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Statistical Evaluation of Minimum Zone Method in Coordinate Metrology

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Abstract

Minimum zone method, minimum circumscribing method and maximum inscribing method are used for calculation of form deviations and determination of datum system in ISO standards. However, there are no researches for statistical evaluations of results from these methods. This is mainly because the results from these methods are determined by positions of only few contact points. The novel statistical evaluation for minimum zone method has been proposed in this article. Using this method, an average and a distribution of the first order (the smallest or the largest) of measured points are calculated from the probability density function and the cumulative distribution function of measured points. From the distribution of the first order of the measured points, the range of the largest value and the smallest value of measured points are calculated. The series of simulations for minimum zone method show that the evaluation methods in this article are useful to estimate the results of minimum zone method in coordinate metrology.

Keywords: minimum zone method, coordinate metrology, coordinate measuring machine, least squares method, uncertainty of measurement

1. Introduction

Minimum zone method, minimum circumscribing method and maximum inscribing method are used for calculation of form deviations and determination of the datum system according to ISO standards. One reason for this is that functions of mechanical parts are strongly effected by contact conditions of each part [1]. There are many researches on minimum zone method [2]-[5], however every researches do not deal with the statistical evaluations of results by minimum zone method. This is mainly because the results of minimum zone methods are determined by positions of only few contact points on a target feature.

When minimum zone method is used in coordinate metrology, the density of measured points is low and the number of measured points is small. These factors influenced strongly the statistical properties of minimum zone calculations in coordinate metrology [6][7].

In this article, novel statistical evaluation for minimum zone method has been proposed. Using this method, an average and a distribution of the first order (the smallest or the largest) of measured points are calculated from the probability density function and the cumulative distribution function of measured points. From the distribution of the first order of the measured points, the range of the largest value and the smallest value of measured points are calculated. For the straight lines, the limits and the distribution of line slope are also calculated using these methods.

The series of simulations for minimum zone method show that the evaluation methods in this paper are useful to estimate the results of minimum zone method in coordinate metrology. And we will consider following items in this article;

(1) statistical evaluation of minimum zone method in coordinate metrology,
(2) relationship between the uncertainty of measurements and the measured results of minimum zone method, and
(3) the statistical properties of minimum zone method.

2. Estimation of Positions of Measured Points in Discrete Measurement

2.1 Statistical Evaluation

In coordinate metrology, the number of measured points on each features (flat plane, spherical plane, cylindrical plane and so on) is very limited because of limitations of measuring time, data volume, time for processing of data. Under this condition, the statistical properties of measured points on a straight line are estimated using the following methods.

Figure 1 shows 10 measured points on a straight line at even intervals. The straight line is expressed by the equation $y = 0$, length a and a probability density function $p\ (y)$ and a cumulative distribution function q (y), where the relation between $p\ (y)$ and q (y) is expressed as equation (1).

$$q(y) = \int_{-\infty}^{y} p(t)\, dt \qquad (1)$$

From now on, the range h as the difference between the largest measured value minus the smallest measured value, is estimated. For this purpose, the probability density function $p_1\,(y)$ of the first order point (the smallest point) is defined as equation (2), and the average ey_1 and the standard deviation sy_1 of the first order point are also defined as equations (3) and (4), respectively.

$$p_1(y) = n\,(q(y))^{n-1}\ p(y) \qquad (2)$$

$$ey_1 = n\int (q(y))^{n-1}\ p(y)\ y\ dy \qquad (3)$$

$$sy_1^2 = n\int (q(y))^{n-1}\ p(y)\,(y - ey_1)^2\ dy \qquad (4)$$

Here we are interested in the probability density function $p_i\,(y)$ and the average ey_i of the i-th order point which are defined by same way as equations (5) and (6). Where, ${}_nC_k$, combination of k in n, is defined as equation (7).

$$p_i\,(y) = {}_nC_i\ {}_iC_{i\text{-}1}\ q^{n-i}(y)\,(1-q(y))^{i-1}\ p(y) \qquad (5)$$

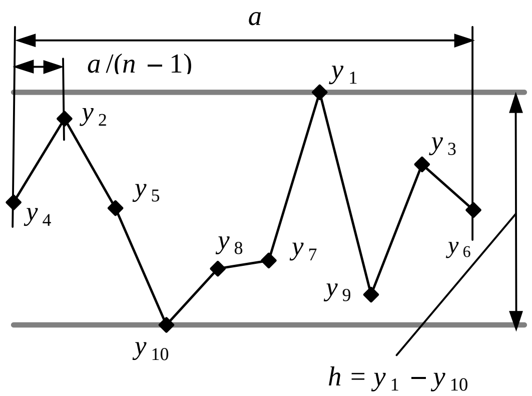


Fig. 1 Orders of measured points (y_1 - y_{10}) and range h in straight line (length = a).

$$ey_i = {}_nC_i\ \ {}_iC_{i-1}\int q^{n-i}(y)\,(1-q(y))^{i-1}\ p(y)\ y\ dy \qquad (6)$$

$${}_nC_k = \frac{n(n-1)...(n-k+1)}{k\cdot(k-1)...1} \qquad (7)$$

Using equations (5) and (6), figure 2 shows the probability density functions of the first to 5th order points when the number of measured points is 5 and the probability density function of measurement is Gaussian distribution of the average is 0 and the standard deviation is 1. And figure 3 also displays the probability functions when the probability density function of measurements is triangular distribution.

We can estimate the probability density function of the positions of measured points by these methods. In this article, the statistical properties of minimum zone methods will be evaluated using these equations.

2.2 Calculation of Range of the Largest Value Minus the Smallest Value

The range h is defined as the difference between the largest values minus the smallest values in the measured points. The probability density function of the range $p\ (h)$ and the average of the range eh are defined as equations (8) and (9). Because equation (9) is too complex, the average eh is approximated using equation (3) to the last term of equation (9).

$$p(h) = {}_nC_2 \int (q(y) - q(y-h))^{n-2}\ p(y)\ p(y-h)\, dy \tag{8}$$

$$\begin{aligned} eh &= \int p(h)\, h\, dh \\ &\cong ey_1 - ey_n = 2ey_1 = 2n\int (q(y))^{n-1}\ p(y)\ y\, dy \end{aligned} \tag{9}$$

The standard deviation of range *sh* is also calculated by the same method, and equation (10) defines the approximate values of *sh*.

$$\begin{aligned} sh^2 &= \int b(h)\,(h-eh)^2\, dh \\ &< sy_1^2 + sy_n^2 = 2sy_1^2 \\ &= 2n\int (q(y))^{n-1}\ p(y)\,(y-ey_1)^2\, dy \end{aligned} \tag{10}$$

Figure 4 (a) shows the relation between the average of range *eh* and the number of measured points *n* for the probability density function of measurement is Gaussian distribution, quadratic distribution, triangular distribution and uniform distribution as equations (8), (9), (10) and (11), respectively.. Figure 4 (b) displays the averages of range *eh* after the normalization by the value of the cumulative distribution function *q* (*y*) = 0.98. The

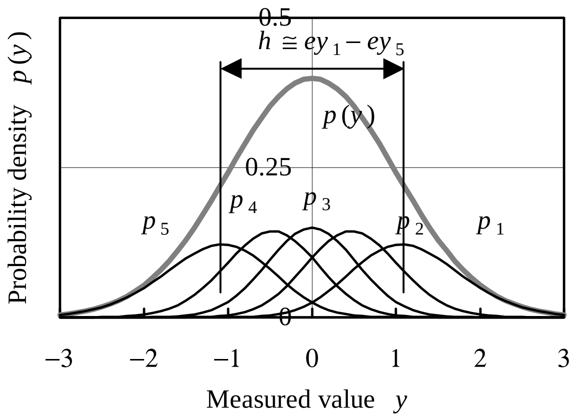


Fig. 2 Probability density $p_1(y)$ - $p_5(y)$ of each order of measured points by normal distribution function (number of measured points = 5).

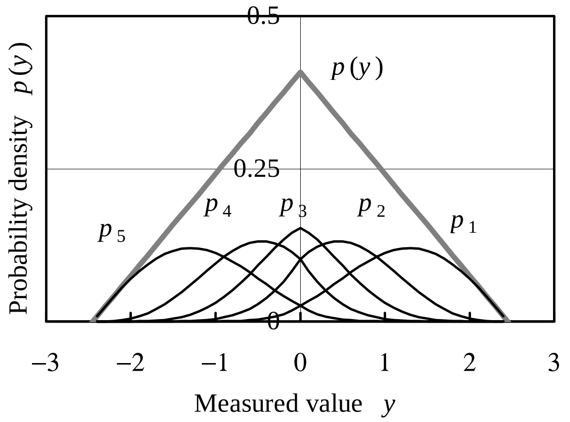
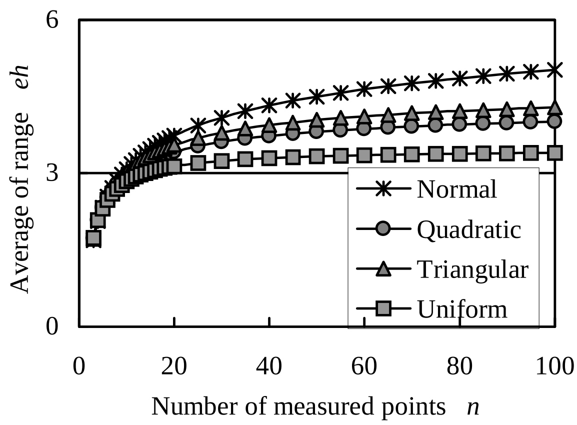
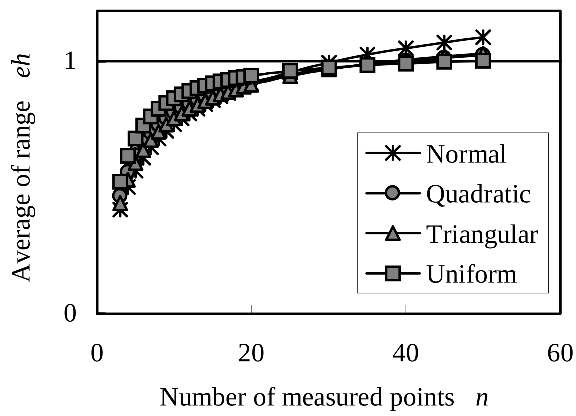


Fig. 3 Probability density of each order of measured points for triangluar distribution functions (number of measured points = 5).



(a) Befor normalization



(b) Normalization by cumulative distribution function

Fig. 4 Relation between average of range *eh* and number of measured points *n* for 4 styles of probability density functions.

properties of the averages of range agree with each other after the normalization. We conclude that the average of range is influenced by the lower or the upper foot (ex. $q(y) = 0.98$) of the probability density function.

Figure 5 displays calculated values of $eh \pm sh$ and $eh \pm 2sh$ by equations (9) and (10), when the probability density function is the quadratic function. Figure 6 shows the results of simulation of relation between the range h which is calculated from 10 sets of simulated measurements for each number of measured points n to compare with $eh \pm sh$ and $eh \pm 2sh$. Figures 5 and 6 show good agreements for the statistical estimations from equations (9) and (10) and the results of simulations of range h.

$$p(y) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{y^2}{2}\right) \tag{11}$$

$$p(y) = \begin{cases} \dfrac{3}{4\sqrt{5}}\left(1 - \dfrac{y^2}{5}\right) & |y| \le \sqrt{5} \\ 0 & |y| > \sqrt{5} \end{cases} \tag{12}$$

$$p(y) = \begin{cases} \dfrac{1}{6}\left(\sqrt{6} - |y|\right) & |y| \le \sqrt{6} \\ 0 & |y| > \sqrt{6} \end{cases} \tag{13}$$

$$p(y) = \begin{cases} \dfrac{1}{2\sqrt{3}} & |y| \le \sqrt{3} \\ 0 & |y| > \sqrt{3} \end{cases} \tag{14}$$

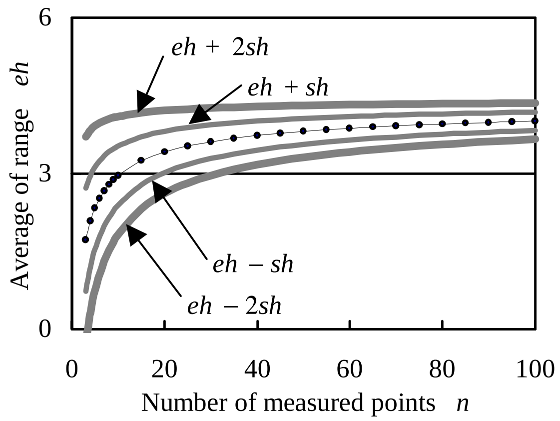


Fig. 5 Average of range eh with standard deviation sh.

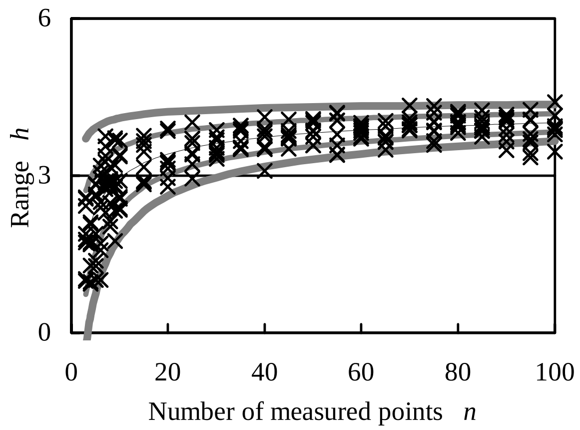


Fig. 6 Simulation of range h and average of range eh with standard deviation sh.

3. Calculation of Straightness of Straight Line

3.1 Minimum Zone Calculation for Straightness

In this section, the straightness of line will be analyzed by the same methods in section 2. Figure 7 indicates the CLRS (Control Line Rotation Scheme) for the calculation of straightness [8]. Firstly, two parallel lines are positioned at the largest and the smallest points as figure 7 (a). Then each line rotates to the direction of the other contact point, and the new contact point by the smaller rotation is assigned as the third contact point. These schemes will be repeated until the relation of three contact points is the same condition of figure 7 (b).

Using this scheme, the line slope under the condition of minimum zone is defined the slope of y_1 and y_2 or y_3 or y_4 ... for the upper line, and the slope of y_n and y_{n-1} or y_{n-2} or y_{n-3} ... for the lower line.

3.2 Distribution of Line Slope

In the same manner, the probability density function of line slope c on the first and the second order points is estimated in disregard of the correlation between these points. The probability density function of line slope $p(c)$ is defined by equation (15).

$$p(c) = \iiint p_1(y)\, p_2(y - c(a_1 - a_2))\, dy\, da_1\, da_2 \qquad (15)$$

Since equation (15) is too complex and difficult to integrate, the integration of Equation (15) is approximated to equations (16) and (17) using the standard deviations of the first and the second order points, where a is the length of line and k is a suitable constant from the distribution of order of points.

$$sc^2 = \int p(c)\, c^2\, dc \cong \frac{sy_1 + sy_2}{a} \cong \frac{2sy_1}{a} \qquad (16)$$

$$sc \cong \frac{k}{a} sy_1 \qquad (17)$$

The relation between the line slope c and the number of measured points n is illustrated in figure 8, when 100 sets of line features are simulated. In figure 8, 30 sets of line slope and the range of $ec \pm sc$ and $ec \pm 2sc$ are displayed, where ec is the average line slope and sc is the standard deviation of 100 sets of line features. The plotted values by the simulation have good agreements with the estimation ranges from equation (17), where k is approximated to 3.

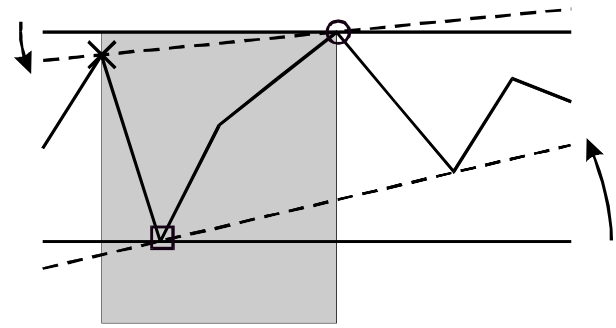
4. Conclusion

In this article, we proposed a novel statistical method for evaluating minimum zone method in coordinate metrology. Firstly, we show that the calculating method of the distribution function of the first order of measured points. Using this distribution, we estimate the results of minimum zone. From simulations, we can note that this method can precisely estimate the results of minimum zone method of line, and we consider that this method also can be applied to minimum zone of plane, circle and so on.

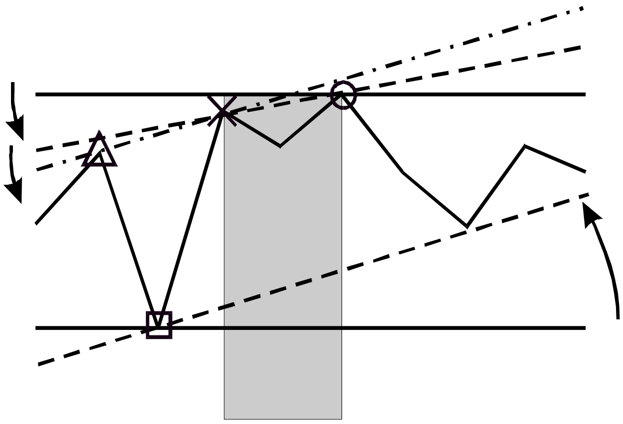
Further more, we show that the distribution of minimum zone values is defined by the lower and the upper foots of distribution functions. This directly shows that large number of measured points should be used for minimum zone method.

From these discussion and simulations, we conclude as the following;

(1) the range of measured points can be estimated by the probability density function of the first order points,



(a) First condition of two parallel lines on the largest and the smallest points and rotating to the direction of the other contact point.



(b) Final condition of two parallel lines.

Fig. 7 Concept of CLRS (Control Line Rotation Scheme) [8].

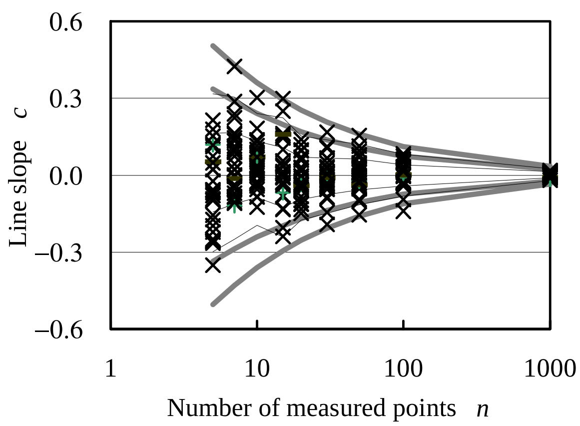


Fig. 8 Distribution of line slope and estimations by equation (17)

(2) the range of line slope by minimum zone method can be also estimated by the probability density function first order points.

The future works as follows;

(1) these methods are applied on minimum zone of other features such as plane, circle, cylinder and so on,

(2) the relation between the uncertainty of measurement of minimum zone method and the uncertainty of measured points is analyzed, and

(3) the strategy of minimum zone methods in coordinate metrology is clearly defined.

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