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Evaluation of Uncertainty by Form Deviations of Measured Workpieces in Specified Measuring Strategies

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Abstract:
In coordinate metrology, the evaluation methods of the uncertainty of measurement in the specific measuring strategy are key techniques. The evaluation method for uncertainties of measured parameters has been already proposed when the only random errors are put in the consideration. In this paper, the effects of systematic errors are theoretically analyzed to evaluate the uncertainties in feature-based metrology. We estimated the uncertainty for a circle feature with form deviations is measured by a CMM (coordinate measuring machine). When form deviations have an autocorrelation function in position of measured points, the autocorrelation function influences the results of measurements as unknown systematic errors. The method to calculate the error matrix is statistically derived. Using this method, the uncertainties of the measured parameters can also be calculated dealing with the autocorrelation function of the measured circle.

1. Feature based metrology

In coordinate metrology, associated features and associated derived features are calculated from measured data sets on real features by CMMs (Coordinate Measuring Machines). Then, the associated features are compared with the nominal features indicated on the drawings. In this data processing, the features are primal targets to calculate, to evaluate and to process. Consequently, this process is called as “Feature-Based Metrology” [1].

In the feature-based metrology, it is a key technique to estimate the uncertainty of measurement [2] in the specific measuring strategy [3-5]. The estimation method for uncertainties of measured parameters has been already proposed when the random errors are put in the consideration [6-7]. The uncertainty of each measured point is defined by error analysis of the CMM and the probing system. From the uncertainty of measured point, the uncertainty of measured feature can be calculated statistically using following equations. Equations (1) and (2) show the calculation method of an error matrix of measured parameters **P**, where **A** is Jacobian matrix, **S** is an error matrix of measured points. From the error matrix **P**, we estimate uncertainties of measured parameters such as a diameter and a coordinate of center in the specified measured strategy.

$$\mathbf{C} = (\mathbf{A}^t\mathbf{S}^{-1}\mathbf{A})^{-1}\mathbf{A}^t\mathbf{S}^{-1} \tag{1}$$

$$\mathbf{P} = \mathbf{CSC}^t = (\mathbf{A}^t\mathbf{S}^{-1}\mathbf{A})^{-1} \tag{2}$$

2. Circle features measurement

2.1 Parameters and error matrix of circle features

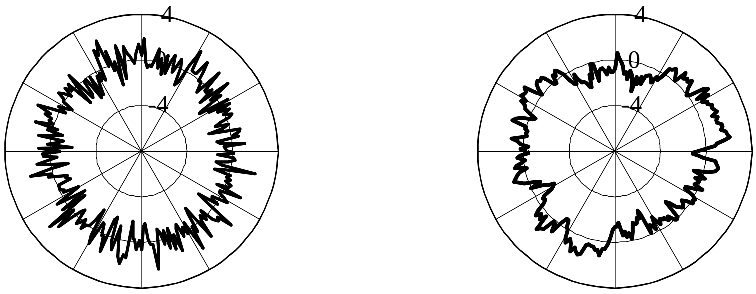
In this paper, we examine the method of estimation of measured parameters for circle feature measurement. For circle features, we calculate the diameter and, X and Y coordinate of center of the circle. Therefore, the error matrix of measured parameter of circle measurement includes variance of X and Y coordinate of center s_x^2 and s_y^2, variance of diameter s_d^2 and covariance of these parameters s_{xy}, s_{xd} and s_{yd} (see Eq. 3)

$$\mathbf{P} = \begin{pmatrix} s_x^2 & s_{xy} & s_{xd} \\ s_{xy} & s_y^2 & s_{yd} \\ s_{xd} & s_{yd} & s_d^2 \end{pmatrix} \tag{3}$$

Fig. 1 displays (a) a circle feature with random errors and (b) a circle feature with errors from an autocorrelation function. When the form deviation is defined by the random function, the error matrix $\mathbf{S}_{\mathrm{ran}}$ is the unit matrix multiplied by the variance s_f^2 of form deviation in Eq. 4. In the other hand, when the form deviation has the specified correlation function, the error matrix $\mathbf{S}_{\mathrm{cov}}$ is defined by the autocorrelation matrix $\mathbf{R}_{\mathrm{cov}}$ and s_f^2 in Eq. 5.

$$\mathbf{S}_{\mathrm{ran}} = \begin{pmatrix} s_f^2 & & 0 \\ & \ddots & \\ 0 & & s_f^2 \end{pmatrix} = s_f^2 \begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix} = s_f^2\mathbf{E} \tag{4}$$

$$\mathbf{S}_{\mathrm{cov}} = \begin{pmatrix} s_f^2 & s_{12} & \cdots & s_{1n} \\ s_{12} & \ddots & s_{ij} & \vdots \\ \vdots & s_{ij} & \ddots & s_{n-1n} \\ s_{1n} & \cdots & s_{n-1n} & s_f^2 \end{pmatrix} = s_f^2 \begin{pmatrix} 1 & r_{12} & \cdots & r_{1n} \\ r_{12} & \ddots & r_{ij} & \vdots \\ \vdots & r_{ij} & \ddots & r_{n-1n} \\ r_{1n} & \cdots & r_{n-1n} & 1 \end{pmatrix} = s_f^2\mathbf{R}_{\mathrm{cov}} \tag{5}$$



(a) random error (b) error with specified autocorrelation function

Fig. 1: Circle features with form deviation (s_f^2 = 1 µm)

2.2 Calculation of uncertainty for circle feature

From two types of error matrices $\mathbf{S}_{\mathrm{ran}}$ and $\mathbf{S}_{\mathrm{cov}}$, there are three types of uncertainties of measured parameters $\mathbf{P}_{\mathrm{ran}}$, $\mathbf{P}_{\mathrm{cov}}$ and $\mathbf{P}_{\mathrm{r+c}}$ are defined in Eqs. 6, 7 and 8. $\mathbf{P}_{\mathrm{ran}}$ is the uncertainty matrix of parameters when the form deviation is assumed as the random function. $\mathbf{P}_{\mathrm{cov}}$ is the uncertainty matrix of parameters when the form deviation has the specified autocorrelation function and calculated using the autocorrelation function. $\mathbf{P}_{\mathrm{r+c}}$ is the uncertainty matrix of parameters when the form deviation has the specified autocorrelation function and calculated using the normal least squares method without the autocorrelation function. Usually the calculating program in CMM can not handle the autocorrelation function. Therefore, uncertainties of the normal calculating situation in measuring by CMM are defined by $\mathbf{P}_{\mathrm{r+c}}$.

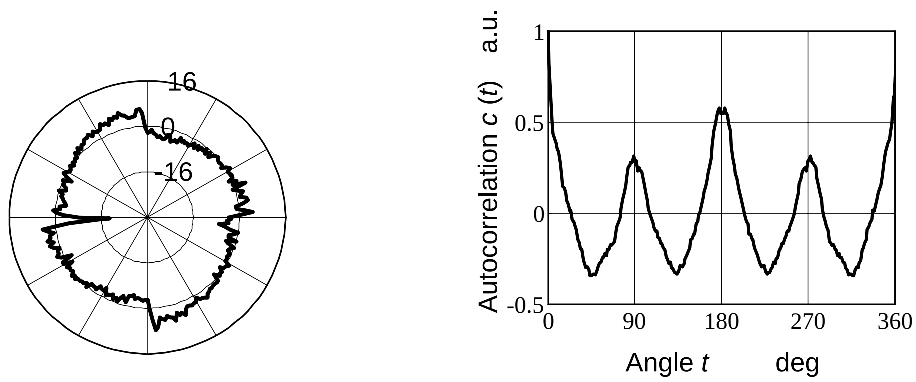
$$\mathbf{P}_{\mathrm{ran}} = (\mathbf{A}^t \mathbf{S}_{\mathrm{ran}}^{-1} \mathbf{A})^{-1} = s_f^2 (\mathbf{A}^t \mathbf{A})^{-1} \tag{6}$$

$$\mathbf{P}_{\mathrm{cov}} = (\mathbf{A}^t \mathbf{S}_{\mathrm{cov}}^{-1} \mathbf{A})^{-1} = s_f^2 (\mathbf{A}^t \mathbf{R}_{\mathrm{cov}}^{-1} \mathbf{A})^{-1} \tag{7}$$

$$\mathbf{P}_{\mathrm{r+c}} = ((\mathbf{A}^t \mathbf{A})^{-1} \mathbf{A}^t)\, \mathbf{S}_{\mathrm{cov}}\, ((\mathbf{A}^t \mathbf{A})^{-1} \mathbf{A}^t)^t = s_f^2 ((\mathbf{A}^t \mathbf{A})^{-1} \mathbf{A}^t)\, \mathbf{R}_{\mathrm{cov}}\, ((\mathbf{A}^t \mathbf{A})^{-1} \mathbf{A}^t)^t \tag{8}$$

3. Examples of uncertainty estimation for circle feature measurement

Using a hole with circularity of 28.8 µm in Fig. 2 (a) as example, we calculate uncertainty of measurement using Eqs. 6 - 8. Fig. 2 (b) illustrates the autocorrelation function of the hole. The autocorrelation function has large 2 and 4 order frequency values.

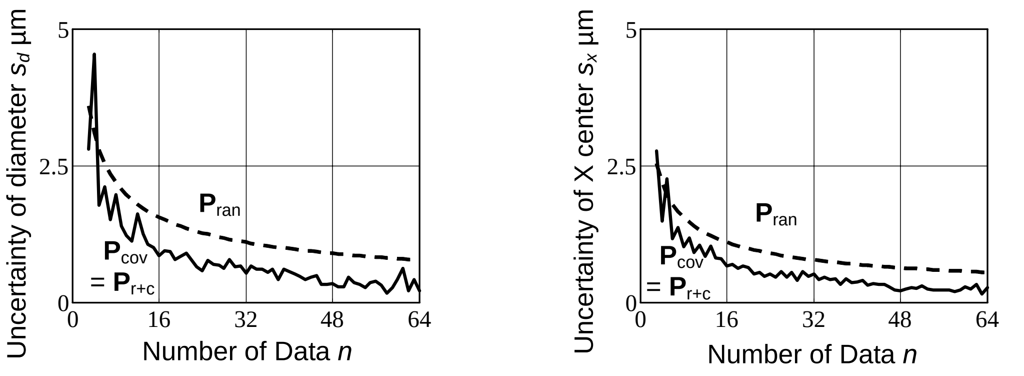


(a) Circle feature for calculation (b) Autocorrelation function of circle feature (a)

Fig. 2: Hole on aluminum board: diameter of 20 mm, standard deviation s_f^2 = 3.1 µm and circularity = 26.8 µm

3.1 Measured points uniformly on the circle

When measuring points are set uniformly on the measured circle, $\mathbf{P}_{\mathrm{cov}}$ and $\mathbf{P}_{\mathrm{r+c}}$ are completely same values, because of $\mathbf{S}_{\mathrm{cov}}$ and $\mathbf{S}_{\mathrm{ran}}$ are the same weight function for least squares calculation. Fig. 3 illustrates relationship between number of data n and uncertainty (standard deviation) of diameter s_d and X coordinate of center s_x. On Fig. 3 (a), the uncertainty of diameter in 4, 6 and 8 measured data is larger than these in odd number of measured data. This is because autocorrelation function of measured circle (Fig. 2) has large 2 and 4 order frequency values. On the other hand, the uncertainty of center (Fig. 3 (b)) in 3 and 5 measured data are larger than these in even number of measured data.



(a) Uncertainty of diameter s_d (b) Uncertainty of X coordinate of center s_x

Fig. 3: Relationship between number of data n and uncertainty (standard deviation) of diameter s_d and X coordinate of center s_x in uniformly measurment

3.2 Measured points partially on the part of the circle

When the measured data are in the measured area a for partial circle measurement in Fig. 4, $\mathbf{S}_{cov}$ and $\mathbf{S}_{ran}$ are not same weight functions. Fig. 5 shows the relationship between the uncertainty of diameter s_d and the number of data n in the partial circle measurement of angle a = 180 and 90 deg, by 3 calculated methods $\mathbf{P}_{r+c}$, $\mathbf{P}_{cov}$ and $\mathbf{P}_{ran}$. For the partial circle measurement, $\mathbf{P}_{ran}$ is under estimation and $\mathbf{P}_{r+c}$ is over estimation for the uncertainty, when the number of measured data is large.

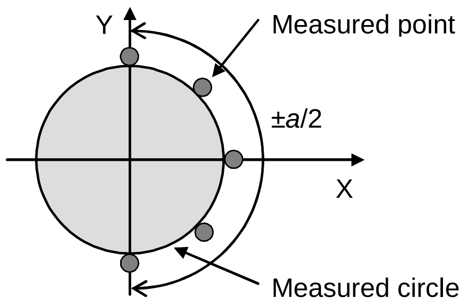
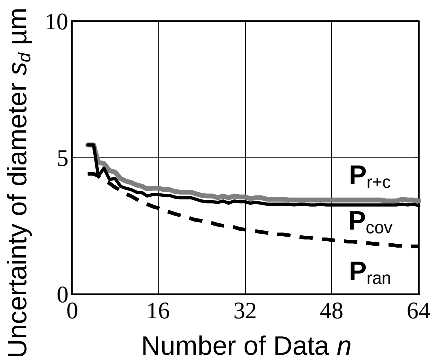
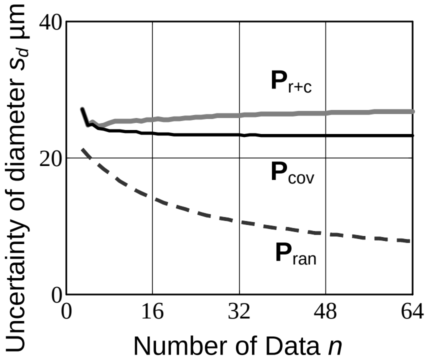


Fig. 4: Measuerd area a for partial circle measurement, 5 measured points in ±90 deg (a = 180 deg)



(a) a = 180 deg



(b) a = 90 deg

Fig. 5: Relationship between uncertainty of diameter s_d and number of data n in partial circle measurement of angle a = 180 and 90 deg, by three calculated methods $\mathbf{P}_{r+c}$, $\mathbf{P}_{cov}$ and $\mathbf{P}_{ran}$

4. Conclusions

In this paper, we theoretically analyzed the effects of the unknown systematic errors in feature-based metrology. The form deviation of circular feature treated as the unknown systematic errors. These errors propagate as unknown systematic errors to the uncertainties of measured parameters, such as the center position and the diameter of a measured circle. The method to calculate the error matrix **S** was derived when the center position and the diameter of the circle are measured.

We derived three types of uncertainties of measured parameters as follows:

(1) $\mathbf{P}_{ran}$: form deviation is assumed as the random function,

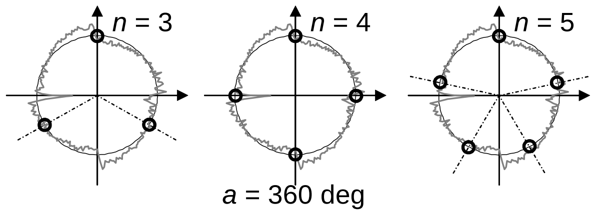
(2) $\mathbf{P}_{cov}$: form deviation has correlation and calculated using the autocorrelation function, and

(3) $\mathbf{P}_{r+c}$: form deviation has correlation and calculated without the autocorrelation function.

In these three types, $\mathbf{P}_{cov}$ is the theoretical true estimation and, $\mathbf{P}_{ran}$ and $\mathbf{P}_{r+c}$ have problems, such as over or under estimation in the specified measuring conditions.

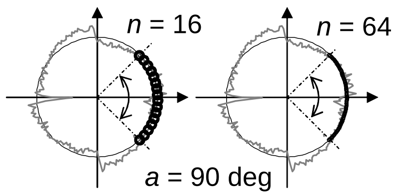
From these calculations, we pointed the measuring cases in which more point of measurement does not give small uncertainties. Fig. 6 (a) shows the uncertainty of diameter of n = 4 is larger than that of n = 3, and uncertainty of X coordinate of n = 5 is larger than that of n = 4, because of the feature has large even order frequency values. Fig. 6 (b) shows the uncertainty of diameter by $\mathbf{P}_{r+c}$ of n = 16 is larger than that of n = 64 in the partial circle measurement, because of $\mathbf{P}_{r+c}$ can not handle the autocorrelation function of the circle feature.

Using the calculation methods by $\mathbf{P}_{cov}$, the uncertainties of the measured parameters can be estimated with the autocorrelation function of the measured features. The series of simulations for the method in statistical way directly implies that the basic data processing method in this paper are useful of the feature based metrology.



number of data n	3	4	5
uncertainty of diameter s_d	2.80 µm	4.54 µm	1.78 µm
uncertainty of X coordinate of center s_x	2.78 µm	1.48 µm	2.26 µm

(a) a = 360 deg, n = 3, 4 and 5



number of data n	16	64
uncertainty of diameter s_d by $\mathbf{P}_{r+c}$	25.5 µm	26.7 µm
uncertainty of diameter s_d by $\mathbf{P}_{cov}$	23.5 µm	23.2 µm

(b) a = 90 deg, n = 16 and 64

Fig. 6: Relationship between number of data and uncertainties: more point of measurement does not give small uncertainties

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