - 2N \sum_{i=1}^N x_i Y_i \right) \left( 2 \left( \sum_{i=1}^N z_i \right)^2 - 2N \sum_{i=1}^N z_i^2 \right) \Bigg) \\
 & \left( \left( 2 \left( \sum_{i=1}^N z_i \right)^2 - 2N \sum_{i=1}^N z_i^2 \right) \left( N \sum_{i=1}^N Y_i x_i^2 - \left( \sum_{i=1}^N x_i^2 \right) \sum_{i=1}^N Y_i - \left( \sum_{i=1}^N Y_i \right) \sum_{i=1}^N Y_i^2 + N \sum_{i=1}^N Y_i^3 - \left( \sum_{i=1}^N Y_i \right) \sum_{i=1}^N z_i^2 + N \sum_{i=1}^N Y_i z_i^2 \right) - \right. \\
 & \left( 2 \left( \sum_{i=1}^N Y_i \right) \sum_{i=1}^N z_i - 2N \sum_{i=1}^N Y_i z_i \right) \\
 & \left( N \sum_{i=1}^N z_i x_i^2 + N \sum_{i=1}^N z_i Y_i^2 - \left( \sum_{i=1}^N x_i^2 \right) \sum_{i=1}^N z_i - \left( \sum_{i=1}^N Y_i^2 \right) \sum_{i=1}^N z_i - \left( \sum_{i=1}^N z_i \right) \sum_{i=1}^N z_i^2 + N \sum_{i=1}^N z_i^3 \right) \Bigg) \Bigg) / \\
 & \left( - \left( -2 \left( \sum_{i=1}^N x_i \right) \sum_{i=1}^N z_i - 2N \sum_{i=1}^N x_i z_i \right) \left( 2 \left( \sum_{i=1}^N Y_i \right) \sum_{i=1}^N z_i - 2N \sum_{i=1}^N Y_i z_i \right) + \right. \\
 & \left( 2 \left( \sum_{i=1}^N x_i \right) \sum_{i=1}^N Y_i - 2N \sum_{i=1}^N x_i Y_i \right) \left( 2 \left( \sum_{i=1}^N z_i \right)^2 - 2N \sum_{i=1}^N z_i^2 \right) \Bigg) \wedge 2 + \\
 & \left( -2 \left( \sum_{i=1}^N x_i \right) \sum_{i=1}^N z_i - 2N \sum_{i=1}^N x_i z_i \right)^2 + \left( 2 \left( \sum_{i=1}^N x_i \right)^2 - 2N \sum_{i=1}^N x_i^2 \right) \left( 2 \left( \sum_{i=1}^N z_i \right)^2 - 2N \sum_{i=1}^N z_i^2 \right) \\
 & \left( -2 \left( \sum_{i=1}^N Y_i \right) \sum_{i=1}^N z_i - 2N \sum_{i=1}^N Y_i z_i \right)^2 + \left( 2 \left( \sum_{i=1}^N Y_i \right)^2 - 2N \sum_{i=1}^N Y_i^2 \right) \left( 2 \left( \sum_{i=1}^N z_i \right)^2 - 2N \sum_{i=1}^N z_i^2 \right) \Bigg)
 \end{aligned}$$

$$\begin{aligned}
Y_0 \rightarrow & - \left( - \left( 2 \left( \sum_{i=1}^N x_i \right) \sum_{i=1}^N z_i - 2N \sum_{i=1}^N x_i z_i \right) \left( 2 \left( \sum_{i=1}^N y_i \right) \sum_{i=1}^N z_i - 2N \sum_{i=1}^N y_i z_i \right) + \right. \\
& \left. \left( 2 \left( \sum_{i=1}^N x_i \right) \sum_{i=1}^N y_i - 2N \sum_{i=1}^N x_i y_i \right) \left( 2 \left( \sum_{i=1}^N z_i \right)^2 - 2N \sum_{i=1}^N z_i^2 \right) \right) \\
& \left( \left( 2 \left( \sum_{i=1}^N z_i \right)^2 - 2N \sum_{i=1}^N z_i^2 \right) \left( - \sum_{i=1}^N x_i \sum_{i=1}^N x_i^2 + N \sum_{i=1}^N x_i^3 - \left( \sum_{i=1}^N x_i \right) \sum_{i=1}^N y_i^2 + N \sum_{i=1}^N x_i y_i^2 - \left( \sum_{i=1}^N x_i \right) \sum_{i=1}^N z_i^2 + N \sum_{i=1}^N x_i z_i^2 \right) - \right. \\
& \left. \left( 2 \left( \sum_{i=1}^N x_i \right) \sum_{i=1}^N z_i - 2N \sum_{i=1}^N x_i z_i \right) \right. \\
& \left. \left( N \sum_{i=1}^N z_i x_i^2 + N \sum_{i=1}^N z_i y_i^2 - \left( \sum_{i=1}^N x_i^2 \right) \sum_{i=1}^N z_i - \left( \sum_{i=1}^N y_i^2 \right) \sum_{i=1}^N z_i - \left( \sum_{i=1}^N z_i \right) \sum_{i=1}^N z_i^2 + N \sum_{i=1}^N z_i^3 \right) \right) + \\
& \left( - \left( 2 \left( \sum_{i=1}^N x_i \right) \sum_{i=1}^N z_i - 2N \sum_{i=1}^N x_i z_i \right)^2 + \left( 2 \left( \sum_{i=1}^N x_i \right)^2 - 2N \sum_{i=1}^N x_i^2 \right) \left( 2 \left( \sum_{i=1}^N z_i \right)^2 - 2N \sum_{i=1}^N z_i^2 \right) \right) \\
& \left( \left( 2 \left( \sum_{i=1}^N z_i \right)^2 - 2N \sum_{i=1}^N z_i^2 \right) \left( N \sum_{i=1}^N y_i x_i^2 - \left( \sum_{i=1}^N x_i^2 \right) \sum_{i=1}^N y_i - \left( \sum_{i=1}^N y_i \right) \sum_{i=1}^N y_i^2 + N \sum_{i=1}^N y_i^3 - \left( \sum_{i=1}^N y_i \right) \sum_{i=1}^N z_i^2 + N \sum_{i=1}^N y_i z_i^2 \right) - \right. \\
& \left. \left( 2 \left( \sum_{i=1}^N y_i \right) \sum_{i=1}^N z_i - 2N \sum_{i=1}^N y_i z_i \right) \right. \\
& \left. \left( N \sum_{i=1}^N z_i x_i^2 + N \sum_{i=1}^N z_i y_i^2 - \left( \sum_{i=1}^N x_i^2 \right) \sum_{i=1}^N z_i - \left( \sum_{i=1}^N y_i^2 \right) \sum_{i=1}^N z_i - \left( \sum_{i=1}^N z_i \right) \sum_{i=1}^N z_i^2 + N \sum_{i=1}^N z_i^3 \right) \right) \Bigg) / \\
& - \left( - \left( 2 \left( \sum_{i=1}^N x_i \right) \sum_{i=1}^N z_i - 2N \sum_{i=1}^N x_i z_i \right) \left( 2 \left( \sum_{i=1}^N y_i \right) \sum_{i=1}^N z_i - 2N \sum_{i=1}^N y_i z_i \right) + \right. \\
& \left. \left( 2 \left( \sum_{i=1}^N x_i \right) \sum_{i=1}^N y_i - 2N \sum_{i=1}^N x_i y_i \right) \left( 2 \left( \sum_{i=1}^N z_i \right)^2 - 2N \sum_{i=1}^N z_i^2 \right) \right) \wedge 2 + \\
& \left( - \left( 2 \left( \sum_{i=1}^N x_i \right) \sum_{i=1}^N z_i - 2N \sum_{i=1}^N x_i z_i \right)^2 + \left( 2 \left( \sum_{i=1}^N x_i \right)^2 - 2N \sum_{i=1}^N x_i^2 \right) \left( 2 \left( \sum_{i=1}^N z_i \right)^2 - 2N \sum_{i=1}^N z_i^2 \right) \right) \\
& \left( - \left( 2 \left( \sum_{i=1}^N y_i \right) \sum_{i=1}^N z_i - 2N \sum_{i=1}^N y_i z_i \right)^2 + \left( 2 \left( \sum_{i=1}^N y_i \right)^2 - 2N \sum_{i=1}^N y_i^2 \right) \left( 2 \left( \sum_{i=1}^N z_i \right)^2 - 2N \sum_{i=1}^N z_i^2 \right) \right) \Bigg)
\end{aligned}$$

$Z_0 \rightarrow$

$$\begin{aligned}
 & - \left( - \left( - \left( 2 \left( \sum_{i=1}^N x_i \right) \sum_{i=1}^N y_i - 2 N \sum_{i=1}^N x_i y_i \right) \left( 2 \left( \sum_{i=1}^N x_i \right) \sum_{i=1}^N z_i - 2 N \sum_{i=1}^N x_i z_i \right) + \left( 2 \left( \sum_{i=1}^N x_i \right)^2 - 2 N \sum_{i=1}^N x_i^2 \right) \left( 2 \left( \sum_{i=1}^N y_i \right) \sum_{i=1}^N z_i - \right. \right. \\
 & \quad \left. \left. 2 N \sum_{i=1}^N y_i z_i \right) \right) \left( - \left( 2 \left( \sum_{i=1}^N x_i \right) \sum_{i=1}^N y_i - 2 N \sum_{i=1}^N x_i y_i \right) \left( - \sum_{i=1}^N x_i \sum_{i=1}^N x_i^2 + N \sum_{i=1}^N x_i^3 - \right. \right. \\
 & \quad \left. \left( \sum_{i=1}^N x_i \right) \sum_{i=1}^N y_i^2 + N \sum_{i=1}^N x_i y_i^2 - \left( \sum_{i=1}^N x_i \right) \sum_{i=1}^N z_i^2 + N \sum_{i=1}^N x_i z_i^2 \right) + \left( 2 \left( \sum_{i=1}^N x_i \right)^2 - 2 N \sum_{i=1}^N x_i^2 \right) \\
 & \quad \left( N \sum_{i=1}^N y_i x_i^2 - \left( \sum_{i=1}^N x_i^2 \right) \sum_{i=1}^N y_i - \left( \sum_{i=1}^N y_i \right) \sum_{i=1}^N y_i^2 + N \sum_{i=1}^N y_i^3 - \left( \sum_{i=1}^N y_i \right) \sum_{i=1}^N z_i^2 + N \sum_{i=1}^N y_i z_i^2 \right) \Bigg) + \\
 & \quad \left( - \left( 2 \left( \sum_{i=1}^N x_i \right) \sum_{i=1}^N y_i - 2 N \sum_{i=1}^N x_i y_i \right)^2 + \left( 2 \left( \sum_{i=1}^N x_i \right)^2 - 2 N \sum_{i=1}^N x_i^2 \right) \left( 2 \left( \sum_{i=1}^N y_i \right)^2 - 2 N \sum_{i=1}^N y_i^2 \right) \right) \\
 & \quad \left( - \left( 2 \left( \sum_{i=1}^N x_i \right) \sum_{i=1}^N z_i - 2 N \sum_{i=1}^N x_i z_i \right) \left( - \sum_{i=1}^N x_i \sum_{i=1}^N x_i^2 + N \sum_{i=1}^N x_i^3 - \left( \sum_{i=1}^N x_i \right) \sum_{i=1}^N y_i^2 + \right. \right. \\
 & \quad \left. \left. N \sum_{i=1}^N x_i y_i^2 - \left( \sum_{i=1}^N x_i \right) \sum_{i=1}^N z_i^2 + N \sum_{i=1}^N x_i z_i^2 \right) + \left( 2 \left( \sum_{i=1}^N x_i \right)^2 - 2 N \sum_{i=1}^N x_i^2 \right) \right) \\
 & \quad \left( N \sum_{i=1}^N z_i x_i^2 + N \sum_{i=1}^N z_i y_i^2 - \left( \sum_{i=1}^N x_i^2 \right) \sum_{i=1}^N z_i - \left( \sum_{i=1}^N y_i^2 \right) \sum_{i=1}^N z_i - \left( \sum_{i=1}^N z_i \right) \sum_{i=1}^N z_i^2 + N \sum_{i=1}^N z_i^3 \right) \Bigg) \Bigg) / \\
 & \quad \left( - \left( - \left( 2 \left( \sum_{i=1}^N x_i \right) \sum_{i=1}^N y_i - 2 N \sum_{i=1}^N x_i y_i \right) \left( 2 \left( \sum_{i=1}^N x_i \right) \sum_{i=1}^N z_i - 2 N \sum_{i=1}^N x_i z_i \right) + \right. \right. \\
 & \quad \left. \left( 2 \left( \sum_{i=1}^N x_i \right)^2 - 2 N \sum_{i=1}^N x_i^2 \right) \left( 2 \left( \sum_{i=1}^N y_i \right) \sum_{i=1}^N z_i - 2 N \sum_{i=1}^N y_i z_i \right) \right) \wedge 2 + \\
 & \quad \left( - \left( 2 \left( \sum_{i=1}^N x_i \right) \sum_{i=1}^N y_i - 2 N \sum_{i=1}^N x_i y_i \right)^2 + \left( 2 \left( \sum_{i=1}^N x_i \right)^2 - 2 N \sum_{i=1}^N x_i^2 \right) \left( 2 \left( \sum_{i=1}^N y_i \right)^2 - 2 N \sum_{i=1}^N y_i^2 \right) \right) \\
 & \quad \left( - \left( 2 \left( \sum_{i=1}^N x_i \right) \sum_{i=1}^N z_i - 2 N \sum_{i=1}^N x_i z_i \right)^2 + \left( 2 \left( \sum_{i=1}^N x_i \right)^2 - 2 N \sum_{i=1}^N x_i^2 \right) \left( 2 \left( \sum_{i=1}^N z_i \right)^2 - 2 N \sum_{i=1}^N z_i^2 \right) \right) \Bigg)
 \end{aligned}$$

$$\mathbf{R} = \mathbf{x}_0^2 + \mathbf{y}_0^2 + \mathbf{z}_0^2 - \frac{2 \mathbf{x}_0 \sum_{i=1}^N \mathbf{x}_i}{N} + \frac{\sum_{i=1}^N \mathbf{x}_i^2}{N} - \frac{2 \mathbf{y}_0 \sum_{i=1}^N \mathbf{y}_i}{N} + \frac{\sum_{i=1}^N \mathbf{y}_i^2}{N} - \frac{2 \mathbf{z}_0 \sum_{i=1}^N \mathbf{z}_i}{N} + \frac{\sum_{i=1}^N \mathbf{z}_i^2}{N}$$

Needless to say, that Mathematica™ helped significantly to derive these expressions.

The expression for the coordinate uncertainty is the same as that for the circle given in equation 1. Again the length dependent term is negligible because the distances involved are less than 100 mm. The results of the Risk™ simulation are shown in figures 13 and 14. Also shown in these figures are the results obtained with the VCMM.

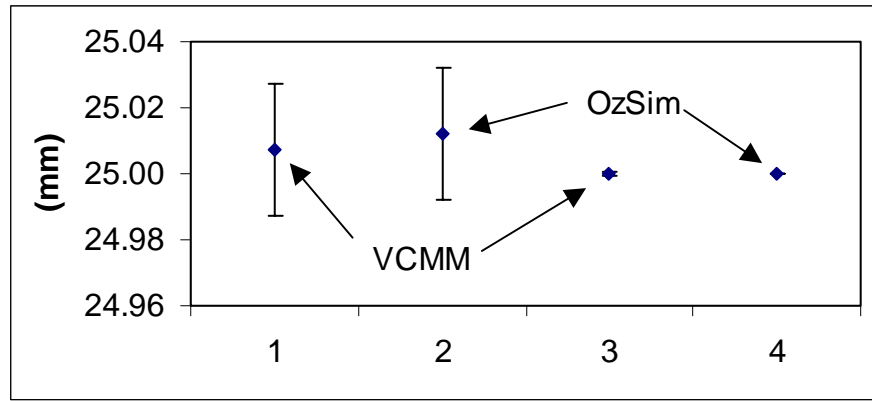


Figure 13: The diameter of the sphere. Cases 1 and 2 are the results when sampling strategy A are used, while cases 3 and 4 show the results with sampling strategy B.

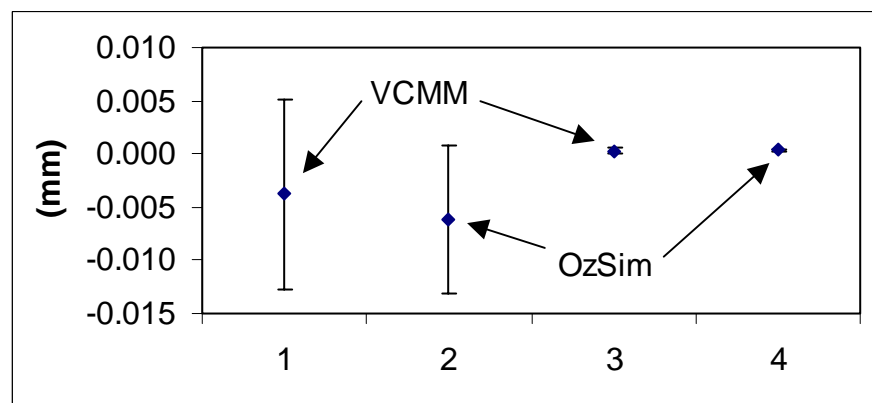


Figure 14: The z coordinate of the origin of the sphere. Cases 1 and 2 are the results when sampling strategy A are used, while cases 3 and 4 show the results with sampling strategy B.

The results again show that OzSim is producing similar mean values as calculated by the CMM which implies that the algorithm used in OzSim is correct. Also, the magnitudes of the uncertainties produced by OzSim are virtually the same as that obtained with the VCMM, giving confidence that the simulations are operating as they should.

### Information obtained from the uncertainty calculation

A number of conclusions can be drawn from the examples given for the simple artefacts of the sphere, circle and plane. Firstly, by direct comparison with the VCMM, it appears that the algorithms and simulator used by OzSim operate correctly. Further confidence of this is drawn by the way that the mean values and their uncertainties overlap for the two different sampling strategies for most of the measured geometric features (see figures 7, 8, 10, 11, 13 and 14). For example, the mean value of the sphere diameter measured with a poor sampling strategy shown in figure 13, differs by 12  $\mu\text{m}$  from the expected value, but this is within the 40  $\mu\text{m}$  uncertainty region calculated for that strategy. This is true for all the geometric features measured in the three examples except for the angles that the plane makes to the two axes (figures 10 and 11).

Figures 10 and 11 demonstrate one of the limitations of the simulation method. The method can calculate uncertainties produced by the many error sources arising from an imperfect CMM and its environment, however, it does not take into account the uncertainty contribution that arises from the workpiece itself. In our example of the measurement of the plane we used a plane that was not ideal (ie large form error). As a result the uncertainty we calculated for the measurement was too small because it does not include the form error. To rectify this the

total uncertainty should be calculated by assuming that the CMM uncertainty and the form uncertainty are independent, and adding them using the standard root-sum-square method.  
eg.

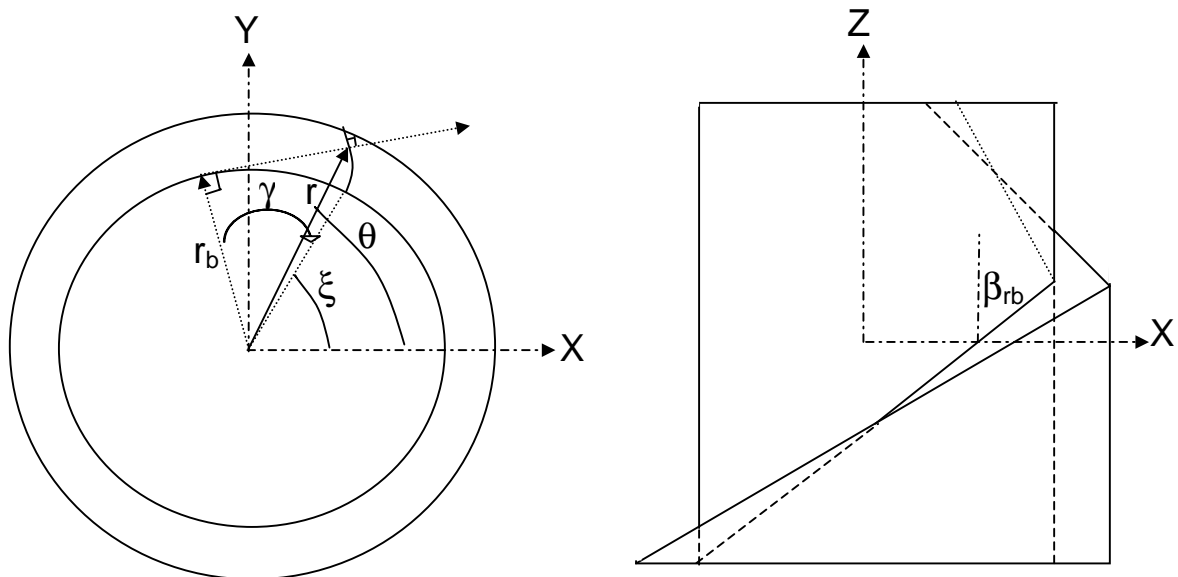
$$\Delta_{tot} = \sqrt{\Delta_{sim}^2 + \Delta_{form}^2}$$

Other uncertainties such as those associated with workpiece temperature should be included in evaluating the coordinate uncertainty (equation 1) and taken into the simulation.

The results from the three examples also clearly show the usefulness of calculating the measurement uncertainty. The operator of the CMM has a major influence on the output result by choosing the measurement strategy. In our examples a poor choice of strategy produced a poorer result, which is reflected by a larger uncertainty. As our examples are relatively simple, the results are more or less intuitive, as is the choice of the 'better' probing strategy. However, that is not always the case as will be seen in the next example of the helical gear. For complicated tasks the measurement uncertainty is a helpful tool in deciding upon the appropriate measurement strategy - one that achieves the desired accuracy level for the feature of interest using the minimum number of probing points and therefore time.

### **Calculating the uncertainty of a helical gear measured to a DIN standard**

A customer wanted the parameters and their uncertainties of a helical gear measured to a German DIN standard. The Quindos application used by our CMM has the necessary procedures to perform the measurement, however, the VCMM version that we possess is not able to calculate the uncertainties. This is because of the huge data manipulation that must be performed to transform the data in order to evaluate the parameters. The points on the involute helical gear must be either wound from the involute for the profile measurements, or from the base cylinder for the trace measurements. Both require intensive number crunching to do the task. Figure 15 shows two plane projections of a single tooth involute helical gear.



*Figure 15: The Plan View and approximate End Elevation of a single tooth involute helical gear. This example shows a right-handed involute and a right-handed helix.*

The surface of the involute helical gear is defined by:

$$\theta_{i,j} = \xi + i.(\gamma - \text{ArcTan}(\gamma)) + j.(z - i.j.r_p.\text{Sin}(\beta)).\frac{\text{Tan}(\beta_{rb})}{r_b}$$

where

$$\gamma = \sqrt{\frac{r^2}{r_b^2} - 1}, \quad \text{Tan}(\beta) = \frac{r.\text{Tan}(\beta_{rb})}{r_b}, \quad \text{and } r_p \text{ is the probe radius}$$

$$i = \begin{cases} 1 & \text{for a right-handed involute} \\ -1 & \text{for a left-handed involute} \end{cases}$$

$$j = \begin{cases} 1 & \text{for a right-handed helix} \\ -1 & \text{for a left-handed helix} \end{cases}$$

The German DIN standard for helical gears evaluates the departure of the measured points from the theoretical surface for tooth trace and tooth profile measurements. A tooth trace measurement attempts to trace along the length of a tooth (ie. vary  $z$ ) while keeping the distance from the gear axis a constant (ie.  $r = \text{constant}$ ). While in a profile measurement, the involute shape is of interest so  $r$  is varied while  $z$  is kept constant. For both types of measurements the departure from the ideal surface is characterized by parameters defined in the DIN standard. For the trace measurement the parameters are defined as shown in figure 16.

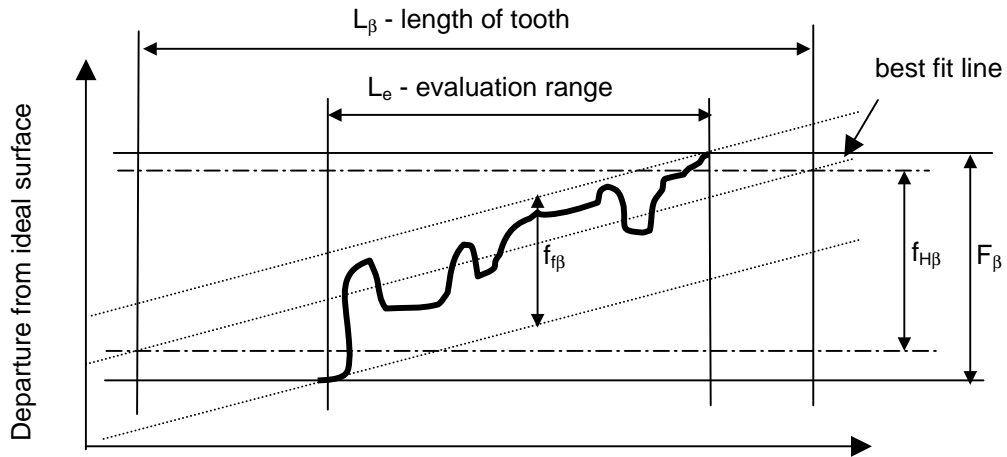


Figure 16: The definitions of some of the parameters in the DIN standard characterizing a trace measurement of a helical tooth.

These parameters are described as:

$F_\beta$  - is the total tooth trace error  
 $f_{H\beta}$  - is the tooth trace error  
 $f_{f\beta}$  - is the tooth longitudinal error

Another parameter, the helix angle error,  $f_\beta$ , is given by:

$$f_\beta \approx \frac{2 \cdot f_{H\beta}}{L_e} \cdot \cos^2(\beta)$$

The DIN parameters for profile measurements are:

$F_\alpha$  - is the total profile error  
 $f_{H\alpha}$  - is the profile angle error  
 $f_{f\alpha}$  - is the profile form error

These parameters are defined in exactly the same way as the trace parameters and are obtained by substituting  $\alpha$  for  $\beta$  in figure 16.

For both types of measurement the departure from the ideal surface is found by evaluating;

$$\Delta = r_{meas} \cdot \sin[i \cdot (\theta_{i,j} - \theta_{meas} + \text{ArcTan}(\gamma))] - [r_b \cdot \gamma + r_p \cdot \sin(\beta)]$$

The gear measured had seven teeth, each having a right and left handed involute surface. All teeth were right-handed helices. Table 1 and table 2 show the DIN trace and profile parameters, respectively, as calculated by the CMM and also those calculated by OzSim for one of the gear's teeth. The table also shows the uncertainties calculated by OzSim using the uncertainty expression given in equation 1.

DIN Parameter	Right Involute			Left Involute		
	CMM	OzSim	$U_{95\%}$	CMM	OzSim	$U_{95\%}$
$f_{H\beta}$	7.4	7.4	$\pm 1.0$	16.5	16.4	$\pm 1.4$
$F_\beta$	4.2	4.1	$\pm 1.0$	8.4	8.4	$\pm 1.4$
$f_{f\beta}$	1.8	1.8	$\pm 0.9$	0.9	1.0	$\pm 0.8$
$f_\beta$	0.6	0.6	$\pm 0.1$	1.3	1.3	$\pm 0.1$

Table 1: Shows the trace DIN parameters calculated for one of the gear's teeth. The evaluation range was 20 mm and the tooth face width is 40 mm. All units are  $\mu\text{m}$  except for  $f$  where the units are  $\mu\text{rad}$ .

DIN Parameter	Right Involute			Left Involute		
	CMM	OzSim	$U_{95\%}$	CMM	OzSim	$U_{95\%}$
$f_{H\alpha}$	-1.2	-1.1	$\pm 0.4$	-1.4	-1.1	$\pm 0.4$
$F_\alpha$	5.0	4.6	$\pm 1.1$	4.4	4.3	$\pm 1.0$
$f_{f\alpha}$	4.7	5.0	$\pm 1.0$	3.9	4.2	$\pm 1.2$

Table 2: Shows the profile DIN parameters calculated for one of the gear's teeth. The evaluation range was 1.5 mm and the involute width is 2 mm. All units are  $\mu\text{m}$ .

These two tables show that OzSim is giving results that are comparable to the CMM, implying that the algorithms used are correct. No such comparison is possible with the uncertainties as no alternative calculation exists. However, a number of measurements of the gear were performed and the standard deviation of the DIN parameters from their mean values were found to be approximately one third of the uncertainty ranges given in tables 1 and 2. This gives some confidence that the calculated uncertainties are at least in the ball park of what is expected.

## **Conclusion**

The OzSim simulation method for calculating CMM measurement uncertainty was described. OzSim uses commercially available software packages to derive the fitting algorithms and to perform the Monte-Carlo simulation. A simple expression is used for the uncertainty of the measured point coordinates. All CMM related error sources that contribute to the measurement uncertainty are combined into a single entity used by the simulation. The probability distribution used to simulate any point coordinate is expanded by a factor of 3 to ensure that it encompasses at least 95% of the total population of possible measured coordinates. This expansion is necessary because a value from a single CMM measurement is used as the mean value for the distribution that generates the simulated points, instead of a true measured mean.

The simplifying assumptions used in OzSim make the approach valid for tasks that have uncertainty requirements of 0.5  $\mu\text{m}$  or larger. The results from OzSim agreed well with those calculated by the CMM and the VCMM for various simple geometric surfaces. These examples also highlight the usefulness of measurement uncertainties to a CMM user in identifying suitable sampling strategies.

OzSim was used to calculate the parameters that characterize an involute helical gear according to the German DIN standard, and their uncertainties. The OzSim mean values were similar to those calculated by the CMM, indicating that the algorithm used to evaluate the parameters was correct. No comparison of the OzSim calculated uncertainties could be made as an alternative to calculate uncertainties was not available. However, the standard deviations of repeat measurements were found to be of the order of the calculated uncertainties giving confidence to the validity of the process.



## 5.4 Discussion on ball plate measurement at PTB

T. Takatsuji, NMIJ

On April 2001, Dr. Takatsuji visited PTB together with Dr. Osawa, who is a colleague in AIST, and had a discussion on the calibration of ball plates and hole-plates.

PTB has started calibration service of these gauges and therefore has a lot of experience and skills. Additionally in Germany traceability system of geometrical gauges has already been established.

The calibration of these gauges is performed in two steps. Firstly the geometrical pattern of the balls (holes) is measured using the CMM, and secondly the standard of the length is transferred using gauge blocks or a laser interferometer. Since the standard of the length is transferred in the second step, we don't have to be careful for the temperature condition in the first step.

In the first step, the geometrical pattern of the gauges is measured by means of the reversal method which is commonly used in many national metrological institutes (NMIs).

In the second step, the laser interferometer is an ideal tool to transfer the standard of the length, because the laser is closer to the standard of the length than the gauge blocks. Nevertheless PTB, AIST, and a few NMIs are making use of the laser interferometer. Both PTB and AIST are using interferometric CMMs. The largest difference between the two is the position of the mirror used in the interferometer. In PTB system a corner cube mirror is placed on the moving table, whereas in AIST system a plane mirror is fixed on the measuring probe. The AIST method is more ideal from the point of the view of metrology, since it has shorter measuring loop than the PTB's. On the contrary the AIST method demands finer laser alignment and the compensation of the inclination of the mirror.

Measurement results obtained using different method showed good agreement, and it will be shown in section g in detail.

## 5.5 Collaboration Report on Two New Hole-Plate Measurement

T. Takatsuji, NMIJ

The hole-plate is one of the most appropriate gauges used for characterizing the geometrical error of the CMM. The number of the measurement is minimized when a hole-plate of the same size of the measuring volume of the CMM is used.

NML and AIST own the same model CMMs made by Leitz Company. PTB made two identical hole-plate of 600 mm × 600 mm, which fit to the measuring volume of the CMMs. To minimize influence of the heat, these were made of low thermal expansion glass Zerodur.

The hole-plates were calibrated by PTB, and then by AIST again. Currently these are being calibrated by NML, and the result will be obtained shortly.

The results by PTB and AIST agree each other within almost 0.01 μm. This value is far smaller than the uncertainty of the calibration. This comparison verified the ability of the calibration of PTB and AIST. In addition, the influence of the different measurement system between PTB and AIST was proved to be negligible. The AIST system may achieve the best calibration; on the other hand the PTB system is more appropriate for daily calibration task.

### 1. Calibration Object

The calibration objects are two Zerodur hole-plates with 44 holes. The serial number of these objects are PTB5.32-01/01 and PTB5.32-01/02. The measurands are the coordinates of the hole centers. The coordinate system is defined by hole 1 ( $x = 0$ ,  $y = 0$ ), hole 12 ( $y = 0$ ) and the plane of the plate ( $z = 0$ ).

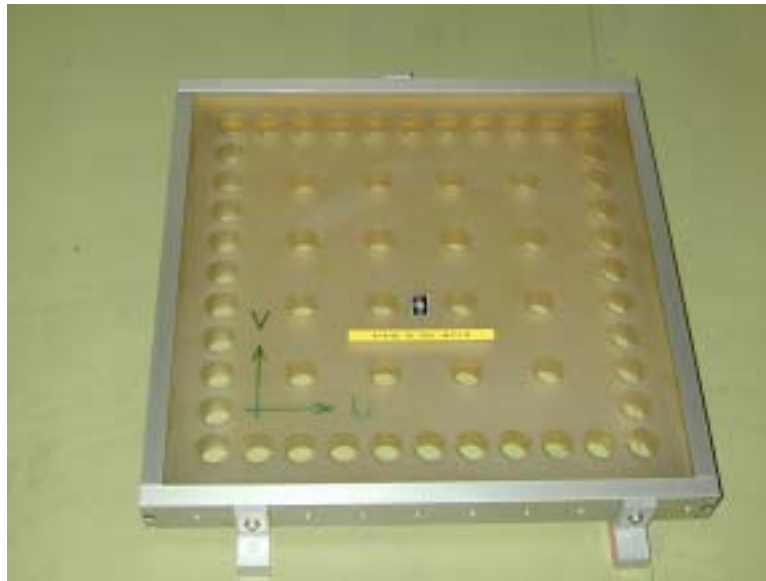


Fig. 1 Calibration Object

## 2. Calibration Method

The plates were measured in four orientations using a coordinate measuring machine (CMM). The CMM is a Leitz PMM12106. The coordinates of holes are measured by the CMM using swing round method. Reference to the unit of length was achieved by calibrating the center distances in the two hole rows 1-12 and 1-33 with interferometric measurements.

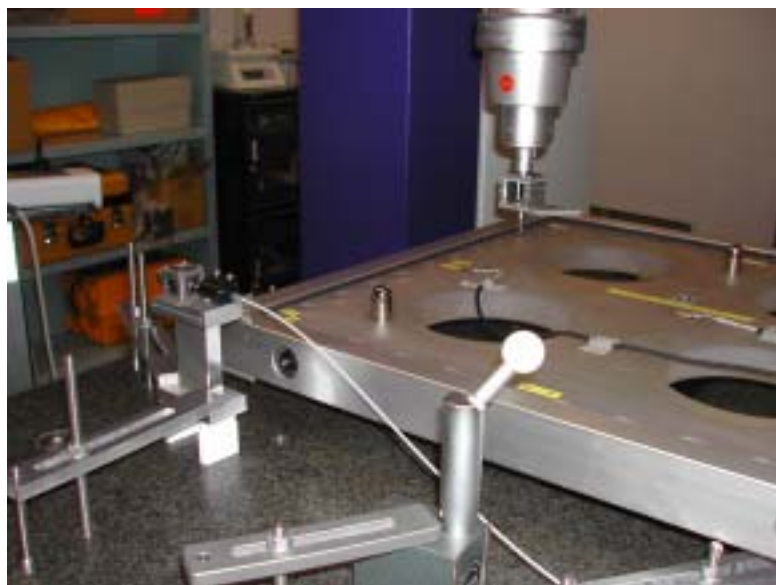


Fig. 2 Calibration by CMM

### 3. Measurement conditions

The temperatures during the reversal measurements were  $20\text{ }^{\circ}\text{C} \pm 0.4\text{ }^{\circ}\text{C}$  and during the interferometry measurements  $20\text{ }^{\circ}\text{C} \pm 0.2\text{ }^{\circ}\text{C}$ . The measurement results are valid for  $20\text{ }^{\circ}\text{C}$ . For corrections of the thermal expansion, we used  $\alpha_{\text{scale}} = 8.3\text{E-}6\text{ [K]}$  for scale and  $\alpha_{\text{hole-plate}} = 0.05\text{E-}6\text{ [K]}$  for hole-plate.

### 4. Measurement results

The measurement results are listed on page 2 and 3.

### 5. Measurement uncertainty

The expanded uncertainty for the coordinates of holes is:

$$U = 2 \times \sqrt{0.25^2 + (0.25L)^2} \quad [\mu\text{m}; \text{L:m}] \quad \text{coverage factor (k = 2)}$$

Table 1 Measurement results of **PTB 5.32-01/01**

Hole Number	X [mm]	Y [mm]
1	0.0000	0.0000
2	50.0014	-0.0003
3	100.0011	0.0012
4	150.0007	0.0022
5	200.0025	0.0018
6	250.0201	0.0105
7	300.0179	0.0123
8	350.0061	0.0036
9	400.0041	0.0015
10	450.0035	0.0039
11	500.0072	-0.0022
12	550.0043	0.0000
13	-0.0004	50.0027
14	550.0054	50.0019
15	-0.0008	100.0048
16	550.0074	100.0057
17	-0.0005	150.0078
18	550.0111	150.0076

19	-0.0003	200.0115
20	550.0113	200.0107
21	-0.0031	250.0136
22	550.0131	250.0139
23	-0.0150	300.0001
24	550.0010	299.9995
25	-0.0154	350.0021
26	550.0017	350.0017
27	-0.0167	400.0048
28	550.0042	400.0057
29	-0.0181	450.0100
30	550.0036	450.0092
31	-0.0183	500.0109
32	550.0034	500.0121
33	-0.0172	550.0151
34	49.9955	550.0096
35	99.9977	550.0116
36	150.0029	550.0126
37	200.0000	550.0157
38	250.0082	550.0141
39	300.0076	550.0119
40	350.0029	550.0188
41	400.0031	550.0183
42	450.0045	550.0176
43	500.0050	550.0158
44	550.0042	550.0133

Table 2      Measurement results of **PTB 5.32-01/02**

Hole Number	X [mm]	Y [mm]
1	0.0000	0.0000
2	49.9960	-0.0010
3	99.9961	0.0002
4	149.9991	0.0003

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5	200.0021	0.0021
6	250.0226	0.0088
7	300.0157	0.0087
8	350.0022	0.0007
9	400.0011	-0.0022
10	450.0002	-0.0015
11	500.0001	-0.0004
12	549.9997	0.0000
13	-0.0012	50.0017
14	550.0007	50.0037
15	0.0005	100.0042
16	550.0055	100.0061
17	-0.0010	150.0064
18	550.0075	150.0095
19	-0.0010	200.0057
20	550.0108	200.0107
21	0.0081	250.0111
22	550.0128	250.0145
23	-0.0103	299.9949
24	550.0098	299.9991
25	-0.0083	349.9937
26	550.0123	349.9987
27	-0.0094	399.9955
28	550.0143	400.0005
29	-0.0083	449.9996
30	550.0131	450.0027
31	-0.0089	499.9991
32	550.0157	500.0028
33	-0.0088	550.0007
34	49.9978	550.0039
35	100.0026	550.0067
36	150.0068	550.0067
37	200.0070	550.0079
38	250.0133	550.0072
39	300.0134	550.0019

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40	350.0156	550.0091
41	400.0173	550.0097
42	450.0180	550.0077
43	500.0203	550.0059
44	550.0222	550.0020

## 5.6 Collaboration Plan on Workpiece Measurements

T. Takatsuji, NMIJ

VCMM must be one of the best simulation methods, which takes many error sources into account in detail. It, however, requires demanding preparing measurements and therefore cannot be said to be very simple. Simpler simulation methods are easy to use, however these likely to deliver overestimating measurement uncertainty. An economical point of view should be considered on the selection of the methods.

In this project, Prof. Furutani made a simulation method using MatLab software and Dr. Jaatinen did using Visual Basic software. Although these two are simpler than VCMM, these still can be useful under specific conditions. To check the validity and usefulness of these methods, we planned an international comparison. Two sample workpieces (see Fig. 1) which are similar to actual industrial objects were made and circulated in members of the project. To observe the influence of the material, one was made of brass and other low thermal expansion steel. Each member measures them and assesses the measurement uncertainty. As a result, sample workpieces are measured using different CMMs and the measurement uncertainties are assessed by different simulation methods.

The workpieces were designed and the protocol (see Protocol of measuring the workpiece) of the international comparison was made by AIST. Currently the sample workpieces are being measured in NML and then will be circulated in participant laboratories.

This experiment will identify the uncertainty contributors which at least should be considered in the simulation methods, and the results will be reflected on the ISO standard.



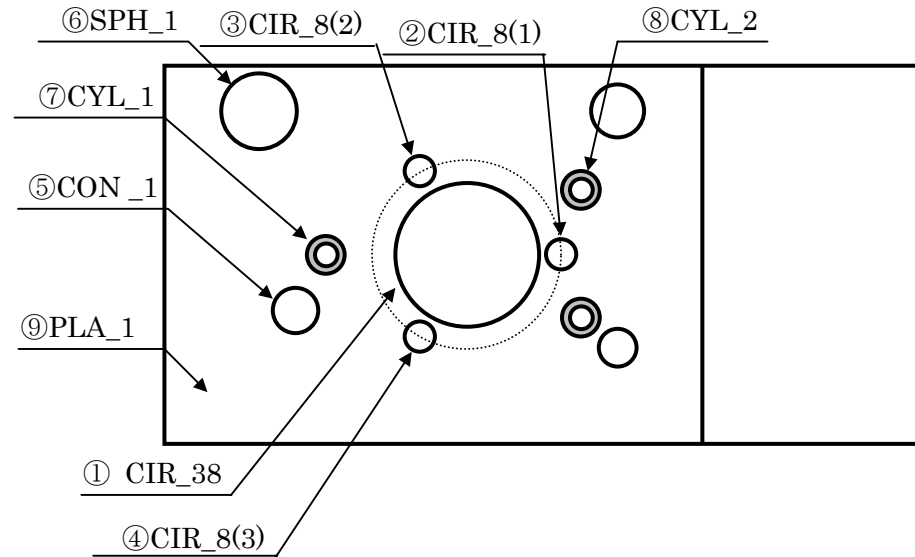
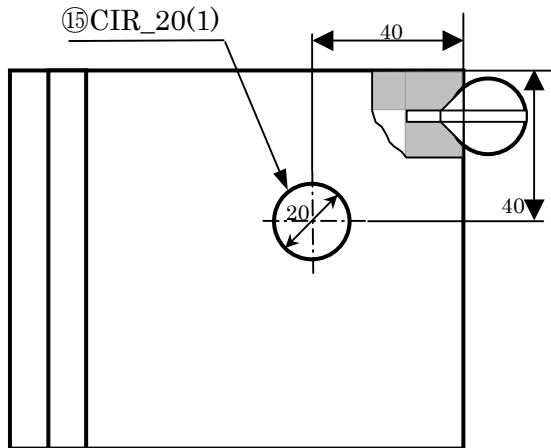
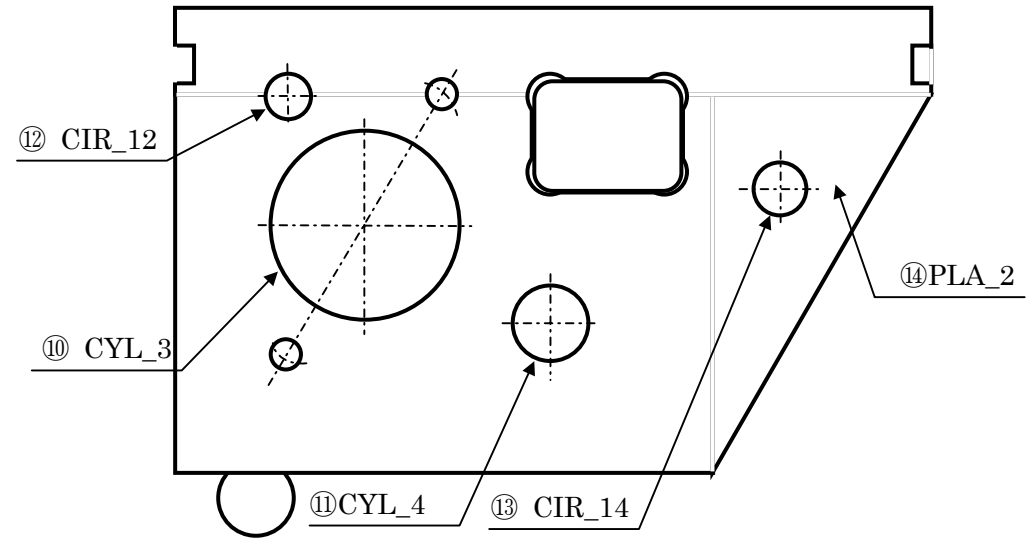


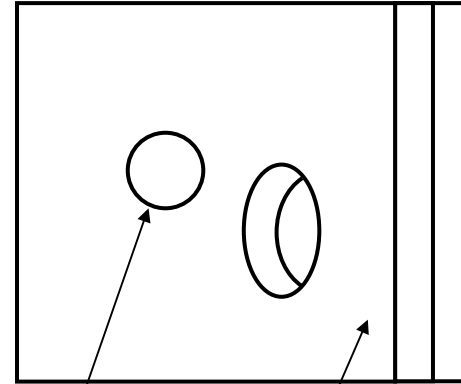
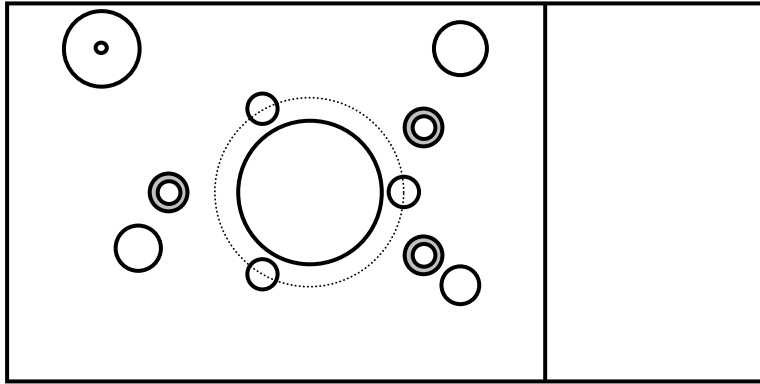
Fig. 1 Workpiece for Workpiece Measurement



Fig. 2 Measurement of Workpiece by CMM

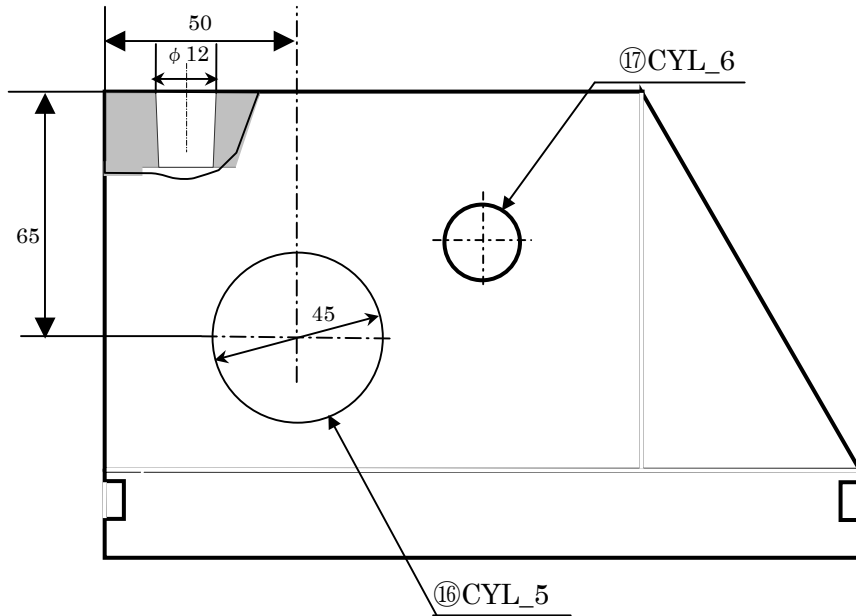
Measurement Plane Number	Object	Form	Object Name
1	1	Circle	CIR_38
1	2	Circle	CIR_8(1)
1	3	Circle	CIR_8(2)
1	4	Circle	CIR_8(3)
1	5	Cone	CON_1
1	6	Sphere	SPH_1
1	7	Cylinder	CYL_1
1	8	Cylinder	CYL_2
1	9	Plane	PLA_1





⑱CIR\_20(2)

⑲PLA\_5



Measurement Plane Number	Object	Form	Object Name
2	10	Cylinder	CYL_3
2	11	Cylinder	CYL_4
2	12	Circle	CIR_12
2	13	Circle	CIR_14
2	14	Plane	PLA_2
3	15	Circle	CIR_20(1)
4	16	Cylinder	CYL_5
4	17	Cylinder	CYL_6
5	18	Circle	CIR20_(2)
5	19	Plane	PLA_5

## Protocol of measuring the workpiece

22/ Mar. /'02 AIST

### ---Workpiece information---

Thermal expansion coefficient:

Steel (Lex 5)       $0.5\text{E-}6$  (/K)

Brass               $18.18\text{E-}6$  (/K)

### ---Probe configuration---

Use Star Probe system (5 styli)

( Diameter =3 mm, Length =40 mm)

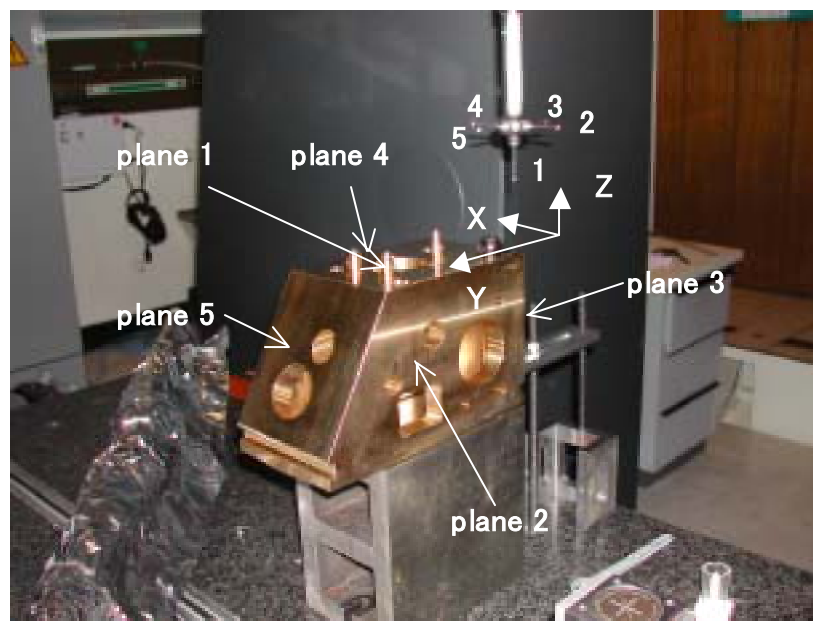


Fig. 1 Workpiece and probe configuration

### ---Plane Number---

Plane Number are shown in fig. 1

### ---Stylus Number information---

Stylus Number 1 is used of the measurement of the plane 1

Stylus Number 2 is used of the measurement of the plane 4

Stylus Number 3 is used of the measurement of the plane 5

Stylus Number 4 is used of the measurement of the plane 2

Stylus Number 5 is used of the measurement of the plane 3

### ---BUILD COORDINATE SYSTEM---

#### \*\*\* First orientation \*\*\*

Measure three points on the plane 1 (stylus 1)

The surface normal vector is used for spatial orientation

Measure two points on the plane 3 (stylus 1 \*Don't mistake direction)

The axis vector is used for planer orientation

Measure one point on the plane 2 (stylus 1)

Element for Spatial alignment, axis of spatial alignment Z

Element for Planar alignment, axis of planar alignment X

Intersection of the plane and the axis and the point is origin of the direction of the coordinate

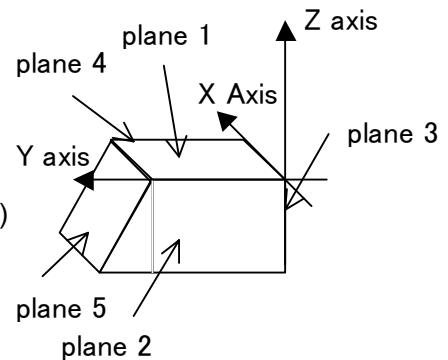


Fig.2 coordinate system

#### \*\*\* Second orientation (automatically)\*\*\*

Measure six or more points on the plane 1 (stylus 1)

The surface normal vector is used for spatial orientation (show fig.2)

Measure six or more points on the plane 2 (stylus 4)

The surface normal vector is used for planer orientation (show fig.2)

Measure six or more points on the plane (stylus 5)

Element for Spatial alignment, axis of Spatial alignment Z

Element for Planar alignment, axis of Planar alignment -X

Intersection of the three planes is origin of the direction of the coordinate

○Measurement : (for plane 1; stylus 1)

1. Measurement circle CIR\_38

Measure 8 points in  $Z = -5 \text{ mm}$  plane

Evaluate center position X, Y, Diameter, Roundness

2. Measurement circle CIR\_8(1)~(3)

Measure 8 points in  $Z = -5 \text{ mm}$  plane

Create a circle (PCD\_50) using CIR\_8(1)~(3)

Evaluate concentricity between CIR\_38 and PCD\_50

3. Measurement cone CON\_1

Measure 16 points in  $Z = -3 \sim -13 \text{ mm}$  plane

Evaluation parameters are position X, Y, Z and position of the apex of the Cone,  
angle of the axis with respect to the Z axis

4. Measurement sphere SPH\_1

\*Measure sphere SPH\_1 in 5 points and evaluate diameter, center position X, Y, Z

\*Measure sphere SPH\_1 in 25 points and evaluate sphericity

5. Measurement cylinder CYL\_1, CYL\_2

Measure cylinder CYL\_1 (Z=12.5, 17.5), CYL\_2 (Z=12, 14.5) in two planes.

For each plane, measure 8 points.

so that 16 points are measured for CYL\_1, CYL\_2.

Evaluate cylindricity, diameter, X, Y, Z

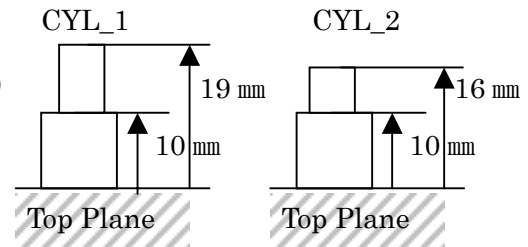


Fig. 3 Cylinder information

6. Measurement plane PLA\_1

Measure 16 points

transform RE\_PLA\_TOP to PLA\_1 (for QUINDOS user)

(for plane 2; stylus 4)

7. Measurement cylinder CYL\_3, CYL\_4

Measure cylinder CYL\_3 (X=2, 10), CYL\_4 (X=2, 10) in two planes

For each plane, measure 8 points, so that 16 points are measured for CYL\_3, CYL\_4.

Evaluate cylindricity, diameter, X, Y, Z

8. Measurement circle CIR\_12, CIR\_14

Measure 8 points in x=5 mm plane

Evaluate center position Y, Z

Evaluate distance between the center position of CIR\_12 and the center position of CIR\_14

9. Measurement plane PLA\_2

Measure 16 points

transform RE\_PLA\_MX to PLA\_2 (for QUINDOS user)

Evaluate angle between PLA\_1 and PLA\_2

(for plane 4; stylus 2)

10. Measurement cylinder CYL\_5

Measure cylinder CYL\_5 (X=98, 88) in two planes

For each plane, measure 8 points, so that 16 points are measured for CYL\_5.

Evaluate cylindricity, diameter, X, Y, Z

Evaluate concentricity between CYL\_3 and CYL\_5

(for plane 5; stylus 3)

11. Measurement plane PLA\_5

Measure 16 points on plane 5.

Estimate flatness

Estimate angle between PLA\_1 and Pla\_5

12. Measurement circle CIR\_20(2)

Measure eight points in Y=156 mm plane

Evaluate center position X, Z

(for plane 3; stylus 5)

13. Measurement circle CIR\_20(1)

Measure eight points in Y=5 mm plane

Evaluate center position X, Z

Evaluate concentricity between CIR\_20(1) and CIR\_20(2)

Table 1 Evaluation components

	Element	Evaluation	Num. of evaluation
1	CIR_38	center position, diameter, roundness	3
2	CIR8_(1)~(3)	concentricity (CIR_38, PCD_50)	1
3	CON_1	position, angle of axis	2
4	SPH1	center position, sphericity	2
5	CYL_1	position, cylindricity, diameter	3
6	CYL_2	position, cylindricity, diameter	3
7	PLA_1	-	0
8	CYL_3	position, cylindricity, diameter	3
9	CYL_4	position, cylindricity, diameter	3
10	CIR_12	center position	1
11	CIR_14	center position, distance (CIR_12, CIR_14)	2
12	PLA_2	angle (PLA_1, PLA_2)	1
13	CYL_5	position, cylindricity, diameter, concentricity (CYL_3,CYL_5)	4
14	PLA_5	flatness, angle (PLA_1, PLA_5)	2
15	CIR_20(2)	center position	1
16	CIR_20(1)	center position, concentricity (CIR_20(1), CIR_20(2))	2
			total 33



## 5.7 Activities on ISO/TC213/WG10

Ryoshu Furutani, Tokyo Denki University

### 1 Background of ISO/TC213

The homepage of ISO/TC213 says the scope and the task.

#### 1.1 Scope

Standardization in the field of geometrical product specifications (GPS) i.e. macro- and microgeometry specifications covering dimensional and geometrical tolerancing, surface properties and the related verification principles, measuring equipment and calibration requirements including the uncertainty of dimensional and geometrical measurements. The standardization includes the basic layout and explanation of drawing indications (symbols).

#### 1.2 Task of ISO/TC213

Dimensioning and tolerancing leave a lot to be desired! I.e. the technical drawings are not unambiguous. Experience shows that the average costs involved amount to as much as 20% of the production turnover.

The reason is not that the designer does not know what he or she wants, or that the workshop does not know how to comply with the drawing. The reason is lack of effective communication, resulting in misunderstanding from idea to the real thing. The poor communication arises from the fact that the parties do not know the "grammar" of the drawing and in particular from the fact that the available standards have not adequately kept pace with development.

The technical drawing serves its purpose only if it is unique and results in the production of one single type of identical products with one single type of functional characteristics. The designer is responsible for ensuring that the technical drawing is unambiguous. The standardizers are responsible for ensuring that the designer has available to him the proper tools - namely the standards.

There is an apparent need for improving the communication between the designer and the workshop. Several elements of the communication can be improved:

1. The human understanding and knowledge of the symbol language used on the drawing which expresses the functional characteristics of the workpiece by means of geometry.
2. Drawings shall be made more precise and unique which means that they shall specify all requirements that are essential for the function.
3. A complete, highly developed, systematic and standardized "language" is needed to express and translate the function of the workpiece into geometrical requirements on the drawing.

Other geometrical characteristics, such as form, orientation, location and run-out and the macro and micro geometrical form characteristics of surfaces, are quite another story. These characteristics cannot be controlled during the process as they usually depend on parameters which cannot be controlled during the process. It is often the choice and order of the process and the material which have a decisive influence. Today the resulting deviations are relatively larger than the dimensional deviations, the difference being greater than before - the deviations are in fact so considerable that they obstruct the function of the workpiece as well as dimensioning and tolerancing. Deviations of form must consequently be limited to a higher degree than before by means of tolerances to ensure correct function of the workpiece and the relevance to dimensioning and tolerancing.

The situation is that a marked shift has taken place between dimensional deviations and deviations of form, orientation, location and run-out. The problem is that the way of drawing and tolerancing has not changed. The ISO dimensioning and tolerancing system is based on the old ISA system which dates back to the 1940s, the ISO roughness system is from the 1950s. And because the ISO dimensioning and tolerancing system works only on theoretically correct workpieces - and not if deviations of form occur - the result is a drawing which does specify sufficiently precisely what is required of the geometry to obtain the desired function. As a consequence, far too much is left to the (random) decisions of other departments within the organization than the design/engineering department.

It is therefore perfectly permissible to maintain the following:

- a) It is not the designer who decides the end function of the workpiece;
- b) It is the measuring and gauging people who decide the end function of the

workpiece by their (random) choice of measuring methods and equipment or the function is decided by the software integrated into the equipment;

- c) Where coincidence reigns, quality cannot be controlled;
- d) Subcontractors are at a loss.

### 1.3 Structure of ISO/TC213

ISO/TC213 had 11 Advisory Groups and 14 Working Groups.

Some AG and WG had completed their missions and disbanded as shown in Table 1 and Figure 1.

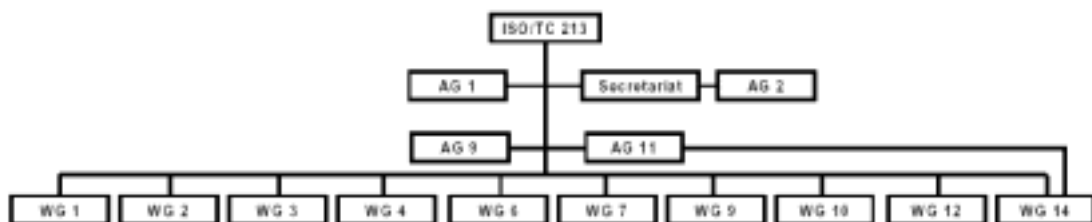


Figure 1 Structure of ISO/TC213

Table 1 Advisory Groups and Working Groups of ISO/TC213

Advisory Groups	Working Groups
AG 1 Strategic planning	WG 1 Roundness, Cylindricity, Straightness, Flatness
AG 2 Final auditing standards team	WG 2 Datums and datums systems
AG 3 Disbanded, transferred into WG 14	WG 3 Reference temperature
AG 4 Disbanded, transferred into WG 13	WG 4 Uncertainty
AG 5 Disbanded, work postponed	WG 5 Disbanded, task completed
AG 6 Disbanded, transferred into WG 14	WG 6 General requirements for GPS-measuring equipment
AG 7 Disbanded, transferred into WG 12	WG 7 ISO/DIS 2692
AG 8 Disbanded, transferred into AG 2	WG 8 Disbanded, task completed
AG 9 GPS-extraction techniques	WG 9 Dimensional and geometrical tolerancing for castings
AG 10 Disbanded, transferred into WG 14	WG 10 Coordinate measuring machines
AG 11 Underlying global concepts	WG 11 Disbanded, transferred into WG 6
	WG 12 Size
	WG 13 Disbanded
	WG 14 Vertical GPS principles

## 2 Status

ISO/TC213/WG10 have two major tasks.

- 1) ISO 10360-series: Acceptance and reverification tests for Coordinate Measuring Machines
- 2) 2)ISO 15530-series: Techniques of Determining the Uncertainty of Measurement in Coordinate Metrology

The whole ISO 10360-Series were issued in 1999 and 2000.

The parts of ISO 15530 are

15530-1: Terms

15530-2: Expert Judgment

15530-3: Substitution Method

15330-4: Simulation Method

15330-5: Historical Estimation

15530-6: Estimation using Uncalibrated objects.

The 15530-3 will be issued as Technical Specifications as soon as Dr. Schwenke will give it some examples.

The 15530-4 and -6 are now on the status of working draft. The project leader of the task force for ISO 15530-4 is Dr. Shakarji(NIST) and the members of the task force are Dr. Schwenke(PTB) and Prof. Takamasu(UT). We have had the task force meeting every WG10 and annual NEDO meeting in Tsukuba and Sydney since 1999. The concept of ISO 15530-4 have been presented every WG10 meeting since 1999. So, All experts from member bodies are familiar with the concept of simulation method. Most of them are considering ISO 15530-4 is similar test procedure to ISO 10360-6.

The task force prepared following documents and had discussed,

WG10N409 in Milan, in September 2000

WG10N424 in Bordeaux, in January 2001

WG10N454 in Madrid, in February 2002

N37-Annex-1(Our documents), in February 2002

Through the discussion of the task force meeting,

- Test 1 and Test 2 were clearly distinguished.
- How to evaluate the simulation software, Test 1, is physical test.

- We will plan the developing Test 1 first.

### 3 Future

WG10 will have the meeting in September,2002 in Ottawa and in January,2003 in Mexico.

In both meeting, ISO 15530-4 will be presented according to the resolution in NEDO annual meeting in Sydney. We will continue to support Dr.Shakarji and make ISO 15530-4 be issued as soon as possible.

#### Attached Documents:

- ISO/TC 213/WG 10 N 409, Revision: September, 2000  
Working Draft International Standard ISO/ 15530-4  
Geometrical Product Specifications (GPS) - Techniques of Determining the Uncertainty of Measurement in Coordinate Metrology - Part 4: Estimating Task-Specific Measurement Uncertainty Using Simulation
- ISO/TC 213/WG 10 N 424, Revision: January 2001  
Working Draft International Standard ISO/ 15530-4  
Geometrical Product Specifications (GPS) - Techniques of Determining the Uncertainty of Measurement in Coordinate Metrology - Part 4: Estimating Task-Specific Measurement Uncertainty Using Simulation
- ISO/TC 213/WG 10 N 454, Revision: September 2001  
Working Draft ISO/WD 15530-4  
Geometrical Product Specifications (GPS) - Techniques of Determining the Uncertainty of Measurement in Coordinate Metrology - Part 4: Estimating Task-Specific Measurement Uncertainty Using Simulation
- Some Conclusions at the NEDO Meeting in Sydney (1-Mar.-2002)

Revision: September, 2000

Note: This working draft is in an early form.

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**Working Draft International Standard****ISO/ 15530-4**

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**Geometrical Product Specifications (GPS)  
- Techniques of Determining the Uncertainty of  
Measurement in Coordinate Metrology -  
Part 4: Estimating Task-Specific Measurement Uncertainty  
Using Simulation**

**Preliminary remark**

For coordinate measuring machines (CMMs) used to control tolerances, the task-specific uncertainties of measurement according to ISO 14253-1 must be taken into account when tests for conformity/non-conformity are carried out. Knowledge of the uncertainty of measurement thus is of utmost importance. Up to the present, there have been only a few procedures which allow the task-specific uncertainty of measurement to be stated.

For simple measuring devices, this uncertainty can be estimated by an uncertainty budget according to the recommendations of the Guide to the Expression of Uncertainty in Measurement (GUM). However, in the case of CMM, the formulation of a classical uncertainty budget is, impossible for the majority of the measurement tasks due to the complexity of the measuring process.

Alternate methods can be used to determine the task-specific uncertainty of coordinate measurements. One of them, which estimates the uncertainty by numerical simulation of the measuring process allowing for uncertainty influences is described in the present standard.

**1. Scope**

It is the objective of the present standard to describe general conditions for the uncertainty determination by simulation for specific measurement tasks carried out on CMMs, taking into account the measuring device, the environment, the measurement strategy and the object. The standard is to unify the general procedure without restricting the possibilities of the technical realization. A procedure for verification and evaluation of the simulation package is included. Measures are recommended which increase the procedure's transparency for the user, and methods described which the user may apply to monitor it.

The sheet is not aimed at defining new parameters for the general evaluation of the accuracy of measurement. The functional capability of the CMMs is taken for granted, agreement with

specifications is checked on the basis of sheets 1 - 6 of VDI 2617 and of the ISO series of standards 10360.

## **2. Procedure**

### **2.1 Principle**

The functional principle is based on a computer-aided mathematical model of the measuring process. In this model, the measuring process is represented from the measurand to the measurement result, taking important influence quantities into account. In the simulation, these influences are varied within their possible or assumed range of values (described by probability distributions), and the measuring process is repeatedly simulated, using combinations of the spectrum of all states considered possible. The uncertainty is determined from the variation of the final result. This procedure is compatible with the fundamental principles of the internationally valid Guide to the Expression of Uncertainty in Measurement (GUM).

### **2.2 Elements of the procedure**

#### **2.2.1 Uncertainty contributors**

The measuring process on a CMM is subject to a great number of uncertainty contributors that affect the measurement result. The following quantities (not complete) may contribute to the uncertainty of the measurement result when measurements are carried out on a CMM:

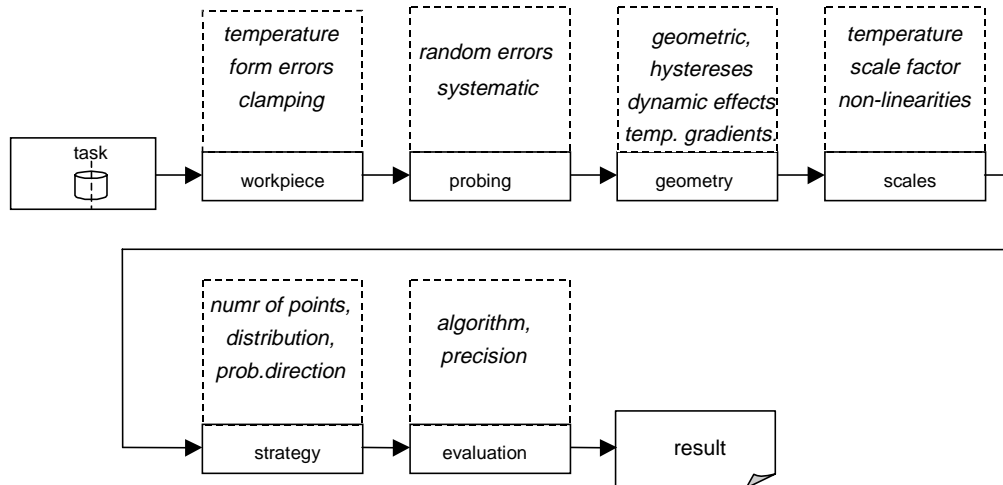
- Geometry                      • Hysteresis                      • Long-term variation                      • Clamping
- Temperature    • Dynamics                      • Specimen                      • ....
- Contacting system    • Elasticity                      • Dirt

When the uncertainty of a measuring process is evaluated, these influence quantities must be recognised and their effect on the measurement result described by a model.

#### **2.2.2 Model**

The model of the measuring process describes the mathematical relationship between the input quantities (measurand and influence quantities) and the output measurement result. The simulation procedure does not require that the model be described by a closed mathematical expression. Numerical algorithms, such as the calculation of derived features or filtering of measurement points can, therefore, be included in the model. This makes the simulation procedure particularly suitable for complex measuring processes like coordinate measurements. The model of the measurement on a CMM can be described by a signal flow chart, in which the quantities influencing the measuring process are plotted. Figure 1 shows such a signal flow chart.





**Figure 1: Measurement on a coordinate measuring machine represented in the form of a signal flow chart**

### 2.2.3 Simulator

With the aid of the simulator, all influence quantities are varied within their assumed probability distribution and numerical calculation of the measurement result is repeatedly carried out. The random values are generated in the assumed probability distributions by random number generators which are to furnish approximately uncorrelated sequences of numbers.

### 2.2.4 Evaluation

By evaluation, the uncertainty of measurement is determined from the simulated measurement results. The result of the evaluation of the simulation should be a statement in the form  $\pm U$  which describes the uncertainty range of the measurement result with a specified confidence level (cf. also GUM).

## 2.3 Realisation

The simulation can be integrated into a control and evaluation software of a CMM (on-line) or implemented as an independent system on an external computer (off-line). Both variants are covered by this standard.

## 2.4 Limits of the procedure

The input quantities are estimated on the basis of best knowledge. The input quantities and their uncertainties must, therefore, be well known. Usually, not all possible uncertainty influences are taken into account in the model. Influence quantities which have not been considered are to be estimated by other procedures and added to the total uncertainty.

### 3 Requirements and information to be provided by the manufacturer

#### 3.1 Uncertainty contributors

The manufacturer of the simulation software must explicitly indicate which uncertainty contributors (cf. 2.2.1) have been taken into account in his software and which information is required from the user. In particular, the manufacturer should specify by means of this checklist, which uncertainty contributors the software claims to take into account:

##### Influence Factors

CMM Types : ☐ moving bridge      ☐ moving table      ☐ horizontal arm   ☐ gantry   etc...  
☐ Rotary Tables

CMM Errors: ☐ static machine geometry errors    ☐ dynamic machine geometry errors  
☐ part loading effects                      ☐ Non 20 C temperature      ☐ spatial gradients  
☐ temporal gradients                      ☐ algorithm software accuracy  
☐ hysteresis

Probe Types: ☐ contact touch trigger                      ☐ contact analog                      ☐ non-contact optical

Probe Errors: ☐ probe response (lobing)      ☐ multiple styli                      ☐ multiple probe  
☐ articulating head                      ☐ styli/probe changing                      ☐ scanning

Nominal Parts: ☐ circles      ☐ planes      ☐ sphere      ☐ cylinder      ☐ splines      etc.

GD&T: ☐ datum ref frames   ☐ form                      ☐ size                      ☐ location      ☐ orientation

Real Part Effects:      ☐ surface roughness   ☐ waviness      ☐ form error  
                                 ☐ non 20 C temp      ☐ spatial gradients      ☐ temporal gradients  
                                 ☐ contamination      ☐ fixturing

Operator Effects:      ☐ sampling strategies, i.e. the number and location of points in the  
                                 workpiece coordinate system  
                                 ☐ workpiece position and orientation in machine coordinate system  
                                 ☐ other operator effects...

The manufacturer shall state what measurements or other quantities are needed to characterize the CMM and its environment in order for the software to produce uncertainty statements (3.7).

The following contributors must at least be taken into account:

- The geometrical deviations of the CMM
- Deviations of the contacting system
- Influences of temporal and spatial temperature gradients on specimen and CMM

[Note: The details of which items should be on this list are yet to be determined]

#### 3.2 Model

The model of the measuring process must take the most important principles of coordinate measurement into account. Processing of the coordinates of the measurement points and evaluation of the geometrical quantities are to be modelled as integral parts of the measuring process. The result must be a task-specific statement of the uncertainty for the final result of a measurement.

The essential features of the model must be documented. Transparency of the model increases the user's confidence in the statement of the uncertainty. Documentation of model and procedure should be sufficient to enable the user to furnish proof of a statement of uncertainty in compliance with the standards. This is important in particular in connection with the requirements of ISO 9000 foll. requiring the documentation of the procedure used for the uncertainty determination.

### **3.3 Simulator**

The manufacturer shall describe how the influence quantities are varied. As a rule, the probability distribution should be documented.

### **3.4 Statistical evaluation**

The manufacturer must document how the uncertainties are derived from the simulated samples.

### **3.5 Statement of the uncertainty**

It must be ensured that the statement of the uncertainty complies with the internationally valid principles of the expression of the uncertainty (GUM). This includes the statement of a confidence level or a coverage factor. The combined standard uncertainty may be indicated in addition to the expanded uncertainty.

### 3.6 Operating conditions

Each factor “checked” in the declarations section (3.1) implies that the simulation software will appropriately address this issue over a specified range of conditions. (For example, “non-standard temperature” might be defined as homogenous temperature in space and time, within the limits of 15 °C to 30 °C.) The software manufacturer shall specify these ranges defining the spectrum of measurement tasks and the environmental conditions for which the simulation is valid. Examples of these ranges to be specified includes but is not limited to:

- Permissible part spectrum (e.g. exclusion of flexible sheet-metal parts, a minimum arc length for circles, maximum cone apex angles, etc.)
- Permissible task spectrum (e.g. exclusion of scanning or form measurement)
- Permissible temperature range
- Permissible temporal temperature gradients  $dT/dt$
- Permissible spatial temperature gradients  $dT/dx$
- Other permissible environmental conditions

Within the scope of these restrictions, computer-aided verification and evaluation can be performed, as well as the user checks, both described in section 5.

### 3.7 Input Quantities and How They Are Obtained

Along with the declarations described in section (3.1), the simulation software manufacturer must specify in detail (or reference appropriate documents) what input quantities are required to characterise the measurement system and how these quantities are obtained. These are the values that are used by the simulation software to characterize the CMM, the environment, operator effects etc. (Operator effects might be assessed from gauge repeatability and reproducibility studies i.e., GR&R; analysis of variance, i.e. ANOVA; and from expert judgement, i.e. "type B estimation").

## 4 Determination of the task-specific uncertainty of measurement

The parameters of the simulated measurement which are important from the metrological point of view should be as similar as possible to those of the real measurement. The standard uncertainty of a measurement result  $y$  is composed of

- the uncertainty  $u_{sim}$  determined by the simulation, and
- the uncertainties  $u_i$  from the influence quantities which have not been taken into account in the simulation and have been manually estimated.

The combined standard uncertainty is then calculated with

$$u_{ges} = \sqrt{u_{sim}^2 + \sum u_i^2} . \quad (1)$$

With the aid of coverage factors, this standard uncertainty can be brought to the desired confidence level. As a rule, the following is valid

$$U_{ges} = 2 \cdot u_{ges} \quad (2)$$

for a confidence level of 95%. If the uncertainty stated by the simulation already is an expanded uncertainty  $U_{sim}$ , the simulated uncertainty  $u_{sim}$  is to be calculated by division with the respective coverage factor.

## **5 Verification and Evaluation of Simulation Software**

### **5.1 Principle**

The verification and evaluation of simulation software consists of two parts: A computer-aided verification and evaluation procedure and a test using a physical object covering the whole system composed of CMM, evaluation software and simulation software. The first is conducted only once on a specific software release, while the latter is conducted multiple times on each software/CMM combination.

### **5.2 Computer-Aided Verification and Evaluation (CVE)**

This procedure uses computer simulation to verify and evaluate the simulation software. The concept is to simulate a measuring instance, based on the claims in the declarations section (3.1). Since the measuring instance is simulated and thus fully known in the CVE process, the error of the simulated measurement can be found. The software under test produces an uncertainty statement for this measurement and a simple comparison can determine if the error of the simulated measurement was contained in the uncertainty region reported by the software under test. This procedure can be repeated hundreds of times with varying conditions and statistics can be determined regarding how often the errors of measurement were contained within their corresponding reported uncertainty ranges.

#### **5.2.1 Creation of a Simulated Measuring Instance used in CVE**

When a CMM probes, there is generally a difference between the target contact point and the measured point, this difference being a vector. So a simulated measuring instance can be created by defining a vector field over the measuring volume of the CMM. That is, associated with each point in the CMM's measuring volume is a vector that represents the difference between the target contact point and the simulated measured point.

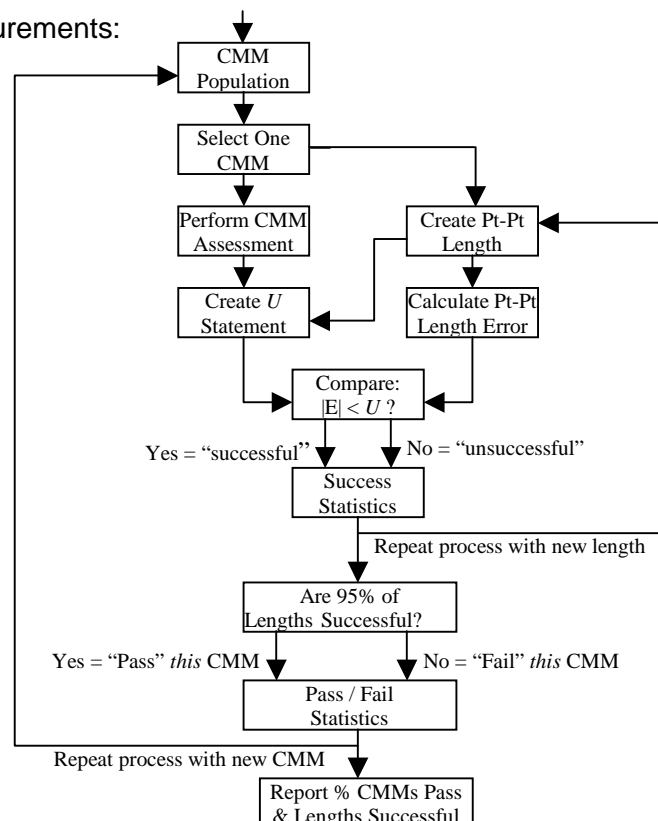
For the purposes of CVE, the declarations section (3.1) determines what influences the definition of the vector field. For instance, if hysteresis is claimed in section (3.1) then the vector associated with a particular point on one query would, in general, be different than on a subsequent query. If part form errors are included in the declarations section (3.1) then the vector associated with a particular point would depend on the placement of the part in the measuring volume. This allows for testing the software's reported uncertainties without combining them with other uncertainties.

### 5.2.2 Creation of Input Quantities

The declarations section includes the indication of the input quantities required by the software (3.1, 3.7). These input quantities might arise from probes of special calibrated workpieces. Appropriate input quantities can be obtained as follows: A simulated measuring instance can be created in accordance with the specifications of the manner in which input quantities are obtained (3.1, 3.7). Note, these conditions might be different than the ranges given in the checklist (e.g. the input quantities might be measured close to 20°C, while the software allows for measurements over a wider temperature range.) The target contact points used in the procedure to determine input quantities are provided to the testing body and the testing body returns the corresponding measured contact points from the simulated measuring instance. The software under test must be able to exchange this information.

### 5.2.3 CVE Testing

The CVE testing proceeds as shown in the following diagram given as an illustrative example using point-to-point length measurements:



#### **5.2.4 Reporting CVE Results**

CVE results consist of the following information:

- The percent of time true value lies in uncertainty interval; e.g. for "good" software the threshold might be 95%.
- The average amount of over-estimation of uncertainty, i.e. when the true value is contained within the uncertainty interval, on average how far is it from the nearest uncertainty interval limit.
- The average amount of under-estimation of uncertainty, i.e. when the true value is outside the uncertainty interval, on average how far is it from the nearest uncertainty interval limit.

### **5.3 Verification Using a Calibrated Workpiece**

In addition to the CVE, this procedure provides verification for a specific task for a specific software/CMM combination. Here, the statement of the uncertainty is checked by a test covering the whole system composed of CMM, evaluation software, and simulation software. The test is based on real measurements performed on calibrated objects. Each object permitted according to section 3.5 may be used. The object must have been calibrated by an independent procedure. In the Annex, an object and a procedure are given as an example, and show a number of measurement tasks to be simulated and which can also be calibrated with sufficient accuracy by independent procedures. For the measurement of such an object it is recommended to vary also the measurement strategy (position and orientation of the test object, distribution of measurement points) in order to check the influence on the measurement uncertainty stated.

The following test objects may also be used: gauge blocks, step gauges, ball plates and other standards. However any specific object is only suited to a limited extent to test the statements of task-specific uncertainty.

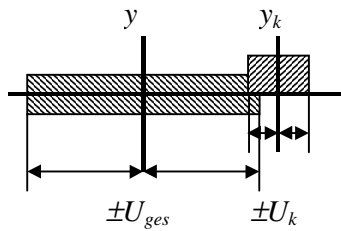
#### **5.3.1 Procedure**

The measurements on the calibrated test objects are carried out on the real CMM for which the uncertainty of measurement is to be determined. The real measurement results  $y$  are calculated and the related task-specific uncertainties of measurement  $U_{\text{ges}}$  determined by simulation.

#### **5.3.2 Calculation of the test result**

Performing a number of measurements on calibrated objects, the coverage of the uncertainty ranges is checked. A statement of uncertainty will be plausible if:

$$|y_k - y| \leq (U_k + U_{ges}) \text{ or } |y_k - y| / \sqrt{U_k^2 + U_{ges}^2} \leq 1 \text{ (to be discussed)}$$



$y$ : measurement result

$y_k$ : calibrated value

$U_k$ : calibration uncertainty

$U_{ges}$ : Task specific uncertainty of the measurement

A reasonable relationship between the uncertainty of the calibration and the uncertainty of the individual measurement is to be aimed at. As a rule, the following should be valid:  $U_k \ll U_{ges}$ . The higher the calibration uncertainty  $U_k$  of the test object, the smaller the meaningfulness of the test.

### 5.3.3 Re-verification

The Re-verification is to be carried out at regular intervals. Its type and scope comply with the procedure described in para. 5. In addition, a Re-verification is to be carried out

- when the coordinate measuring machine has been modified,
- when one or several input parameters of the simulation model have been changed,
- when, in addition, the environmental conditions have changed beyond the specified range,
- when there are doubts about the uncertainties determined for other reasons.

After the first installation on the CMM concerned, short intervals ( $\leq 3$  months) should be selected for the Re-verification. The positions of the test object in the measurement volume should, if possible, be varied for each intermediate test to guarantee as high a number of independent samples as possible. The intervals may be prolonged when sufficient experience has been gained regarding the stability of the measurements.

### 5.3.4 Interim check of the input quantities

In the course of the intermediate test it is to be determined to what extent the present state of the CMM complies with the assumptions. The procedure has to state whether or not the estimation of the influence quantities is still valid. The following influence quantities should in particular be monitored:

- Scale factors
- Rectangularities
- Probing errors
- Temperature and temperature gradients



The input quantities should preferably be monitored by the procedures usually applied in coordinate measurement technology.

## Annex

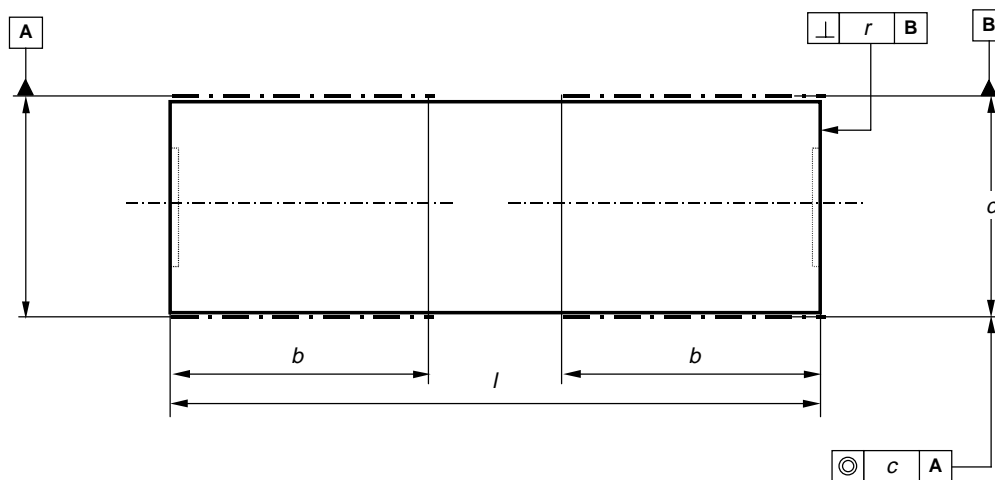


Figure A-1: Test cylinder for verification of the simulation

Symbol	Brief description of the feature
$l$	Distance of end faces
$d$	Diameter of a cylinder
$r$	Rectangularity deviations of the end faces with respect to a cylinder axis
$c$	Coaxiality of the cylinder axes
$(b)$	Reference length for the measurement of coaxiality and rectangularity according to ISO 1101 (no feature)

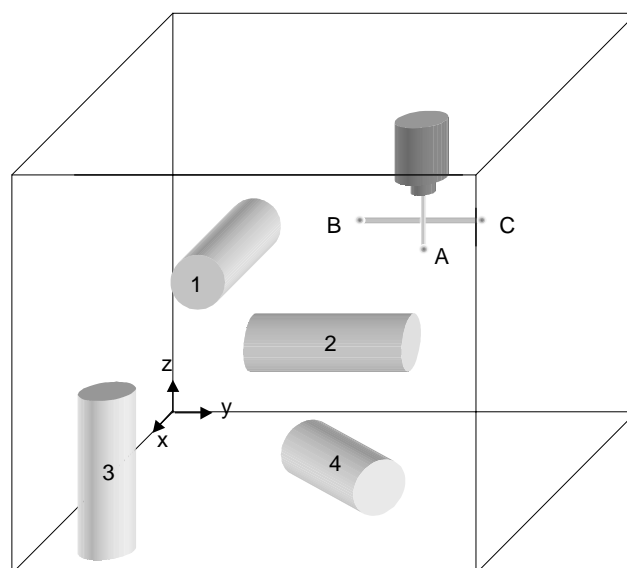


Figure A-2: Positions of the test cylinder in the measurement volume and probe configurations.

Revision: January 2001

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**Working Draft International Standard**

**ISO/ 15530-4**

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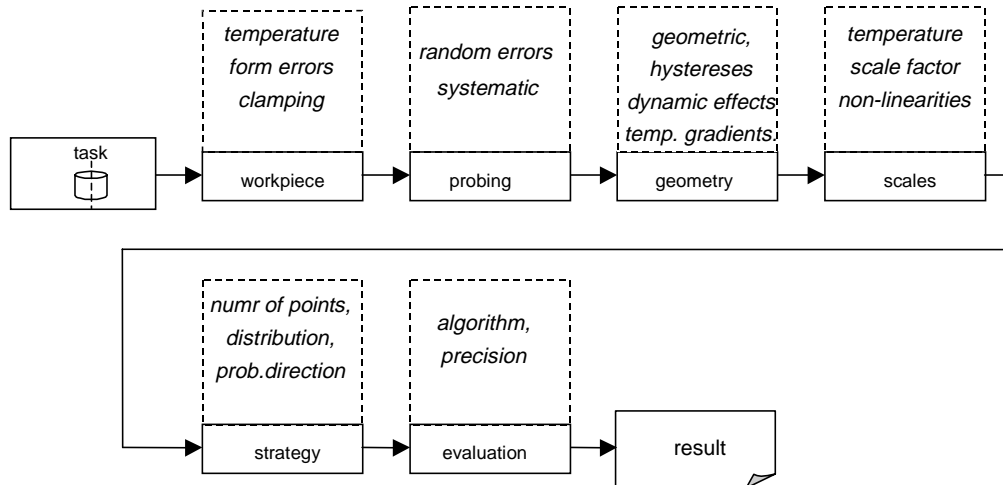
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#### **2.2.1 Uncertainty contributors**

The measuring process on a CMM is subject to a great number of uncertainty contributors (Section 3.1) that affect the measurement result. When the uncertainty of a measuring process is evaluated, these influence quantities must be accounted for, recognizing that, in general, the simulator accounts for only some of these influence quantities.

#### **2.2.2 Model**

The model of the measuring process describes the mathematical relationship between the input quantities (measurand and influence quantities) and the output measurement result. The simulation procedure does not require that the model be described by a closed mathematical expression. Numerical algorithms, such as the calculation of derived features or filtering of measurement points can, therefore, be included in the model. This makes the simulation procedure particularly suitable for complex measuring processes like coordinate measurements. The model of the measurement on a CMM can be described by a signal flow chart, in which the quantities influencing the measuring process are plotted. Figure 1 shows such a signal flow chart.



**Figure 1: Measurement on a coordinate measuring machine represented in the form of a signal flow chart**

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With the aid of the simulator, all influence quantities (within its scope) are varied within their assumed probability distribution and numerical calculation of the measurement result is repeatedly carried out. The random values are generated in the assumed probability distributions by random number generators which are to furnish approximately uncorrelated sequences of numbers.

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Probe Errors: ☐ probe response (lobing)      ☐ multiple styli                      ☐ multiple probe  
☐ articulating head                      ☐ styli/probe changing              ☐ scanning

Nominal Parts: ☐ circles      ☐ planes      ☐ sphere      ☐ cylinder      ☐ splines      etc.

GD&T: ☐ datum ref frames   ☐ form                      ☐ size                      ☐ location                      ☐ orientation

Real Part Effects:      ☐ surface roughness   ☐ waviness      ☐ form error  
                                 ☐ non 20 C temp      ☐ spatial gradients      ☐ temporal gradients  
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The manufacturer shall state what measurements or other quantities are needed to characterize the CMM and its environment in order for the software to produce uncertainty statements (3.7).

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Each factor “checked” in the declarations section (3.1) implies that the simulation software will appropriately address this issue over a specified range of conditions. (For example, “non-standard temperature” might be defined as homogenous temperature in space and time, within the limits of 15 °C to 30 °C.) The software manufacturer shall specify these ranges defining the spectrum of measurement tasks and the environmental conditions for which the simulation is valid. Examples of these ranges to be specified includes but is not limited to:

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Within the scope of these restrictions, computer-aided verification and evaluation can be performed, as well as the user checks, both described in section 5.

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## 4 Determination of the task-specific uncertainty of measurement

The parameters of the simulated measurement which are important from the metrological point of view should be as similar as possible to those of the real measurement. The standard uncertainty of a measurement result  $y$  is composed of

- the uncertainty  $u_{sim}$  determined by the simulation, and
- the uncertainties  $u_i$  from the influence quantities which have not be taken into account in the simulation and have been estimated by other appropriate means.

The combined standard uncertainty is then calculated with

$$u_{ges} = \sqrt{u_{sim}^2 + \sum u_i^2} . \quad (1)$$



With the aid of coverage factors, this standard uncertainty can be brought to the desired confidence level. As a rule, the following is valid

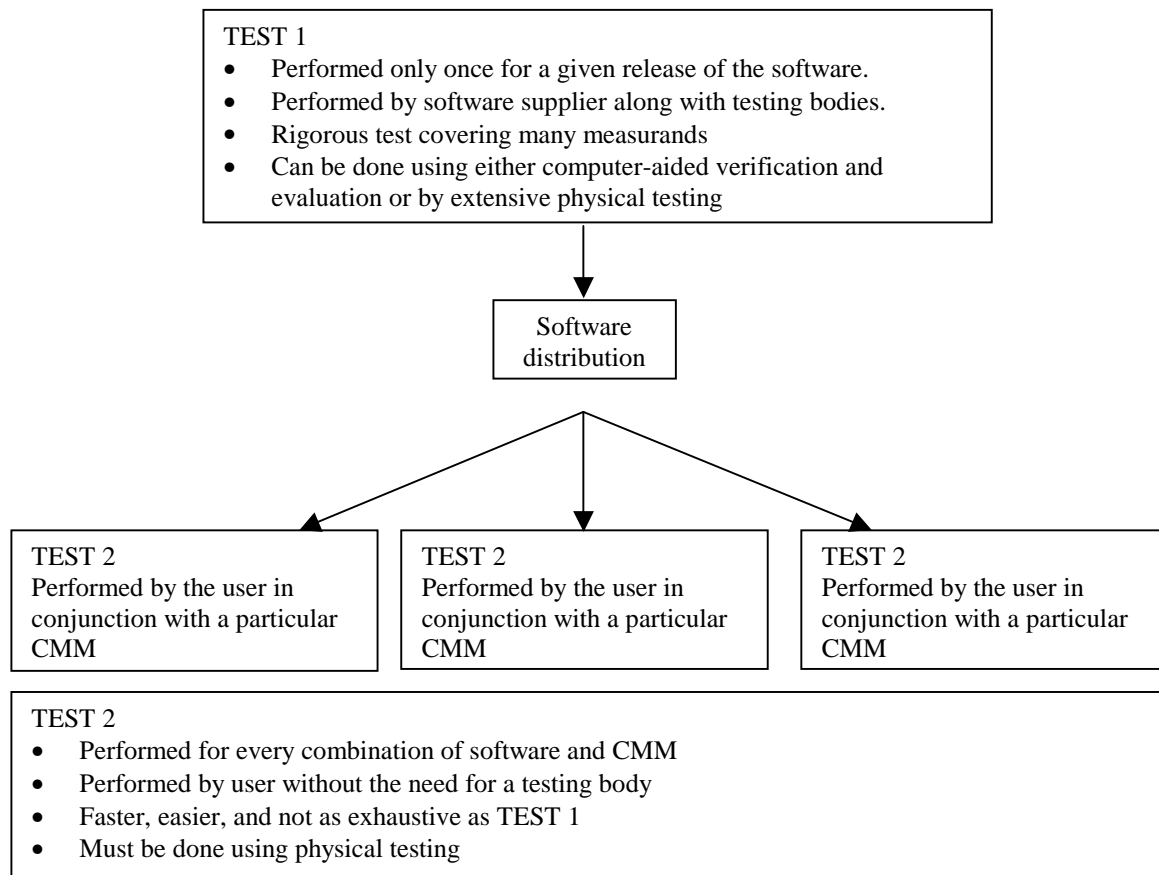
$$U_{ges} = 2 \cdot u_{ges} \quad (2)$$

for a confidence level of 95%. If the uncertainty stated by the simulation already is an expanded uncertainty  $U_{sim}$ , the simulated uncertainty  $u_{sim}$  is to be calculated by division with the respective coverage factor.

## 5 Verification and Evaluation of Simulation Software

### 5.1 Principle

The verification and evaluation of simulation software consists of two parts: TEST 1: A thorough, general software test covering a wide range of capabilities of the simulation software, and TEST 2: a test using a physical object on a specific implementation of the software on a particular CMM and covering the whole system composed of CMM, evaluation software, and simulation software. TEST 1 is conducted only once on a specific software release, while TEST 2 is conducted multiple times on each software/CMM combination.



## **5.2 TEST 1**

**(DRAFT NOTE:** Currently this standard only describes the computer-aided verification and evaluation (CVE) for test 1. An alternate means for achieving TEST 1 by using extensive physical measurements is being investigated.)

This procedure uses computer simulation to verify and evaluate the simulation software. The concept is to simulate a measuring instance, based on the claims in the declarations section (3.1). Since the measuring instance is simulated and thus fully known in the CVE process, the error of the simulated measurement can be found. The software under test produces an uncertainty statement for this measurement and a simple comparison can determine if the error of the simulated measurement was contained in the uncertainty region reported by the software under test. This procedure can be repeated hundreds of times with varying conditions and statistics can be determined regarding how often the errors of measurement were contained within their corresponding reported uncertainty ranges.

### **5.2.1 Creation of a Simulated Measuring Instance used in CVE**

When a CMM probes, there is generally a difference between the target contact point and the measured point, this difference being a vector. So a simulated measuring instance can be created by defining a vector field over the measuring volume of the CMM. That is, associated with each point in the CMM's measuring volume is a vector that represents the difference between the target contact point and the simulated measured point.

For the purposes of CVE, the declarations section (3.1) determines what influences the definition of the vector field. For instance, if hysteresis is claimed in section (3.1) then the vector associated with a particular point on one query would, in general, be different than on a subsequent query. If part form errors are included in the declarations section (3.1) then the vector associated with a particular point would depend on the placement of the part in the measuring volume. This allows for testing the software's reported uncertainties without combining them with other uncertainties.

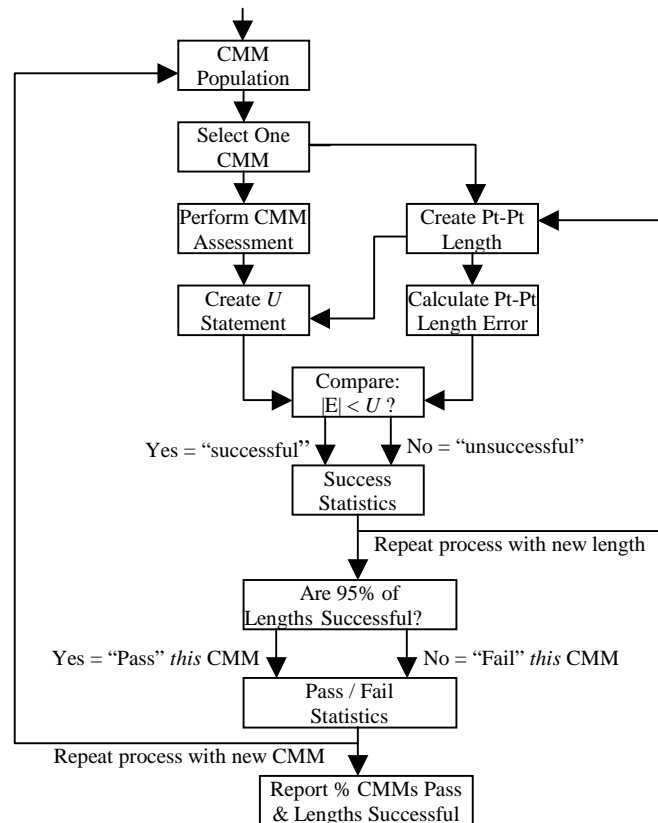
### **5.2.2 Creation of Input Quantities**

The declarations section includes the indication of the input quantities required by the software (3.1, 3.7). These input quantities might arise from probes of special calibrated workpieces. Appropriate input quantities can be obtained as follows: A simulated measuring instance can be created in accordance with the specifications of the manner in which input quantities are obtained (3.1, 3.7). Note, these conditions might be different than the ranges given in the checklist (e.g. the input quantities might be measured close to 20°C, while the software allows for measurements over a wider temperature range.) The target contact points used in the procedure to determine input quantities are provided to the testing body and the testing body returns the corresponding

measured contact points from the simulated measuring instance. The software under test must be able to exchange this information.

### 5.2.3 CVE Testing

The CVE testing should be performed over the entire spectrum of measurands for which the software is claimed to be valid. The CVE testing proceeds as shown in the following diagram given as an illustrative example using point-to-point length measurements:



### 5.2.4 Reporting CVE Results

CVE results consist of the following information:

- The percent of time true value lies in uncertainty interval; e.g. for "good" software the threshold might be 95%.
- The average amount of over-estimation of uncertainty, i.e. when the true value is contained within the uncertainty interval, on average how far is it from the nearest uncertainty interval limit.
- The average amount of under-estimation of uncertainty, i.e. when the true value is outside the uncertainty interval, on average how far is it from the nearest uncertainty interval limit.

### 5.3 Verification Using a Calibrated Workpiece

In addition to the CVE, this procedure provides verification for a specific task for a specific software/CMM combination. Here, the statement of the uncertainty is checked by a test covering the whole system composed of CMM, evaluation software, and simulation software. The test is based on real measurements performed on calibrated objects. Any object permitted according to section 3.5 may be used. The object must have been calibrated by an independent procedure. In the Annex, an object and a procedure are given as an example, and show a number of measurement tasks to be simulated and which can also be calibrated with sufficient accuracy by independent procedures. For the measurement of such an object it is recommended to also vary the measurement strategy (position and orientation of the test object, distribution of measurement points) in order to check the influence on the measurement uncertainty stated.

The following test objects may also be used: gauge blocks, step gauges, ball plates, ball bars and other standards. However any specific object is only suited to a limited extent to test the statements of task-specific uncertainty.

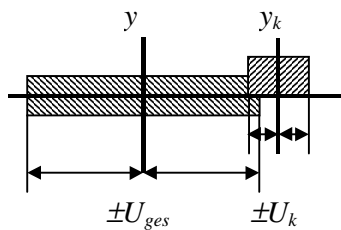
#### 5.3.1 Procedure

The measurements on the calibrated test objects are carried out on the real CMM for which the uncertainty of measurement is to be determined. The real measurement results  $y$  are calculated and the related task-specific uncertainties of measurement  $U_{ges}$  are determined by simulation.

#### 5.3.2 Calculation of the test result

Performing a number of measurements on calibrated objects, the coverage of the uncertainty ranges is checked. A statement of uncertainty will be plausible if:

$$|y_k - y| / \sqrt{U_k^2 + U_{ges}^2} \leq 1$$



$y$ : measurement result

$y_k$ : calibrated value

$U_k$ : calibration uncertainty

$U_{ges}$ : Task specific uncertainty of the measurement

A reasonable relationship between the uncertainty of the calibration and the uncertainty of the individual measurement is to be aimed at. As a rule, the following should be valid:  $U_k \ll U_{ges}$ . The higher the calibration uncertainty  $U_k$  of the test object, the smaller the meaningfulness of the test.

#### 5.3.3 Re-verification

The Re-verification is to be carried out at regular intervals. Its type and scope comply with the procedure described in para. 5. In addition, a Re-verification is to be carried out

- when the coordinate measuring machine has been modified,
- when one or several input parameters of the simulation model have been changed,
- when, in addition, the environmental conditions have changed beyond the specified range,
- when there are doubts about the uncertainties determined for other reasons.

After the first installation on the CMM concerned, short intervals ( $\leq 3$  months) should be selected for the Re-verification. The positions of the test object in the measurement volume should, if possible, be varied for each intermediate test to guarantee as high a number of independent samples as possible. The intervals may be prolonged when sufficient experience has been gained regarding the stability of the measurements.

#### **5.3.4 Interim check of the input quantities**

In the course of the intermediate test it is to be determined to what extent the present state of the CMM complies with the assumptions. The procedure has to state whether or not the estimation of the influence quantities is still valid. The following influence quantities should in particular be monitored:

- Scale factors
- Rectangularities
- Probing errors
- Temperature and temperature gradients

The input quantities should preferably be monitored by the procedures usually applied in coordinate measurement technology.

## Annex

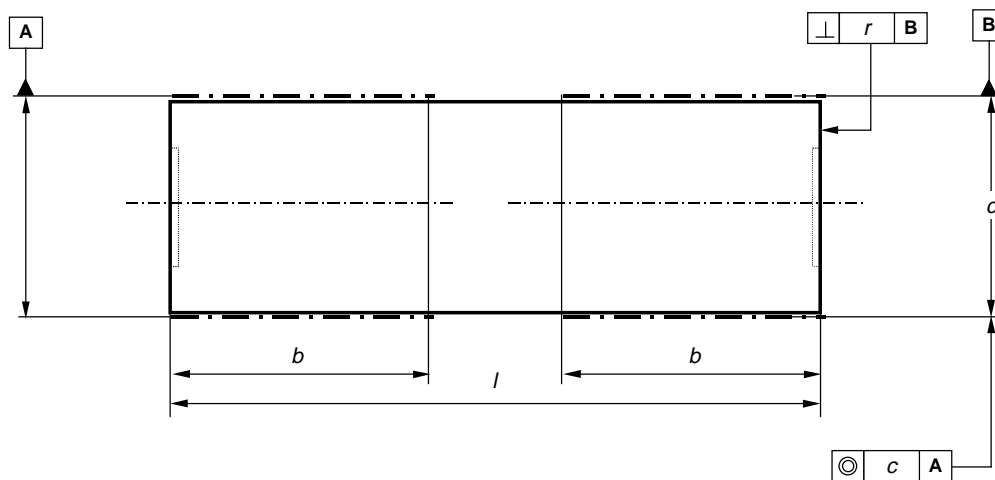


Figure A-1: Test cylinder for verification of the simulation

Symbol	Brief description of the feature
$l$	Distance of end faces
$d$	Diameter of a cylinder
$r$	Rectangularity deviations of the end faces with respect to a cylinder axis
$c$	Coaxiality of the cylinder axes
$(b)$	Reference length for the measurement of coaxiality and rectangularity according to ISO 1101 (no feature)

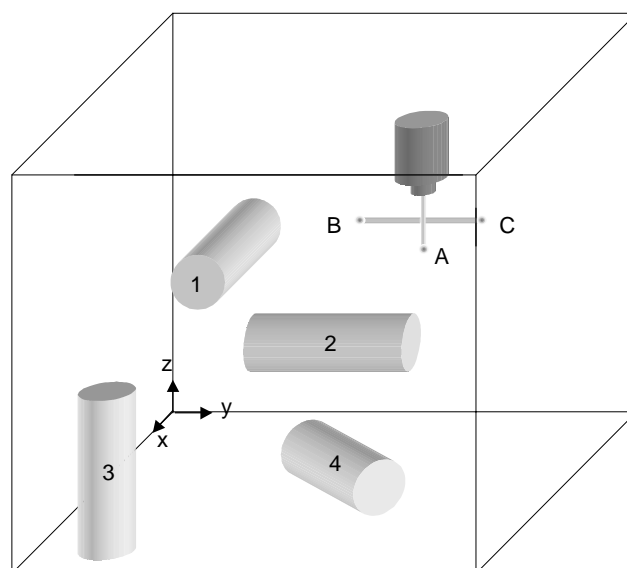


Figure A-2: Positions of the test cylinder in the measurement volume and probe configurations.

Revision: September 2001

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**Working Draft**

**ISO/WD 15530-4**

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**Geometrical Product Specifications (GPS)  
- Techniques of Determining the Uncertainty of  
Measurement in Coordinate Metrology -  
Part 4: Estimating Task-Specific Measurement Uncertainty  
Using Simulation**

**Preliminary remark**

For coordinate measuring machines (CMMs) used to inspect tolerances, the task-specific uncertainties of measurement according to ISO 14253-1 must be taken into account when tests for conformity/non-conformity are carried out. Thus knowledge of the uncertainty of measurement is of utmost importance. Up to the present, there have been only a few procedures that allow the task-specific uncertainty of measurement to be stated.

For simple measuring devices this uncertainty can be estimated by an uncertainty budget according to the recommendations of the Guide to the Expression of Uncertainty in Measurement (GUM). However, in the case of a CMM, the formulation of a classical uncertainty budget is impossible for the majority of the measurement tasks due to the complexity of the measuring process.

Alternate methods that are consistent with the GUM can be used to determine the task-specific uncertainty of coordinate measurements. One of them, which estimates the uncertainty by numerical simulation of the measuring process allowing for uncertainty influences, is described in this standard.

**1. Scope**

It is the objective of this standard to describe testing procedures for the evaluation of task specific uncertainty determination by simulation for specific measurement tasks carried out on CMMs, taking into account the measuring device, the environment, the measurement strategy and the object. The standard is to unify the general procedures without restricting the possibilities of the technical realization. A procedure for verification and evaluation of the simulation package is included. Measures are recommended that increase the procedure's transparency for the user, and methods are described that the user may apply to monitor it.

The standard is not aimed at defining new parameters for the general evaluation of the accuracy of CMM measurements. The functional capability of the CMMs is taken for granted, as agreement with specifications is checked on the basis of sheets 1 - 6 of VDI 2617 and of the ISO series of standards 10360.

## **2. Terminology: (To be defined; descriptions are given when used first)**

Influence Quantity

Input Quantity

Uncertainty Evaluating Software (UES)

Uncertainty Evaluating Software Evaluation (UESE)

Uncertainty Evaluating Software Implementation Test (UESIT)

Computer-aided Verification and Evaluation (CVE)

## **3. Overview**

The generation of task specific uncertainty statements for CMM measurements is a complex issue. To allow CMM users to easily create uncertainty statements, CMM suppliers and other third party companies have developed Uncertainty Evaluating Software (UES). UES is based on a computer-aided mathematical model of the measuring process. In this model, the measuring process is represented from the measurand to the measurement result, taking important influence quantities into account. In the simulation, these influences are varied within their possible or assumed range of values (described by probability distributions), and the measuring process is repeatedly simulated, using possible combinations of the influence quantities. The uncertainty is determined from the variation of the final result. This procedure is compatible with the fundamental principles of the internationally valid Guide to the Expression of Uncertainty in Measurement (GUM). The details of the UES are often hidden in compiled computer code making it difficult for the user to assess the reliability of the calculated uncertainty statements. This standard sets forth terminology and testing procedures for both the UES supplier and the CMM user to communicate and quantify the capability of UES.

This standard divides the problem into three major parts. The first part (section 4) is the declaration of influence quantities. The declarations identify which influence quantities, along with their ranges of values, the UES can account for in its uncertainty evaluation. For example, some UES can include the effects of using multiple styli during a CMM measurement, while others cannot. Similarly, some UES can include the effects of temporal or spatial temperature gradients, while others cannot. The purpose of the declaration section is to clearly identify to the CMM user what influence quantities, and their ranges of values, the UES will consider in its uncertainty evaluation. This will allow the user to be able to make informed decisions. Purchasing a UES product with limited capabilities that do not include some influence quantities present during the CMM measurements requires the CMM user to independently evaluate these unaccounted-for influence quantities and combine them appropriately with those that are evaluated by the UES in order to produce a GUM compliant uncertainty statement.



The second part describes the testing procedure for the uncertainty evaluating software evaluation (UESE). The UESE is performed by a testing body and is only conducted once for each version of the UES. The UESE is a major evaluation that tests the UES for its ability to produce appropriate uncertainty statements under any combination of influence quantities permitted in the declaration section.

The third part describes the final test which is performed by the CMM user on a particular CMM. This Uncertainty Evaluating Software Implementation Test (UESIT) checks that the UES and its associated input values are correctly installed and working properly. The UESIT may also detect a large uncertainty source that is not taken into account by the UES but is present in the CMM system.

#### **4.0 Elements of the UES**

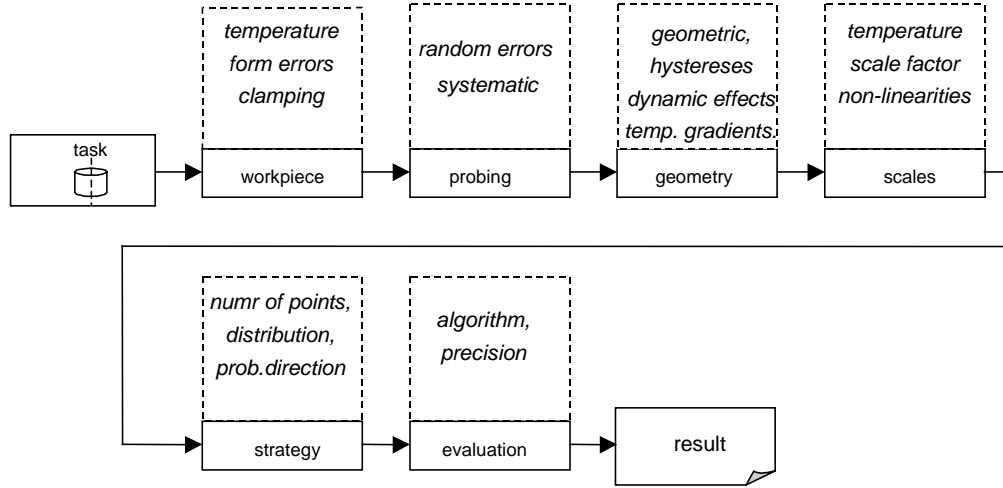
The simulation can be integrated into a control and evaluation software of a CMM (on-line) or implemented as an independent system on an external computer (off-line). Both variants are covered by this standard.

##### **4.1 Uncertainty Contributors**

The measuring process of a CMM is subject to a great number of uncertainty contributors (influence quantities) that affect the measurement result. When the uncertainty of a measuring process is evaluated using the UES, these influence quantities must be accounted for, recognizing that, in general, the UES accounts for only some of these influence quantities.

##### **4.2 UES Model**

The model of the measuring process employed by the UES describes the mathematical relationship between the input quantities (measurand and influence quantities) and the output measurement result. The UES does not require that the model be described by a closed mathematical expression. Numerical algorithms, such as the calculation of derived features or filtering of measurement points can, therefore, be included in the model. This makes UES particularly suitable for complex measuring processes like coordinate measurements. The model used by UES of the measurement on a CMM can be described by a flow chart, in which the quantities influencing the measuring process are plotted. Figure 1 shows a typical flow chart.



**Figure 1: Measurement on a coordinate measuring machine represented in the form of a flow chart**

Usually, not all possible uncertainty influences are taken into account in the model. Influence quantities which have not been considered are to be estimated by other procedures and added to the total uncertainty as follows:

#### 4.3 Determination of the task-specific uncertainty of measurement

The parameters of the simulated measurement which are important from the metrological point of view should be as similar as possible to those of the real measurement. The standard uncertainty of a measurement result  $y$  is composed of

- the uncertainty  $u_{\text{sim}}$  determined by the simulation, and
- the uncertainties  $u_i$  from the influence quantities which have not been taken into account in the simulation and have been estimated by other appropriate means.

The combined standard uncertainty is then calculated with

$$u_{\text{ges}} = \sqrt{u_{\text{sim}}^2 + \sum u_i^2} . \quad (1)$$

With the aid of coverage factors, this standard uncertainty can be brought to the desired confidence level. As a rule, the following is valid

$$U_{\text{ges}} = 2 \cdot u_{\text{ges}} \quad (2)$$

for a confidence level of 95%. If the uncertainty stated by the simulation already is an expanded uncertainty  $U_{\text{sim}}$ , the simulated uncertainty  $u_{\text{sim}}$  is to be calculated by division with the respective coverage factor.

## 4.4 Requirements and information to be provided by the manufacturer

### 4.4.1 Influence Quantities

The manufacturer of the UES shall explicitly declare which influence quantities (cf. 2.2.1) have been taken into account in the software, and what information is required from the user. In particular, the manufacturer should specify by means of this checklist, which uncertainty contributors the software claims to take into account:

#### Declaration of Influence Quantities

CMM Types : ☐ moving bridge      ☐ moving table      ☐ horizontal arm   ☐ gantry   etc...  
☐ Rotary Tables

CMM Errors: ☐ static machine geometry errors   ☐ dynamic machine geometry errors  
☐ part loading effects      ☐ Non 20 C temperature      ☐ spatial gradients  
☐ temporal gradients      ☐ algorithm software accuracy  
☐ hysteresis

Probe Types: ☐ contact touch trigger      ☐ contact analog      ☐ non-contact optical

Probe Errors: ☐ probe response (lobing)   ☐ multiple styli      ☐ multiple probe  
☐ articulating head      ☐ styli/probe changing      ☐ scanning

Nominal Parts: ☐ circles      ☐ planes      ☐ sphere      ☐ cylinder      ☐ splines      etc.

GD&T: ☐ datum ref frames   ☐ form      ☐ size      ☐ location      ☐ orientation

Real Part Effects:      ☐ surface roughness   ☐ waviness      ☐ form error  
☐ non 20 C temp      ☐ spatial gradients      ☐ temporal gradients  
☐ contamination      ☐ fixturing

Operator Effects:      ☐ sampling strategies, i.e. the number and location of points in the  
workpiece coordinate system  
☐ workpiece position and orientation in machine coordinate system  
☐ other operator effects...

The manufacturer shall state what measurements or other quantities are needed to characterize the CMM and its environment in order for the UES to produce uncertainty statements (3.7).

The following contributors must at least be taken into account:

- The geometrical deviations of the CMM
- Deviations of the contacting system
- Influences of temporal and spatial temperature gradients on specimen and CMM

[Note: The details of which items should be on this list are yet to be determined]

### 4.4.2 Operating conditions

Each factor “checked” in the declarations section (3.1) implies that the simulation software will appropriately address this issue over a specified range of conditions. (For example, “non-standard temperature” might be defined as homogenous temperature in space and time, within the limits of

15 °C to 30 °C.) The software manufacturer shall specify these ranges defining the spectrum of measurement tasks and the environmental conditions for which the simulation is valid. Examples of these ranges to be specified includes but is not limited to:

- Permissible part spectrum (e.g. exclusion of flexible sheet-metal parts, a minimum arc length for circles, maximum cone apex angles, etc.)
- Permissible task spectrum (e.g. exclusion of scanning or form measurement)
- Permissible temperature range
- Permissible temporal temperature gradients  $dT/dt$
- Permissible spatial temperature gradients  $dT/dx$
- Other permissible environmental conditions

Within the scope of these restrictions, computer-aided verification and evaluation can be performed, as well as the user checks, both described in section 5.

#### **4.4.3 Input Quantities and How They Are Obtained**

Along with the declarations described in section (3.1), the simulation software manufacturer must specify in detail (or reference appropriate documents) what input quantities are required to characterise the measurement system and how these quantities are obtained. These are the values that are used by the simulation software to characterize the CMM, the environment, operator effects etc. (Operator effects might be assessed from gauge repeatability and reproducibility studies i.e., GR&R; analysis of variance, i.e. ANOVA; and from expert judgement, i.e. "type B estimation").

#### **4.4.4 Additional UES Requirements**

- The manufacturer shall describe how the influence quantities are varied. As a rule, the probability distribution should be documented.
- The manufacturer must document how the uncertainties are derived from the simulated samples.
- The essential features of the model must be documented. Transparency of the model increases the user's confidence in the statement of the uncertainty. Documentation of model and procedure should be sufficient to enable the user to furnish proof of a statement of uncertainty in compliance with the standards. This is important in particular in connection with the requirements of ISO 9000 foll. requiring the documentation of the procedure used for the uncertainty determination.
- The result of the evaluation of the simulation should be a statement in the form  $\pm U$  which describes the uncertainty range of the measurement result with a specified confidence level (cf. also GUM).
- It must be ensured that the statement of the uncertainty complies with the internationally valid principles of the expression of the uncertainty (GUM). This includes the statement of a

confidence level or a coverage factor. The combined standard uncertainty may be indicated in addition to the expanded uncertainty.

## **5.0 Uncertainty Evaluating Software Evaluation (UESE)**

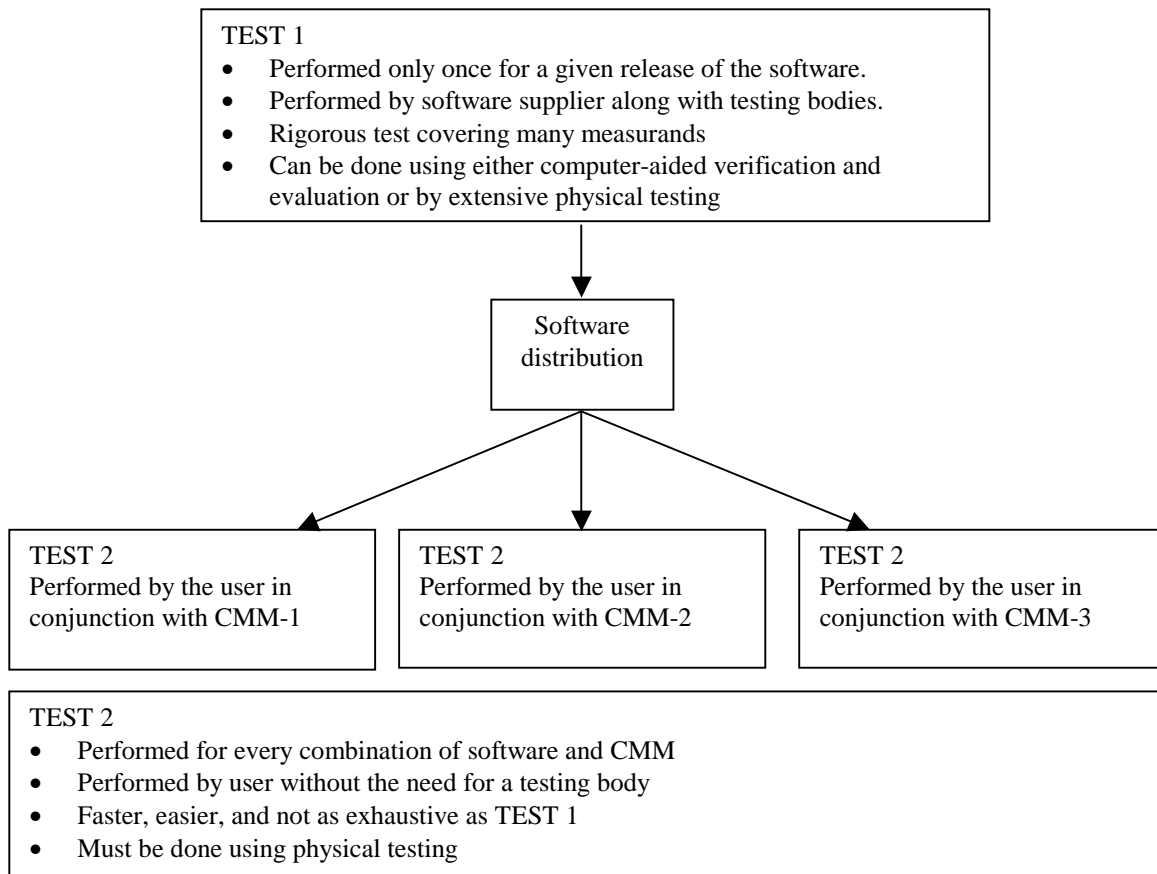
### **5.1 General**

The UES must account for all effects that are specified in the declaration of influence factors. The CMM user can gain confidence that the UES performs as claimed if the UES has passed an UESE performed by a testing body. The UESE test attempts to verify that when all influence quantities that are identified in the declaration section are varied within their permitted ranges, the expanded uncertainty calculated by the UES contains a large fraction (typically 95 %) of the measurement errors. Given the very large number of different measurands and combinations of influence factors that can occur in CMM measurements, each one of which leads to a particular measurement error that is to be compared to the expanded uncertainty as calculated by the UES, the task of the UESE is enormous. In particular, since the measurement error, which is the difference between the measured and true values of a quantity, is to be compared against the UES calculated uncertainty statement, this requires a “true value”, e.g. a calibrated artifact, to be available for every CMM measurement performed in the UESE. Fortunately, if an UES can demonstrate that it can properly calculate the measurement uncertainty under an extensive UESE test, this testing does not need to be repeated unless the UES is revised.

The UESE consists of some combination of physical measurements and software measurements. Ideally, for each measurand, all possible permitted influence quantities are varied over their full permitted extent. To illustrate the magnitude of this task, consider a measurand that is the diameter of a cylinder. Ideally, we would like to measure a calibrated cylinder on a very large number of CMMs, each having a different combination of geometrical, probing error, different thermal, etc. as permitted by the declaration section. On each of these CMMs we would like to measure the cylinder in many locations, orientations, with different probes, sampling strategies, etc. For each of these measurements the error (CMM result minus calibrated value) is compared to the UES calculated expanded uncertainty statement. Obviously this example of a single measurand, involves hundreds, perhaps thousands, of measurements on a large number of CMMs and becomes too expensive as a practical test.

### **5.2 Evaluation**

The UESE consists of two parts: TEST 1: A thorough, general software test covering a wide range of capabilities of the simulation software, and TEST 2: a test using a physical object on a specific implementation of the software on a particular CMM and covering the whole system composed of CMM, evaluation software, and simulation software. TEST 1 is conducted only once on a specific software release, while TEST 2 is conducted on each software/CMM combination.



### 5.2.1 TEST 1

**(DRAFT NOTE:** Currently this standard briefly describes only the computer-aided verification and evaluation (CVE) for test 1. An alternate means for achieving TEST 1 by using extensive physical measurements is being investigated.)

This procedure uses computer simulation to verify and evaluate the simulation software. The concept is to simulate a measuring instance, based on the claims in the declarations section (3.1). Since the measuring instance is simulated and thus fully known in the CVE process, the error of the simulated measurement can be found. The software under test produces an uncertainty statement for this measurement and a simple comparison can determine if the error of the simulated measurement was contained in the uncertainty region reported by the software under test. This procedure can be repeated hundreds of times with varying conditions and statistics can be determined regarding how often the errors of measurement were contained within their corresponding reported uncertainty ranges.

#### 5.2.1.1 Creation of a Simulated Measuring Instance used in CVE

When a CMM probes, there is generally a difference between the target contact point and the measured point, this difference being a vector. So a simulated measuring instance can be created by defining a vector field over the measuring volume of the CMM. That is, associated with each

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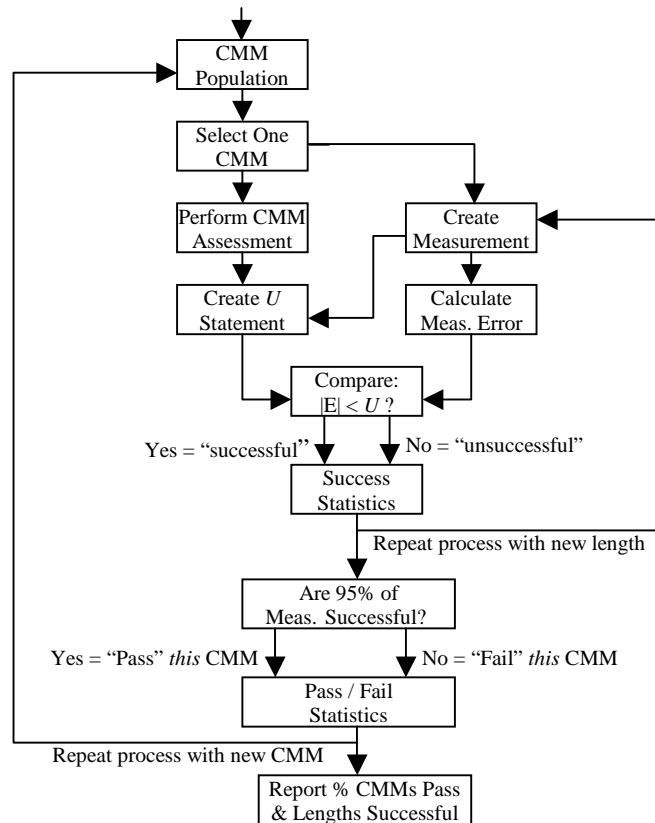
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The CVE testing should be performed over the entire spectrum of measurands for which the software is claimed to be valid. The CVE testing proceeds as shown in the following diagram:



#### 5.2.1.4 Reporting CVE Results

CVE results consist of the following information:

- The percent of time true value lies in uncertainty interval; e.g. for "good" software the threshold might be 95%.
- The average amount of over-estimation of uncertainty, i.e. when the true value is contained within the uncertainty interval, on average how far is it from the nearest uncertainty interval limit.
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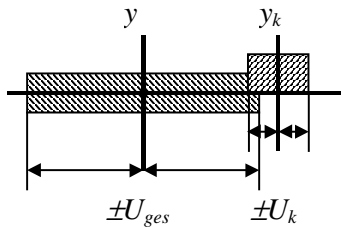
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$$|y_k - y| / \sqrt{U_k^2 + U_{ges}^2} \leq 1$$



$y$ : measurement result

$y_k$ : calibrated value

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A reasonable relationship between the uncertainty of the calibration and the uncertainty of the individual measurement is to be aimed at. As a rule, the following should be valid:  $U_k \ll U_{ges}$ . The higher the calibration uncertainty  $U_k$  of the test object, the smaller the meaningfulness of the test.

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After the first installation on the CMM concerned, short intervals ( $\leq 3$  months) should be selected for the Re-verification. The positions of the test object in the measurement volume should, if possible, be varied for each intermediate test to guarantee as high a number of independent samples as possible. The intervals may be prolonged when sufficient experience has been gained regarding the stability of the measurements.

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The input quantities should preferably be monitored by the procedures usually applied in coordinate measurement technology.

## Annex

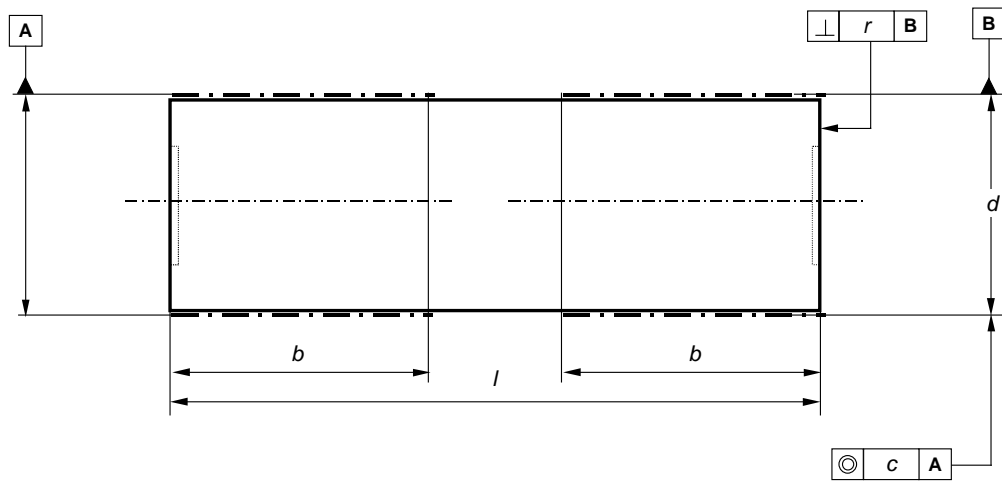


Figure A-1: Test cylinder for verification of the simulation

Symbol	Brief description of the feature
$l$	Distance of end faces
$D$	Diameter of a cylinder
$R$	Rectangularity deviations of the end faces with respect to a cylinder axis
$C$	Coaxiality of the cylinder axes
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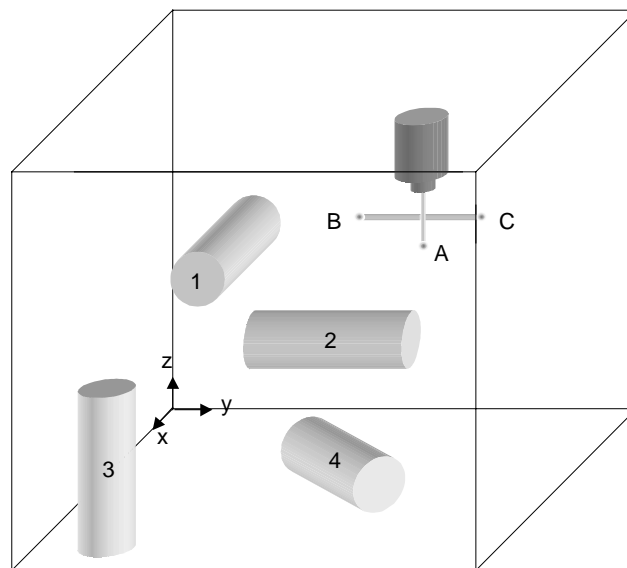


Figure A-2: Positions of the test cylinder in the measurement volume and probe configurations.

NEDO-VCMM team

Virtual Coordinate Measuring Machine

NEDO-VCMM

N37-Annex-1

25-Feb-2002

**Some Conclusions at the NEDO Meeting in Sydney**

(1-Mar.-2002)

We agree on the general concept of the 15530-4 standard, seeing the main open question is that of the details of Test 1.

We also agreed that publication of the standard should not be needlessly delayed.

Test 1 can and should be useful for increasing confidence in uncertainty evaluating software.

Still open for Test 1 are issues of the use of software testing, hardware testing, or a combination of these two (possible by using the decomposition method)

We agreed that we would proceed with physical testing for Test 1 unless we find ourselves compelled to look to computer aided evaluation.

We plan to proceed with developing a Test 1 type procedure for evaluating the ability of the software under test to reflect various sampling strategies using physical testing. This should serve as a starting example. We will try to incorporate this procedure, along with other modifications into a new 15530-4 draft for the next WG 10 meeting.

Currently we will proceed with the thought that Test 1 would be performed by an NMI.

Thus the test designed should reflect the resources and possibilities of an NMI unless we are compelled to consider possibilities beyond NMIs.

The 15530-4 task force would like to thank the attendees of this NEDO meeting for their excellent and kind input.

## 6. Publication List

### 6.1 Publication List by The University of Tokyo (pp. 160 - 303)

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- 1) Kiyoshi Takamasu, Ryoshu Furutani, Shigeo Ozono: Basic Concept of Feature-Based Metrology, *Measurement*, 26 (3), 1999, pp. 151-156.
- 2) 阿部誠, 高増潔, 大園成夫, 沢辺雅二: 空間座標の比較測定による CMM の構成 (第 1 報) - パラメトリックエラーモデルの構成とそのシミュレーション -, *精密工学会誌*, 66 (3), 2000, pp. 483-488 .
- 3) 高増潔, 阿部誠, 古谷涼秋, 大園成夫: 形体計測における不確かさの見積り (第 1 報) - 校正作業で生じる系統誤差の寄与 -, *精密工学会誌*, 67 (1), 2001, pp. 91-95
- 4) 阿部誠, 高増潔, 大園成夫: 空間座標の比較測定による CMM の校正 (第 2 報) - パラメトリックエラー推定値の信頼性 -, *精密工学会誌*, 66 (4), 2000, pp. 578-582
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- 8) M. Fujiwara, A. Yamaguchi, K. Takamasu and S. Ozono: Evaluation of Stages of Nano-CMM, *Initiatives of precision engineering at the beginning of a millennium* (edited by Ichiro Inasaki), Kluwer Academic Publishers (Netherlands), 2001, 634-638
- 9) Sato, M. Hiraki, K. Takamasu and S. Ozono: Calibration of 2-DOF Parallel Mechanism, *Initiatives of precision engineering at the beginning of a millennium* (edited by Ichiro Inasaki), Kluwer Academic Publishers (Netherlands), 2001, 734-738

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- 1) Kiyoshi Takamasu, Masahiko Hiraki, Kazuhiro Enami, Shigeo Ozono: Development of Nano-CMM and Parallel-CMM - CMM in the 21th Century -, Int. Dimensional Metrology Workshop, 1999.
- 2) K. Takamasu, R. Furutani, S. Ozono: Statistical Evaluation of Minimum Zone Method in Coordinate Metrology, IMEKO-XV World Congress, 1999, pp. 101-106.
- 3) M. Hiraki, N. Yoshikawa, K. Takamasu, S. Ozono: Development of 3 DOF Parallel-CMM, IMEKO-XV World Congress, 1999, pp. 123-128.
- 4) K. Takamasu, M. Fujiwara, H. Naoi, S. Ozono: Friction Drive System for Nano-CMM, Proc. Mechatronics 2000, 2000, pp. 565-568
- 5) M. Abbe, K. Takamasu, S. Ozono: Reliability of Parametric Error on Calibration of CMM, IMEKO-XVI World Congress, 2000, pp. 180-184
- 6) G. Olea, M. Hiraki, K. Takamasu, S. Ozono: Kinematics Analysis, Simulation and Design of a Parallel Mechanism with R-L Actuators, Proc. Parallel Kinematic Machines Int. Conf. 2000, 2000, pp. 297-305
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- 1) 佐藤理, 平木雅彦, 高増潔, 大園成夫, 吉川暢広: パラレルメカニズムを用いた三次元測定機の研究 ( 第 9 報 ) - リニアスケールの搭載 - , 1999 年度精密工学会秋季大会学術講演会講演論文集, 1999, p. 346 .
- 2) 高増潔: 座標測定における計測の不確かさ, 1999 年度精密工学会秋季大会学術講演会講演論文集, 1999, p. 438 .
- 3) 阿部誠, 高増潔, 大園成夫: CMM 校正の不確かさに関する 1 考察 ( 第 1 報 ) - 1 次元変位測定によるパラメトリックエラー推定値の信頼性 - , 1999 年度精密工学会秋季大会学術講演会講演論文集, 1999, p. 439 .

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- 7) 佐藤理，平木雅彦，高増潔，大園成夫，吉川暢広：パラレルメカニズムを用いた三次元測定機の研究（第 10 報） - 設置パラメータの導入 - ，2000 年度精密工学会春季大会学術講演会講演論文集，2000，p. 471．
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- 15) 佐野花枝，高増潔，大園成夫，阿部誠，古谷涼秋：形体計測における不確か

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- 16) 山口晃，小林雅康，藤原正直，高増潔：nano-CMM の開発（第 12 報） - Z ステージの精度評価 - ，2001 年度精密工学会秋季大会学術講演会講演論文集（大阪大，2001 年 9 月），2001，J32
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  - 18) 高増潔，野坂健一郎，古谷涼秋，大園成夫：形体計測における不確かさの評価（第 2 報） 形状偏差の影響 ，2001 年度精密工学会秋季大会学術講演会講演論文集（大阪大，2001 年 9 月），2001，N01
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- 2) 高増潔：光を応用した三次元形状計測，画像応用技術専門委員会報告，15（1），2000，pp. 25-34
- 3) 高増潔：測定の不確かさと三次元測定機 東大精研会基礎講座，2001



- 4) 高増潔：バーチャル三次元測定機，計測と制御，40（11），2001，801-804

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- 1) Sonko Osawa, Ryoshu Furutani, Kiyoshi Takamasu, Shigeo Ozono, Hidemitsu Asano: 3-D shape masurement by self-referenced pattern projection method, Measurement, 26(2) 2, 157- 166, 1999
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- 3) Ryoshu FURUTANI, Tomoyuki TAKAMOTO, Cho Young-Won, Development of High Sensitive Probe using optical displacement detector(1st Report)-Estimation of capability of sensitivity of planar displacement-, JSPE, 67(10), 1670- 1674, 2001

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- 5) R.Furutani, : probe system for detecting small displacement, IMEKO World Congress XVI, 89-93, 2000
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- 2) 大澤尊光, 高辻利之, 黒澤富蔵, “幾何学ゲージ持ち回り比較の問題 - ボールプレート 2 国間国際比較を通して -”, 精密工学会誌, Vol.67, No.2, pp.256-261 (2001).
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- 13) Frank Härtig, Franz Wäldele: Verzahnungsmessung auf KMG,
- 14) Heinrich Schwenke: Abschätzung von Meßunsicherheiten durch Simulation an Beispielen aus der Fertigungsmeßtechnik, PTB-Bericht

6.5 Publication List by National Measurement Laboratory (pp. 554 - 557)

- 1) E. Jaatinen, R. Yin, M. Ghaffari, and N. Brown: Optimizing CMM measurement processes by calculating the uncertainty



## 7. Published Papers

Published Papers by The University of Tokyo (pp. 160 - 303)

Published Papers by Tokyo Denki University (pp. 304 - 368)

Published Papers by National Metrology Institute of Japan (pp. 369 - 443)

Published Papers by Physikalisch-Technische Bundesanstalt (pp. 444 - 553)

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