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Development of 3 DOF Parallel-CMM

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Abstract

Coordinate Measuring Machines (CMM) have been developed and widely used to industrial goods. Traditional CMM tends to become large and heavy in order to avoid the influence of the bending and twisting of its components and to decrease measurement errors, because it is based on a serial mechanism. This paper deals with the development of a new type of CMM device used a parallel mechanism. The position of the probe is calculated by solving the forward kinematics. The equations of forward kinematics include some kinematic parameters, such as the length of connecting rods, and it is necessary to decide these kinematic parameters for calculating the position of probe ball accurately. In case of small parallel mechanisms, other larger measuring machines can measure these kinematical parameters, but it is difficult to measure the parameters of huge mechanism using another measuring machines. Therefore, we use artifacts to identify the kinematic parameters.

Keywords: Parallel mechanism, Coordinate measuring machine, Parameter identifying, Calibration

1. Introduction

Coordinate Measuring Machines (CMM) have been developed and widely used to measure “quickly” “complex shapes” with “high accuracy” as improving precision of industrial goods. Traditional CMM is based on a serial mechanism: the components from base unit to end-effector i.e. base unit, *x*-axis, *y*-axis, *z*-axis and measuring probe are connected serially. However, some drawbacks of this mechanism are its weakness against external force and the accumulation of errors. Therefore, CMM tends to become large and heavy in order to avoid the influence of the bending and twisting of its components and to decrease measurement errors.

This paper deals with the development of a new type of CMM device: we used a parallel mechanism where the base unit and the end-effector are connected by many links in parallel. The advantages of this mechanism are its robustness against external force and error accumulation. Therefore, we will be able to make larger measuring machine that can measure quickly large objects like cars or industrial devices.

We have already built a famous Stewart platform type of parallel mechanism that has 6 degrees of freedom (DOF). This mechanism is based on spherical magnetic joints using steel balls and magnets that allow higher repeatability. However, the movable area of this mechanism is not wide, because the stroke of each leg is short. Moreover, it is too difficult to solve the forward kinematics of this mechanism.

To resolve above problems, we are currently developing new 3 DOF Parallel CMM, whose forward kinematics can be solved analytically. The equations of forward kinematics include some kinematical parameters, such as the length of connecting rods, and it is necessary to decide these kinematical parameters for calculating the position of probe ball accurately. In case of small parallel mechanisms, other larger measuring machines can measure these kinematical parameters, but it is difficult to measure the parameters of huge mechanism using another measuring machines. Therefore, we use artifacts to identify the kinematical parameters.

In this paper, firstly, we compare the characteristics of parallel mechanism and serial mechanism for measurement, and introduce two parallel mechanisms: one is 6-DOF and another is 3-DOF mechanism. Next, the equations of kinematics are shown and the method of identifying the unknown kinematical parameters is described.

2. Parallel CMM

2.1 Characteristics

In this section, we compare serial mechanism and parallel mechanism for measurement.

The position errors of joints in a serial mechanism are accumulated. In parallel mechanism, they are averaged because end-effector is connected with base unit in parallel. For the same reason, the parallel mechanism has higher rigidity and accuracy.

In serial mechanism, the dimensions of components depend on their positions: a part close to the base has to be stronger and thus heavier. However, we can use lighter components that have small inertia in a parallel mechanism because of its structure. So, the probe fixed on end-effector can move quickly and the measurement time becomes shorter.

2.1 6-DOF Parallel Mechanism [1]

We built a prototype of parallel mechanism based on the famous Stewart platform [2] shown as Fig.1. It consists of magnetic spherical joints, DC motors and Cylinoids, which transform the rotational movement of motors into linear movement. This prototype has 6 DOF: the position and the posture of upper triangular plate (end-effector) are controlled. Kinematics equations include the position and the posture of plate and the position of the actuators, and solving forward kinematics is calculating the position and the posture of plate from the position of actuators. In parallel mechanism, solving forward kinematics is difficult and this problem is a drawback for measurement. But some structures are known for an easy solving of forward kinematics. DELTA mechanism [3] is one of the structures which forward kinematics can be solved analytically.

2.2 3-DOF Parallel CMM

Figure 2 shows new 3-DOF Parallel CMM developed us for the purpose of measuring experiment. This mechanism consists of DC motors, linear mechanisms that transform the rotational movement of motors into the linear movement using ball screw and ball nut, rotational joints, connecting rods, universal joints and end-effector. All heavy components that are DC motors and linear mechanisms are fixed on base unit at the intervals of 120^{O}, so the upper part of this mechanism is very light and can move quickly. The end-effector can move only *x*, *y* and *z*-axis and do not rotate, because the universal joints and a pair of rods make a parallel crank mechanism.

We put the developed mechanism on the table of the general CMM and we checked the movement and accuracy of this mechanism. In this experiment, the end-effector can move in a parallel plane to the base unit and its error is too small to be measured by our general CMM.

This system has a measuring probe that is a touch trigger type under the end-effector and rotary encoders in DC motors. Figure 3 shows the Parallel CMM system include sensors and a controller. The trigger signal from the touch trigger probe is detected through the probe controller and parallel I/O board. The signal from zero switches and phase Z of encoders are detected through parallel I/O board, which are used for deciding the initial position of travelling components on linear mechanisms. The signal of phase A and B from encoders on DC motors are detected through counter board, which are used for calculating the angle of DC motors i.e. the position of travelling components.

The position of a ball that is a tip of the probe is calculated by solving the forward kinematics using a trigger signal from the measuring probe and the angle of DC motors which are detected by the rotary encoders.

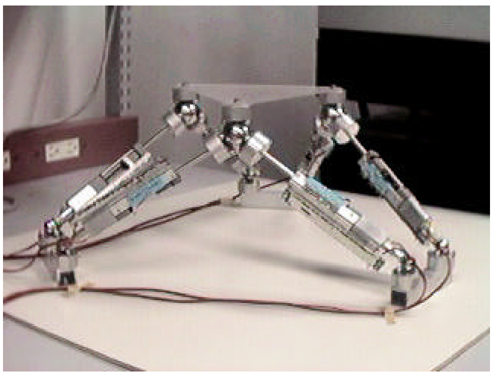

Fig.1 6DOF Parallel mechanism

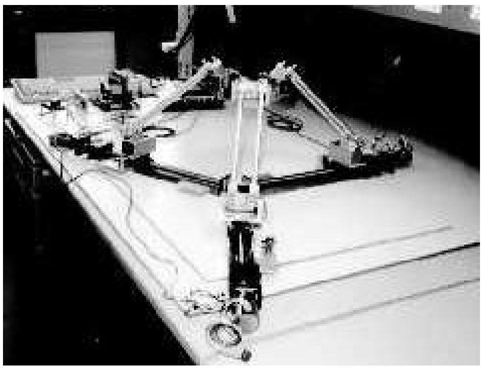

Fig.2 3DOF parallel CMM

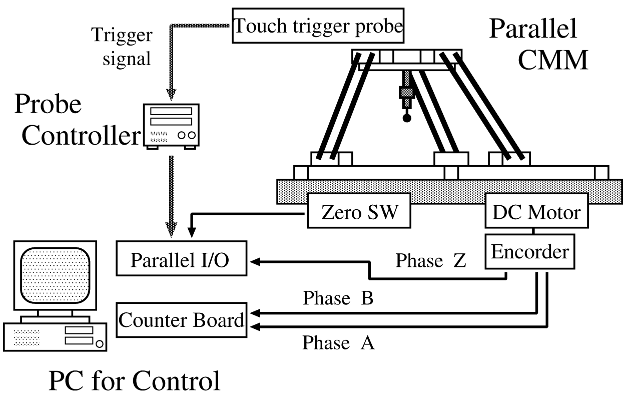

Fig.3 Parallel CMM system

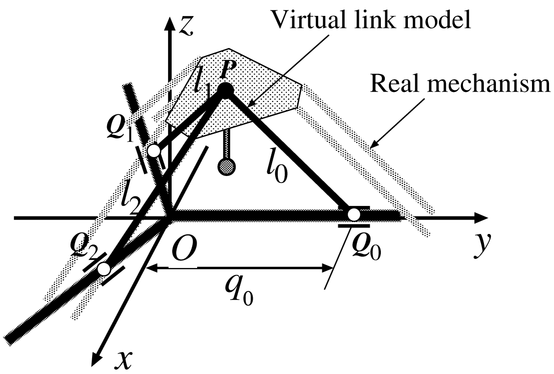

Fig.4 Relationship between the real mechanism and virtual link model

2.3 Kinematics of 3-DOF PCMM

As a result of above experiment, it is found that the end-effector can move in a parallel plane to the base unit. Therefore, each pair of the rods can be replaced by one rod and these three rods are regarded as sides of trigonal pyramid, which upper vertex is the center of end-effector. We call this trigonal pyramid “virtual link model”. Each lower vertex is the point that is inside of the universal joint and the real rods and the virtual rods are in parallel.

Figure 4 shows the relationship between the real parallel mechanism and the virtual link model. At this virtual link model, the position vector of the upper vertex and another vertexes are $\boldsymbol{P}$, $\boldsymbol{Q}_0$, $\boldsymbol{Q}_1$ and $\boldsymbol{Q}_2$, respectively.

$$\begin{aligned}
\boldsymbol{P} &= (\, x, y, z + l_p \,)^T \\
\boldsymbol{Q}_0 &= (\, 0, q_0, 0 \,)^T \\
\boldsymbol{Q}_1 &= (\, -\sqrt{3}q_1 / 2, -q_1 / 2, 0 \,)^T \\
\boldsymbol{Q}_2 &= (\, \sqrt{3}q_2 / 2, -q_2 / 2, 0 \,)^T
\end{aligned} \qquad (1)$$

where x, y and z are the position of the tip of the probe, q_0, q_1 and q_2 are the distance from the origin to the virtual travelling components and l_p is the length of probe. The relationship between the position vectors shown equation (1) and the length of the connecting rods are as follows:

$$\| \boldsymbol{P} - \boldsymbol{Q}_i \| = l_i \qquad (\, i = 0, 1, 2 \,) \qquad (2)$$

where l_i (i=0, 1, 2) are the length of rods. So, the kinematics equations of virtual link model are as follows:

$$\begin{aligned}
&x^2 + (y - q_0)^2 + (z + l_p)^2 = l_o^2 \\
&(x + \sqrt{3}q_1 / 2)^2 + (y + q_1 / 2)^2 + (z + l_p)^2 = l_1^2 \\
&(x - \sqrt{3}q_2 / 2)^2 + (y + q_2 / 2)^2 + (z + l_p)^2 = l_2^2
\end{aligned} \qquad (3)$$

2.4 Singular points

This 3-DOF Parallel CMM has two singular points: one is called under mobility and another is over mobility. The serial mechanisms have only under mobility and over mobility is the special singular point at parallel mechanisms.

When all connecting rods are in same plane, this system is the condition of under mobility. In real system, the touch trigger probe will touch the base unit and will be broken out. So, this condition can be avoided in normal use. Over mobility is the condition that all connection rods are in parallel. We have to avoid this condition, because the end-effector can move freely even if all DC motors are fixed while over mobility.

3. Parameter Identification

Kinematics equations include the position of the probe and the position of the actuators. Solving inverse kinematics is calculating the position of actuators from the position of probe, and it is used for controlling the position of the probe. Conversely, solving forward kinematics is calculating the position of the probe from the position of actuators. In general parallel mechanism, it is too difficult to solve forward kinematics. However, forward kinematics of the PCMM can be solved analytically as follow equations.

$$\begin{aligned}
x &= \frac{(q_2 - q_1)(q_0^2 + 2q_0q_2 + 2q_0q_1 + q_1q_2)}{2\sqrt{3}(q_0q_1 + q_1q_2 + q_2q_0)} \\
&\quad + \frac{(q_2 - q_1)l_0^2 - (2q_0 + q_2)l_1^2 + (2q_0 + q_1)l_2^2}{2\sqrt{3}(q_0q_1 + q_1q_2 + q_2q_0)} \\
y &= \frac{(q_1 + q_2)(q_0^2 - q_1q_2) - (q_1 + q_2)l_0^2 - q_2 l_1^2 - q_1 l_2^2}{2(q_0q_1 + q_1q_2 + q_2q_0)} \\
z &= \sqrt{l_0^2 - x^2 - (y - q_0)^2} - l_p
\end{aligned} \qquad (4)$$

The equations of forward kinematics include some kinematical parameters, such as the length of connecting rods, and it is necessary to decide these kinematical parameters for calculating the position of probe ball accurately. There are some method for identifying kinematical parameters: (a) measuring all kinematical parameters using other larger measuring system, (b) calculating the kinematical parameters using the result of measure by other measuring system and the signal from encoders and (c) calculating the kinematical parameters by measuring artifacts whose size is known.

Method (a) is fit for only small mechanisms and not for large mechanism, because parallel mechanism is suitable for huge system and it is hardly difficult to measure the unknown kinematical parameters of huge mechanism using another measuring machine.

In case of method (b), other larger measuring system measures the position of the tip of probe x, y and z or the position of the end-effector, and the position of the virtual actuators are detected simultaneously. This method is fit for small mechanisms as method (a), but there is a case of using special noncontact measuring system.

We thought that method (c) is most suitable for our Parallel CMM. Therefore, we use artifacts to identify the kinematical parameters.

There are many kinds of artifact: for example, sphere, block gauge, step gauge and so on. We choose sphere whose radius is known, because it is used frequently for compensating the radius of the probe ball and we get it easily. This sphere is fixed on a rod and a magnetic stand. Figure 4 shows the situation of parameter identification by virtual link model.

At the parameter identification, the following equation of sphere is necessary in addition the kinematics equations (1):

$$(x-x_0)^2+(y-y_0)^2+(z-z_0)^2=R^2 \tag{5}$$

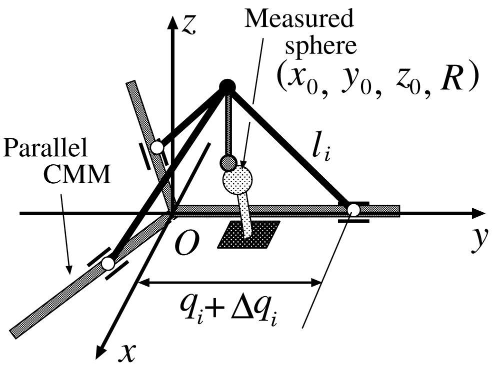

Fig.4 Parameter identification model

where x_0, y_0 and z_0 are the position of center of measured sphere and R is the radius of sphere, but this radius is added the radius of the probe ball.

Here, we define the following equation as evaluating equation:

$$f(p_0, p_1, \ldots)=(x-x_0)^2+(y-y_0)^2+(z-z_0)^2-R^2 \tag{6}$$

where p_0, p_1,… are the unknown parameters and forward kinematics equations (4) are substituted for above x, y and z. All unknown parameters have to satisfy the equation

$$f(p_0, p_1, \ldots)=0 \tag{7}$$

Measuring once the sphere by CMM makes one evaluating equation. Therefore, we have to repeat the measurement and the times of measurement equal the number of parameters. We regard the evaluating equation from the i-th measurement as $f_i(\ )$ and get following simultaneous equations:

$$\boldsymbol{F}=\begin{pmatrix} f_0(p_0, p_1, \ldots, p_k) \\ f_1(p_0, p_1, \ldots, p_k) \\ \vdots \\ f_m(p_0, p_1, \ldots, p_k) \end{pmatrix}=\boldsymbol{O} \tag{8}$$

where k is the number of unknown parameters, m is the times of measurement and $\boldsymbol{O}$ is the zero vector. It is difficult to find solutions analytically because the above simultaneous equations are nonlinear equations. So, we need to use numerical calculation such as Newton-Raphson method.

In fact, it is better that the times of measurement are more than the number of unknown parameters because of measuring errors. In Newton-Raphson method, iteration goes using inverse matrix of Jacobi matrix $\boldsymbol{J}^{-1}$:

$$\boldsymbol{J}=\begin{pmatrix} \frac{\partial f_0}{\partial p_0} & \frac{\partial f_0}{\partial p_1} & \cdots & \frac{\partial f_0}{\partial p_k} \\ \frac{\partial f_1}{\partial p_0} & \frac{\partial f_1}{\partial p_1} & \cdots & \frac{\partial f_1}{\partial p_k} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_m}{\partial p_0} & \frac{\partial f_m}{\partial p_1} & \cdots & \frac{\partial f_m}{\partial p_k} \end{pmatrix} \tag{9}$$

The inverse matrix exists when $k = m$, however, does not exist when $k < m$. Therefore, pseudo-inverse matrix $\boldsymbol{J}^{\#}$ is used.

$$\boldsymbol{J}^{\#}=(\boldsymbol{J}^T\boldsymbol{J})^{-1}\boldsymbol{J}^T \tag{10}$$

The pseudo-inverse matrix is suitable for iteration of Newton-Raphson method with measuring errors, because it is equivalent to using least square method.

Table 1 Results of experiment: 36 points of the upper sphere are measured.

	Designed values	6 parameters	5 parameters
l_0	290.0	360.55	294.69
l_1	290.0	360.42	294.63
l_2	290.0	360.23	296.23
x_0		−4.6845	−6.6159
y_0		−27.0223	−25.4314
z_0	(114.0)	176.41	

(unit : mm)

Table 2 Result of simulations: the number of sphere is one and kinds of kinematical parameters are same as simulations. The measuring error is regarded as 5 μm.

	Set values	3 parameters	6 parameters
l_0	291.0	291.0015	290.9954
l_1	290.0	289.9995	289.9909
l_2	289.0	289.0001	289.0018
x_0	0.0		−0.0102
y_0	0.0		0.0030
z_0	114.0		113.9948

(unit : mm)

4. Experiment

We carried out experiments of kinematical parameter identification using 3-DOF PCMM developed by us. In this experiment, firstly, a sphere whose diameter is 1 inch is fixed in the movable area of PCMM. Secondly, at a moment that the probe hanging on end-effector contacts to the sphere, the position of virtual travelling components q_i (i=0, 1, 2) are calculated from encoders on DC motors. Finally, nonlinear simultaneous equations (8) are solved and the solutions i.e. the values of unknown kinematical parameters are identified.

Table 1 shows the result of experiment when the number of unknown parameters is six include the length of connecting rods and the position of sphere. The values of identified parameters are quite different from designed values. We think that it is caused by measuring in narrow area as only upper part of sphere and big measuring errors at gaps of bearings. Especially, it seems that there are some relationship between identified value z_0 of the z coordinate of the sphere and identified values l_0, l_1 and l_2.

Next, in case that z_0 is known, the results of identifying parameters except z_0 using same data as above experiment are shown in Table 1. We have a good reason for this assumption, which the height of the measured sphere can be measured beforehand because the sphere is fixed on a rod and a magnetic stand.

The result of identification became closer to designed values, but it is wrong that errors between designed values and identifying values is over 6mm. The reasons are that the random deviations of measuring errors by gaps of components are bigger than our estimate and errors of other kinematical parameters are ignored here.

5. Simulation

In this simulation, the parameter identification is simulated by the following method. Firstly, when the probe touches to points on the sphere, the positions of travelling components on linear system are calculated by inverse kinematics. Next, measuring errors are added to the calculated positions of the virtual travelling components q_i (i=0, 1, 2). Here, the parameters of inverse kinematical model are regarded as true values. After this, it is the same as experiment. Nonlinear simultaneous equations (8) are solved to identify the values of unknown kinematical parameters.

Table 2 shows the result of simulations in cases that the numbers of unknown parameters are 3 and 6. The case of 3 parameters is for test of our program and in fact it is impossible to know accurately the position of the sphere except the z coordinate. Therefore, it is necessary to regard the position of sphere as unknown parameters.

Of course, if there are no measuring errors, the solutions of the nonlinear simultaneous equations equal set values. In table 2, the measuring error is regarded as 5 μm and the solutions are very close to set values. It is supposed that 1,000 points inside of movable area of PCMM are measured, and the calculating values by forward kinematics using identified 3 and 6 parameters have maximum errors of 2.7 μm and 15 μm, respectively.

However, in above experiment, the identified parameters have larger errors than these simulations. One of the reasons is that there are another kinematical parameters which should be identified. Therefore, position errors Δq_i (i=0, 1, 2) of the virtual travelling components are added to 6 parameters and next simulation is executed using 9 parameters such as the length of 3 connecting rods, the position errors of 3 virtual travelling components and the position of sphere.

In addition, the numbers of spheres are varied from 1 to 4. There are 9 parameters in case of one sphere and 3 parameters are increased for every one sphere. The number of measured point in each condition is same and 36.

The results of simulations are shown in Table 3. The

Table 3 Results of simulations: the position of only one sphere is mentioned and other positions are omitted.

	Set values	Number of spheres			
		1	2	3	4
l_0	290.20	275.81	287.78	288.88	288.69
l_1	290.10	275.92	289.92	290.01	291.18
l_2	290.05	276.07	288.49	290.20	289.98
Δq_0	–1.0	–4.05	–3.24	–3.11	–4.08
Δq_1	0.8	–7.39	–2.42	2.50	4.48
Δq_2	0.3	–7.30	5.49	3.13	2.66
x_0	0.3	0.41	4.91	0.56	0.63
y_0	–119.5	–115.21	–121.54	–118.09	–122.13
z_0	220.9	208.40	219.18	219.42	219.54

(unit : mm)

position of one sphere is mentioned and other positions are omitted. It seems that the new parameters which are position errors Δq_i (i=0, 1, 2) of the virtual travelling components have great influence on identified parameters. Moreover, the number of measured spheres similarly affect values of identified parameters. In case of using 3 or 4 sphere, the identified parameters are closer to set values.

Here, we discuss about correlation between the identified parameters. It is assumed that the random deviation of every measurement is independent each other and the correlation are calculated. As the result, especially, there is strong correlation between the z coordinate of measured sphere and the length of connecting rods. This is the reason why the identified errors of the length of connecting rods and the errors of the z coordinate of measured sphere are almost same in first experiment.

6. Conclusions

We constructed 3-DOF Parallel Coordinate Measuring Machine whose forward kinematics can be solved analytically. Firstly, we described the characteristics of parallel mechanism and introduced our 6-DOF parallel mechanism and 3-DOF parallel CMM. Next, the virtual link model and its kinematics were described. Finally, we carried out experiments of identifying unknown parameters using developed PCMM and simulations for investigation of the identified errors.

As the result of experiment, the random deviations of measuring errors by gaps of components are bigger than our estimate and there are errors of other kinematical parameters which cannot be ignored. The result of simulations means that added parameters which are position errors of the virtual travelling components have great influence on identified parameters. Moreover, it is seemed that there are strong correlations between kinematical parameters.

In the future, we will try identification of kinematical parameters using other artifacts and we will develop new PCMM whose mechanism has little gaps in components.

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