CALIBRATION OF 2-DOF PARALLEL MECHANISM

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- Introduction
- Calibration of 2-DOF Parallel Mechanism
  - Arrangement of the measuring points
  - Parameter identification and error estimation
- Conclusion

INTRODUCTION

How to calibrate Parallel-CMM ?

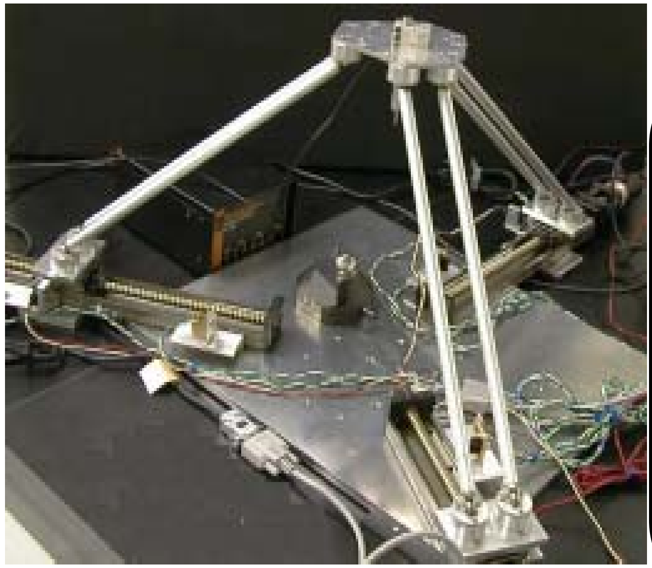
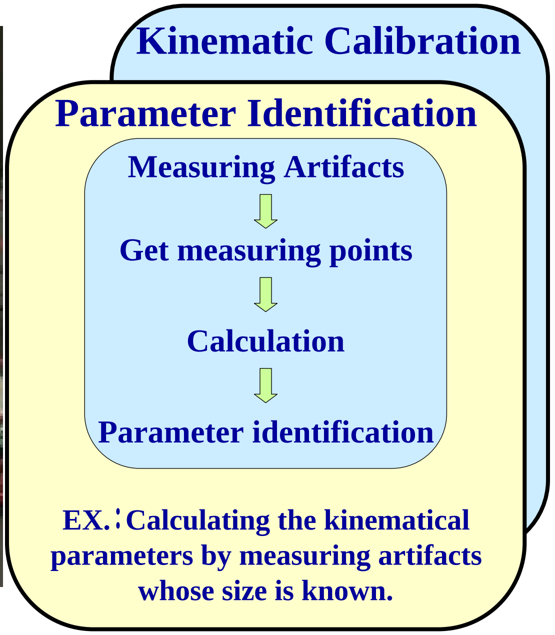


Fig. 1 Prototype of parallel-CMM



We are developing Parallel-CMM, Coordinate Measuring Machine using parallel mechanism. Figure 1 shows the prototype of it.

To improve and evaluate the uncertainty of measurement, there is necessity for parameter identification and kinematic calibration. But the efficient method to calibrate parallel mechanism has not realized yet.

Our goal is to know how to calibrate Parallel-CMM efficiently. So through the easy case, the simulation of calibration of 2-DOF parallel mechanism, we get the basic knowledge about it.

CALIBRATION OF 2-DOF PARALLEL MECHANISM

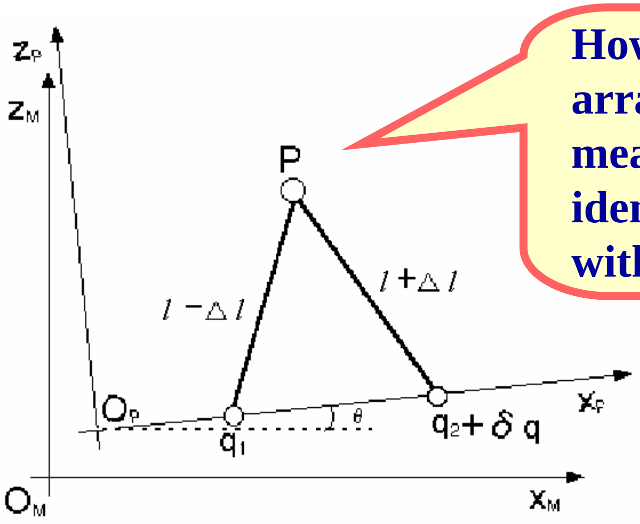


Fig. 2 Calibration model

How should we arranged the measuring points to identify parameters with high accuracy ?

l	**: Average length of links**
Δl	**: Difference length of links**
δq	**: Offset value of encoders**
(x_m, z_m, θ)	**: The parameters by the coordinates transforming from *Xp* - *Zp* to *Xm* - *Zm***

We calibrate the 2-DOF parallel mechanism shown in Fig. 2.

This parallel mechanism is on a Coordinate Measuring Machine, and we can know the coordinates (*x*, *z*), the position of the end-effector (*P*).

Here the parameters are calculated using the result of measuring by the CMM and the signal from encoders. The parameters we need to identify are the length of links ($l + \Delta l$, $l - \Delta l$), the offset value between encoders (δq), and the parameters by the coordinates transforming from *Xp* - *Zp* to *Xm* - *Zm* (x_m, z_m, θ).

When these parameters are calculated with least squares method, what arrangement of measured points gives the good result ?

ARRANGEMENT OF MEASURING POINTS

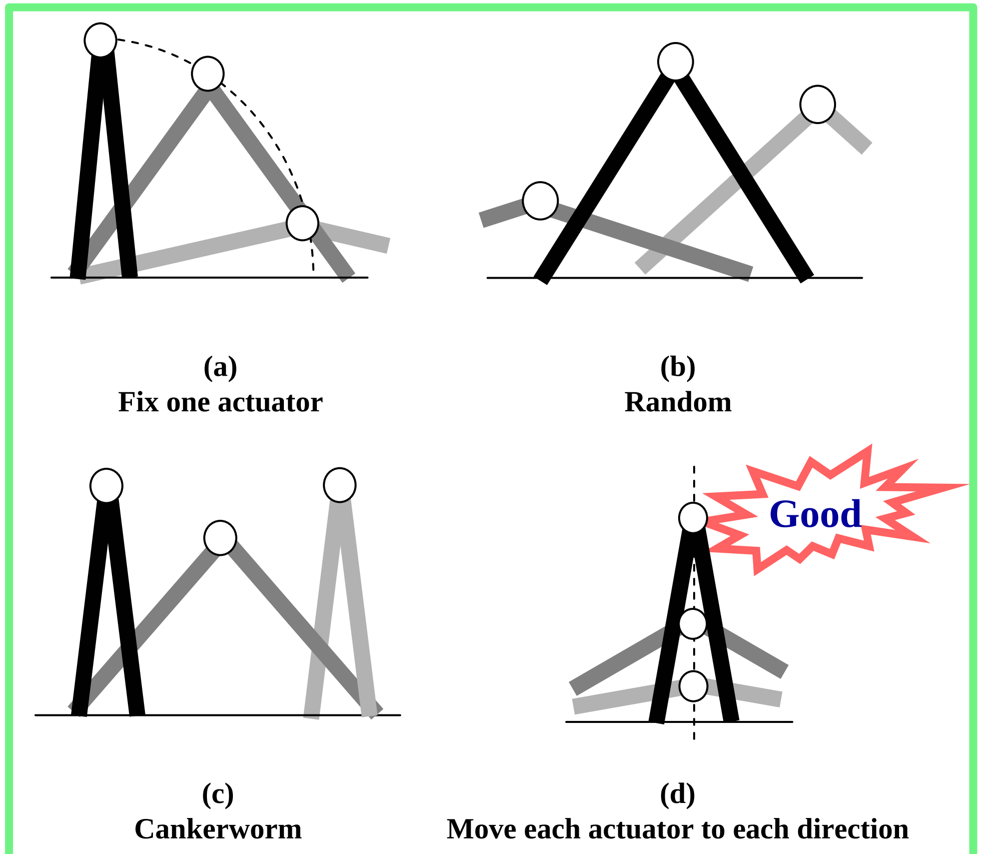


Fig. 3 Arrangements of measured points

Trough the simulation, we tried to identify parameters and estimate the positioning error of end-effector with the several arrangements of measured points. Figure 3 shows the samples of arrangement of measured points.

Each arrangement of the measured points gives each values of kinematic parameters. For example, the arrangement shown in Fig. 3-(d) gives stable calculation and identifies parameters with high accuracy. On the other hand, the arrangement shown in Fig. 3-(a) sometimes gives unstable calculation and fails to achieve high accuracy parameter identification.

What is the difference around these arrangements ? That is the Jacobian which is calculated by arranging the measuring points.

One arrangement gives its own Jacobian. And the result of the calibration and calibrated positioning error depend on the Jacobian.

CONDITION NUMBER OF JACOBIAN AND CALIBRATED POSITIONING ERROR

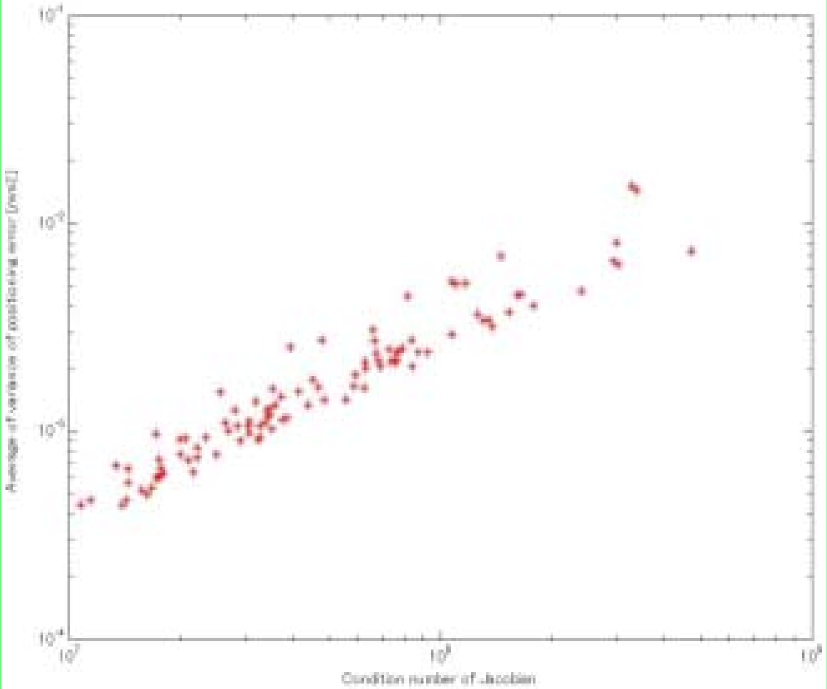


Fig. 4 The condition number of Jacobian and the average variance of positioning error

Here assume that solving the equation

$$y = Ax,$$

and estimating the residual

$$z - z' = G(x - x'),$$

where x is the parameter vector, y is the observed values vector and z is the vector calculated by x and G. In our case, A is the Jacobian calculated by arranging measuring points and G is the matrix whose elements are partial differential coefficient calculated at the point of z. Then

$$z' = GA^{-1} y',$$

where A^{-1} is generalized inverse of A, y' and z' are the error of measurement and the residual. This equation shows that the arrangement of measured point for calibration affect the positioning error after calibration.

Because of the round error, the residual is estimated as

$$\kappa(A)\,\varepsilon \leqq \|z'\| / \|z\| \leqq \kappa(A) \|y'\| / \|y\|,$$

where $\kappa(A)$ is the condition number of A, and ε is the accuracy of calculation. So when the condition number of Jacobian is large, the positioning error is large.

Figure 4 shows the relation between the condition number of Jacobian and the average variance of positioning error of end-effector in workspace.

The arrangement shown in Fig. 3-(d) gives the condition number of Jacobian about 10^6.

PARAMETER IDENTIFICATION

Table 1 Result of simulation

	Set values	Identified values
l	100.555	100.560
Δl	0.613	0.610
δq	0.948	0.959
x_m	-0.172	-0.184
z_m	-1.113	-1.115
θ	-0.00822	-0.00819

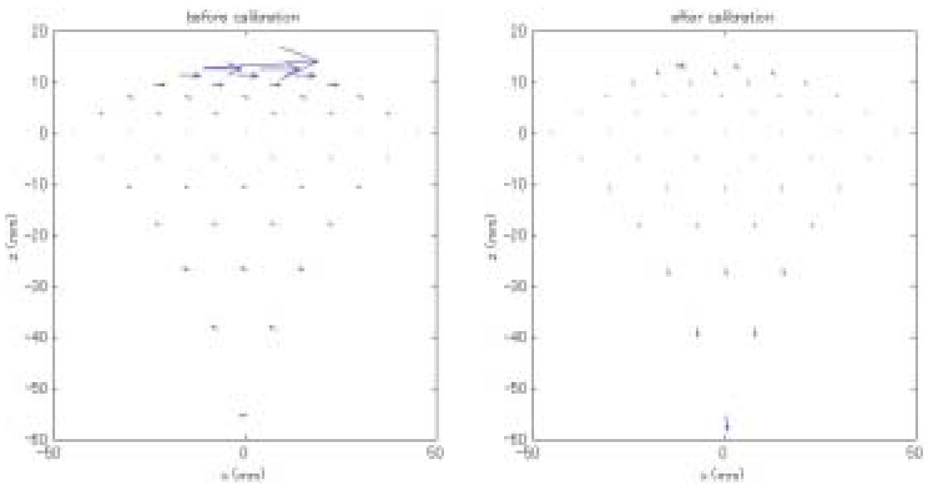
(unit : mm and rad)

Table 1 shows a result of simulation of parameter identification. The condition of simulation is follows.

- **The number of measured points : 20**
- **The error of measuring machine : 0.01 mm**
- **The error of encoder : 0.02 mm**

The measured points are arranged as Fig. 3-(d).
By arranging the measuring points as Fig. 3-(d), parameters are identified with high accuracy.
The result of simulation shows the arrangement of measured points that gives the small condition number of Jacobian gives the parameters with high accuracy.

POSITIONING ERROR



Maximum error	Maximum error
x : 0.575 mm	x : 0.008 mm
z : 0.461 mm	z : 0.021 mm

Fig. 5 Positioning error at each position

Figure 5 shows the positioning error of end-effector at each position. Base point of each vector is the position of end-effector and the length of each vector shows the positioning error at each position.

The coordinate plane is the parallel mechanisms (*Xp* - *Zp*) and the origin is at the position (q_1 , q_2) =($-l/2$, $l/2$).

The error of positions after calibration is at most 10 μm.

The result of simulation shows calibrating by the arrangement of measured points that gives the small condition number of Jacobian, the error of positions after calibration get to small.

CONCLUSION

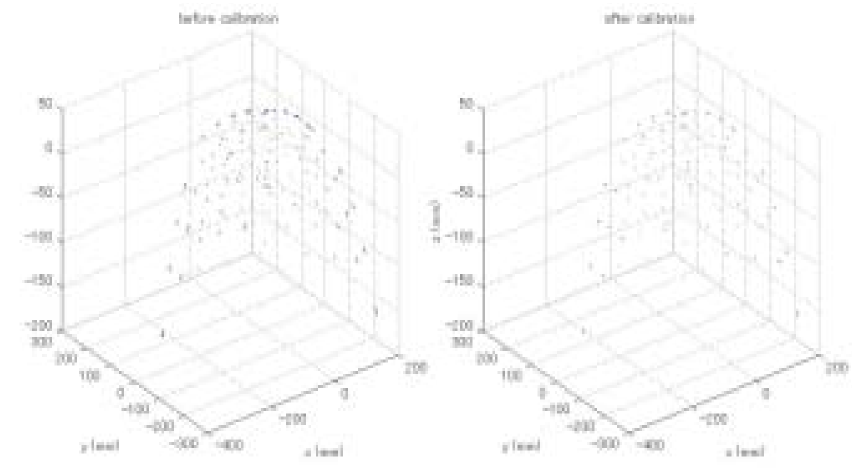


Fig. 6 Positioning error of Parallel-CMM at each position

Through the simulation of calibration of 2-DOF parallel mechanism, we got the knowledge about how to arrange the measuring points to identify parameters with high accuracy. That is

- **The arrangement which gives the small condition number of Jacobian gives the good result for parameter identification and reduce the error of position after calibration.**

Arranging measuring points as that is effective to calibrate Parallel-CMM too.

Figure 6 shows the positioning error of Parallel-CMM through the simulation of parameter identification. After calibration the error of position is reduce to one hundredth or smaller.